Wavelength Assignment and Upgrading Strategies for WDM Rings

Helio Waldman, Divanilson R. Campelo and Raul C. Almeida Jr. (1)

Abstract – We investigate the performance improvements imparted by some wavelength assignment algorithms to optical path WDM rings. A computationally inexpensive algorithm is presented that minimizes: a) the blocking probability immediately after each assignment; and b) the increment of the channel blocking probability at the assigned wavelength. Simple metrics are proposed for the choice of the best wavelength assignment that are based totally on local information. We also discuss upgrading strategies for the WDM ring, showing that a modest wavelength pool size excess with respect to fiber load can produce almost the same improvement as full wavelength conversion.

Keywords – Wavelength division multiplexing, wavelength assignment, optical rings.

I. INTRODUCTION

We consider optical path networks over ring topologies. Multiple link paths are set up and may be taken down under demand from upper layers in the network hierarchy. Routing in the ring is not complex, since there are only two routes between any two points, one of them being often much shorter than the other. Wavelength assignment, however, may imply a choice among many alternatives, which may or may not favor the capacity of the ring to meet future traffic demands. At first, one might think that balancing the load among all \( W \) wavelengths would be a good policy. This can be easily achieved by just randomly choosing among all available wavelengths (or all available wavelength sequences, when conversion is allowed somewhere) when assigning a new path under request: this is called the \textit{random algorithm} \([5],[6]\). Early simulations, however, have shown that other algorithms provide better performance than the random algorithm, even though they put more load on some wavelengths than on others (or rather for this reason) \([3]\).

This paper discusses some criteria to derive good wavelength assignment algorithms. The discussions focus on comparisons between the performance improvements that can be obtained with increasing algorithmic complexity, growing wavelength pool size, and providing wavelength conversion capabilities at the nodes. Once good algorithms are identified in Section 3, they are used as reference in Section 4 to investigate strategies for upgrading the capacity of optical rings. Such strategies may include increasing the wavelength load on the fiber and using multiple fiber rings. In the latter case, one might consider requesting paths to each ring separately, sequentially or not; jointly managing wavelength assignment with a single algorithm on both rings; or interconnecting the ring at the hardware level with cross-connecting nodes in lieu of OADMs.

II. BASIC MODEL

Let \( W \) and \( L \leq W \) be the maximum number of wavelengths allowed in a ring and in each of its links, respectively, and let the wavelengths be numbered \( 1,2,3,...,W \) \([2]\). If there is no wavelength conversion, the network may be thought of as \( W \) separate, but jointly load-constrained, single-wavelength sub-rings. If there is full wavelength convertibility in all nodes, any optical path is free to switch from any wavelength to any other available wavelength in the next link. In this case, the ring is equivalent to a ring of trunks with \( L \) wires in each trunk, hence there is no point in making \( W > L \). Under sparse and/or partial wavelength conversion capabilities, wavelength switching is constrained to the allowable alternatives at each node.

One or more upper layers request paths. A request for a path is called illegal if one of the requested links is already busy on \( L \) wavelengths. Illegal requests are always blocked. If the ring has full conversion capability on all nodes, all legal requests will be served. Otherwise some legal requests will have to be blocked, but good wavelength assignment algorithms may reduce the frequency of this occurrence.

(1) This work was supported by scholarships from FAPESP, CAPES and CNPq. Early versions of this paper were presented in part at IEEE GLOBECOM’99, San Francisco, CA, USA and in part at XVIII Simpósio Brasileiro de Telecomunicações, Gramado, RS, Brazil, 2000.

The authors are with the State University of Campinas, Campinas, SP, Brazil. Address: DECOM/EEC/UNICAMP, Caixa Postal 6101, 13083-970, Campinas, SP, Brazil. Tel: +55-19-37883793, FAX: +55-19-32691395.

E-mail: {waldman, dcampelo, rcamelo}@decom.fee.unicamp.br.
III. WAVELENGTH ASSIGNMENT ALGORITHMS

In this section, we investigate the performance improvements imparted by first-fit wavelength assignment algorithms on blocking WDM rings without wavelength conversion. We still consider optical path networks over the ring topology. Multiple link, single wavelength optical paths are activated and taken down according to the demand for traffic from an upper layer in the network hierarchy. An available wavelength must be assigned to each path.

The performance of these networks is critically dependent on the wavelength assignment algorithm used to set up new paths for the incoming calls. Although no algorithm can produce a higher performance than full convertibility, it is interesting to investigate the maximum performance provided by conversion because the cost of the intelligence present in the algorithm is much smaller than the cost of the conversion.

Several algorithms have been proposed in the literature, each of them having a different heuristics [1]. The next subsections present some definitions and heuristics used to define the algorithm we propose in this work. The simulation results show the relative importance of the proposed heuristics to improve the blocking performance of the ring.

III.1 DEFINITIONS

The number of possible paths that may be requested in a WDM ring is \( N(N - 1) \), where \( N \) is the number of nodes. Each of these paths may be activated in any of the \( W \) colors. In addition, no more than \( L \) paths may traverse the same link at the same time. Given the different meanings that may be implied by the notion of a path, it is useful to define:

Definition 1 - A network path is a sequence of nodes in the network, such that any node is physically connected to the previous and succeeding nodes in the sequence.

Definition 2 - A channel path (or “colored” path) is a path in the sub-network of a given wavelength (“channel” or “color”).

When there is no wavelength conversion, a channel path is available if all its links are free. A network path is available if at least one of its corresponding \( W \) channel paths is available. When two or more channels are available for the requested path, then a wavelength assignment algorithm will choose between them.

Whenever any one wavelength is not being used anywhere in a network, instantaneous blocking probability is zero. This may be the reason why good wavelength assignment algorithms tend to unbalance the load, using some wavelengths more than “others”. Such “good” algorithms are generally comprised in the class of first-fit algorithms, defined below.

Definition 3 - An algorithm is first-fit if it assigns a wavelength that is not being used in the network only when the requested path cannot be accommodated by one of the wavelengths that are already being used somewhere.

Several first-fit algorithms have been studied and compared in the literature [3]. The simplest one uses an \( a \) priori wavelength list: the algorithm will then look up the list and pick the first wavelength under which the path can be accommodated. This will be called the fixed priority algorithm (FP). Other algorithms favor the use of the wavelength that is being most used in the network at assignment time. A good survey of routing and wavelength assignment algorithms for WDM networks is given in [1].

III.2 PATHS IN RING AND LINEAR TOPOLOGIES

Let \( \lambda_1, \lambda_2, \ldots, \lambda_m \) be the \( m \) wavelengths available to accommodate a given path request on a ring. This means that in each of the \( m \) corresponding sub-rings there is a “hole” (i.e. a maximal sequence of free adjacent links) that can accommodate the request. If an available wavelength is currently unused everywhere in the ring, then its corresponding hole is the whole ring. Let \( C_1, C_2, \ldots, C_m \) denote the available holes where a given path request may be accommodated.

Theorem 1 - If \( C_i \subseteq C_j \) for some \( 1 \leq i, j \leq m \) then assigning wavelength \( \lambda_i \) for the requested path minimizes the instantaneous path blocking probability immediately after the assignment.

Proof: All paths in \( C_i \) are also in \( C_j \). Therefore, assigning \( \lambda_i \) for the requested path will not change the set of available network paths, thus keeping the path blocking probability unchanged.

Theorem 1 means that the minimization of path blocking probability will often lead to multiple assignment choices. For example, if one wavelength is currently unused all over the ring, then any of the remaining available wavelengths may be chosen to accommodate the requested path without incrementing the instantaneous path blocking probability.

III.3 MINIMIZATION OF PATH BLOCKING PROBABILITIES

Nevertheless, there may be situations in which more than one wavelength are available, but no hole is contained in any other available hole. This can only happen if each available wavelength is being used somewhere in the ring. Therefore, each hole \( C_i \), if and when it is used to accommodate the requested path, will leave two other holes with (possibly null) sizes \( a_i \) and \( b_i \) to the left and right sides of the path, respectively.

Lemma 2 - Let the available holes be such that no hole is contained in any other, and let them be indexed such that \( a_1 > a_2 > a_3 > \ldots > a_m \). Then:

\[ b_1 < b_2 < b_3 < \ldots < b_m. \]

Figure 1 – Typical situation in which lemma 2 holds.
Proof: If \( b_i \geq b_j \) for some \( i < j \), then \( C_j \subseteq C_i \), which is a contradiction.

Fig. 1 illustrates a typical situation in which Lemma 2 holds. In this situation, any assignment will block some new paths that were not blocked before. The algorithm must then choose that assignment which blocks the least probable set of paths. An inspection of Fig. 1 shows that paths that connect nodes from two disjoint node sets form these sets.

Consider sets of \( s + v + t + 1 \) succeeding nodes in the ring. Let \( f(s,t;v) \) be the probability of a path being requested from any of the first \( s \) nodes to any of the last \( t \) nodes, passing through all of the central \((v+1)\) nodes.

Theorem 3 - Let the available wavelengths be such that no hole is contained in any other, and let them be indexed as in Lemma 2. Then, given a request for an \( H \)-hop path, the assignment that minimizes the ensuing path blocking probability minimizes \( f(s_i,t_i;v_i) \), where:

\[
\begin{align*}
    s_i &= a_i - a_{i+1}, \quad i = 1, 2, \ldots, m - 1 \\
    s_m &= a_m + H \\
    t_i &= b_i - b_{i-1}, \quad i = 2, 3, \ldots, m \\
    t_1 &= b_1 + H \\
    v_i &= |C_i| - s_i - t_i, \quad i = 1, 2, \ldots, m .
\end{align*}
\]

Proof: Paths that are blocked by the assignment of \( \lambda_i \) are those that can only be provided by \( \lambda_i \). Each one connects one of the leftmost \( s_i \) nodes of \( C_i \) to one of its rightmost \( t_i \) nodes.

Theorem 3 shows that the optimal assignment, when a path blocking probability increment must be accepted, results from the minimization of \( f(s_i,t_i;v_i) \), which is dependent on traffic statistics. We now derive specific assignment rules for the cases of uniform and exponential traffics.

### III.3.1 Uniform Traffic

In uniform traffic, we assume that all paths on the ring are equally likely to be requested. Therefore:

\[
f(s,t;v) = \frac{N!}{s!t!(N-s-t)!} ,
\]

where \( N \) is the number of nodes on the ring.

Therefore, the best assignment is the one(s) that minimize \( s_i,t_i \) regardless of \( v_i \).

### III.3.2 Exponential Traffic

In exponential traffic, the probability of a given path being requested decreases exponentially with its number of hops \( H \). For \( 0 < r < 1 \) and \( t = 1, 2, \ldots \):

\[
\text{prob}(H = t) = p(t) = \left( \frac{1-r}{r} \right)^t.
\]

We assume very large \( N \), so that the truncation of the exponential distribution may be neglected. Without loss of generality, let \( s_i \leq t_i \). Then:

\[
f(s_i,t_i;v_i) = N^{-1} [h(v_i) - h(v_i + s_i) - h(v_i + t_i) + h(v_i + s_i + t_i)],
\]

where

\[
h(x) = \sum_{k=0}^{\infty} k p(x+k+1) = \frac{e^{x+1}}{1-r}.
\]

Therefore:

\[
f(s_i,t_i;v_i) = \frac{1}{N} \left( \frac{r}{1-r} \right)^{v_i} \left( 1 - r^{s_i} \right) \left( 1 - r^{t_i} \right).
\]

It is enough, in this case, to assign \( \mu_i \) such that

\[
\mu_i = r^{v_i} \left( 1 - r^{s_i} \right) \left( 1 - r^{t_i} \right)
\]

is minimized, since \( \mu_i \) is a sufficient decision metric. The assignment should then favor large \( v_i \), but small \( s_i \) and \( t_i \). It is easy to show that, when \( r \) approaches 1, minimizing \( \mu_i \) is equivalent to minimizing \( s_i,t_i \), with vanishing influence from \( v_i \), as suggested by (6). However, for small \( r \) the prevailing influence comes from \( v_i \), with vanishing influence from \( s_i \) and \( t_i \).

### III.4 Minimization of Channel Path Capacities

The occurrence of multiple holes contained in other holes will be frequent on a ring operating under a low blocking probability. Since all corresponding wavelengths could be assigned with no path blocking probability increment, some other algorithm must be used to choose between them. Hence the idea of minimizing, among these wavelengths, the increment in channel blocking probability. The motivation is to preserve the ability of remaining channel paths to support future paths requests. In this way, the assignment will not only minimize the current path blocking probability, but also keep the network better prepared to minimize it in the future.

Let \( C_1, C_2, \ldots, C_q \) denote all available holes that are contained in some other hole, and let \( n_j = |C_j| \) be the size of \( C_j \). Let \( H \) be the number of hops in the requested path. Accommodating the requested path in \( C_j \) will generate two new holes on its left and right sides with sizes \( a_j \) and \( b_j \) respectively, with:

\[
a_j + b_j + H = n_j .
\]

Let \( g(n) \) be the probability that a channel path request of any size be accommodated in a hole of size \( n < N \) of the requested wavelength.

Theorem 4 - Minimization of the increment in channel path blocking probability is achieved by assigning wavelength \( \lambda_j \) that minimizes:

\[
\Delta_j = g(n) - g(a_j) - g(b_j) .
\]

Proof: \( \Delta_j \) is the loss in the probability that a request for a \( \lambda_j \) path be accommodated.

The actual metric to be used to guide the assignment choice is derived from Theorem 4 and the traffic first-order statistics. We now derive it for the same two cases considered in the subsection III.3.
III.4.1 Uniform Traffic

There are \( n \) 1-hop, \((n-1)\) 2-hop, \((n-2)\) 3-hop, ..., one \( n \)-hop paths that may be accommodated in an \( n \)-hop hole. Since they are all equally likely to be requested in uniform traffic, we have:

\[
g(n) = \frac{1}{N^2} \left\{ \frac{n(n+1)}{2} \right\}.
\]

(14)

So we have

\[
\Delta_j = \frac{1}{2N^2} \left[ b_j^2 + n_j - a_j^2 - a_j - b_j^2 - b_j \right]
\]

(15)

\[
= \frac{1}{2N^2} \left[ H + n_j - a_j^2 - b_j^2 \right].
\]

(16)

Therefore, it is enough to minimize \( n_j - a_j - b_j^2 \), which is equal to \( 2(H + a_j b_j - H^2) \). A simple and sufficient metric is then:

\[
\rho_j = Hn_j + a_j b_j.
\]

(17)

The assignment should then favor: a) smaller holes; and b) asymmetric insertion of the path in the hole, which yields small \( a_j b_j \). Since \( a_j b_j \) is at least zero and at most \( \left( n_j^2 - H \right)^2 \), we have

\[
Hn_j \leq n_j + a_j b_j \leq H + \frac{(n_j - H)^2}{4}.
\]

(18)

If the maximum possible metric for hole size \( n_j \) is less than the minimum possible metric for hole size \( n_j + 1 \), then the decision may take hole size \( n_j \) as a sufficient metric.

This will happen if and only if:

\[
Hn_j + \frac{(n_j - H)^2}{4} \leq H(n_j + 1)
\]

(19)

or

\[
n_j \leq H + 2\sqrt{H}.
\]

(20)

As long as (20) is met for some available hole, the smallest hole should be assigned. The decision between two holes with the same size should always favor the most asymmetric insertion.

Since all requests are equally likely in uniform traffic, this assignment will also minimize the loss in the number of channel paths available for future requests, which is the objective of the MaxSum algorithm proposed by [4].

III.4.2 Exponential Traffic

The same arguments as in the previous subsection will now provide:

\[
g(n) = \frac{1}{N} \left\{ \sum_{i=1}^{n} (n-i+1) \left( \frac{1-r}{r} \right)^i \right\}.
\]

(21)

As shown in the Appendix:

\[
g(n) = \frac{1}{N} \left( n - \frac{r}{1-r} + \frac{(1-r)^n}{r^n} \right).
\]

(22)

Theorem 4 will then yield:

\[
\Delta_j = \frac{1}{N} \left[ H + \frac{r}{1-r} + \frac{(1-r)^n}{r^n} \left( \frac{n_j^2 - a_j^2 - b_j^2}{r^2} \right) \right],
\]

(23)

yielding the following equivalent metric to be minimized

\[
\sigma_j = r^{n_j} - r^{a_j} - r^{b_j}.
\]

(24)

Let \( d_j = \min\{a_j, b_j\} \). If \( r \leq 1/2 \), the term \( r^{d_j} \) is dominant in this equation, meaning that \( d_j \) is a sufficient metric to be minimized, i.e. the best assignment will place the path as close as possible to another path with the same wavelength. In case of a tie, then the smallest hole should be chosen.

Again, insertion asymmetry and hole size are the important decision parameters. However, asymmetry becomes most important for small \( r \), and sufficient for \( r \leq 1/2 \). As \( r \) exceeds 1/2, size becomes more and more important, but it never reaches overall sufficiency: for \( r = 1 \), size is sufficient only below \( H + 2\sqrt{H} \), as seen in subsection III.4.1. Notice that the metric \( \rho_j \) may be obtained from \( \sigma_j \) in the limit when \( r \) approaches 1.

III.5 First-Fit Algorithmic Gains

The following algorithm, to be applied whenever two or more wavelengths are available for assignment, results from the full application of all results obtained above:

1. Index the available wavelengths in the order of increasing hole size, forming list \( A \);
2. Check if each hole is contained in some succeeding one, and put it in list \( B \) (initially empty) if it does;
3. If \( B \) has only one member, assign it; if \( B \) is empty, go on to step 5; otherwise, continue;
4. Assign a wavelength in \( B \) according to the following rules:
   a) Uniform Traffic (\( r = 1 \)). If the smallest hole(s) is (are) smaller than \( H + 2\sqrt{H} \), assign the wavelength with the most asymmetrical path insertion among the smallest holes. Otherwise, assign \( \lambda_i \) that minimizes \( \rho_i \) from (17);
   b) Exponential Traffic. If \( r \leq 1/2 \), assign a wavelength with an existing path closest to the requested path, choosing the smallest hole in case of a tie. If \( 1/2 < r < 1 \), assign \( \lambda_i \) that minimizes \( \sigma_i \) from (24);
5. Reorder the available wavelengths in the order of decreasing \( a_i \), forming the list \( C \);
6. Calculate \( s_i, t_i, v_j \) from (1),(2),(3),(4) and (5) for each wavelength in \( C \);
7. For uniform traffic, assign the wavelength that minimizes \( s_i t_i \). For exponential traffic, assign the wavelength that minimizes \( v_i \), from (11).

This algorithm, called minimal blocking (MB), minimizes: a) the instantaneous path blocking probability after each assignment; and b) the increment in channel blocking probability induced by the assignment, subject to the minimization of the path blocking probability. Applying step 4 directly to list \( A \) is the MaxSum or the maximal sum of channel capacities (MC) algorithm. Our
simulations compare the performances of the RD, FP, MC and MB algorithms on a 16-node ring. A network with the same topology and full wavelength conversion capability on all nodes gives a lower bound (LB) on the performance of all algorithms.

![Figure 2 – Blocking probability for uniform traffic.](image)

![Figure 3 – Blocking probability for exponential traffic.](image)

Figure 2 and 3 compare the simulated performances of RD, FP, MC, MB and LB for uniform and exponential \((r=1/2)\) traffics, respectively. The simulations were made on a 16-node ring with shortest-path routing. Each data point in the simulation was obtained from \(10^5\) random requests. Both the load \(L\) and the wavelength pool size \(W\) are equal to 4.

Shortest-path routing in the ring effectively truncates the traffic distributions considered above at \(H = N / 2\) instead of \(H = N\), so the criteria derived in the previous sections keep their approximate validity.

**IV. UPGRADING STRATEGIES**

In this section, we discuss two upgrading strategies for the optical path ring [2]:

1. Enhancing the wavelength pool size \(W\), which may be done up to the fiber load \(W = L\) or beyond it; and
2. Duplicating the fiber ring and integrating the resources of both rings, which may be done either by duplicating the OADMs and integrating them or not at the management level, or by replacing the OADMs by OXCs with full routing capability.

The purpose of this discussion is to evaluate the trunking gains obtained by each upgrading strategy. Comparing these gains with the costs of each strategy may then guide the system planning decisions. For the sake of fairness in the comparison, the same algorithm was used in all simulations. Considering the results of previous Section, the MaxSum (MC) algorithm was chosen for this purpose.

**IV.1 UPGRADING THE FIBER LOAD AND THE WAVELENGTH POOL**

Let us consider a four-wavelength ring with no wavelength conversion capability. If the number of wavelengths is upgraded to eight both in the fiber and in the pool of available wavelengths, how much more traffic can now be requested under the same blocking probability? Simulations shown in Fig. 4 address this and related questions. Curve \(WxL_y\) shows the results obtained by simulating the ring performance with a wavelength pool with \(W = x\) wavelengths and a fiber load of \(L = y\) wavelengths. For comparison with the full convertibility bound, curve \(FCB_z\) shows the ring performance with \(z\) wavelengths when all nodes have full conversion capability, in which case there is no gain in making \(W > L\).

Comparing curves \(W4L4\) and \(W8L8\) in Fig. 4 shows that requested traffic for the same blocking probability (and therefore the serviced traffic too) is approximately trebled when the number of wavelengths is doubled. The resulting trunking gain is almost the same as would be obtained with full conversion capability, as can be seen from comparison with \(FCB4\) and \(FCB8\).

**IV.2 UPGRADING THE FIBER PLANT**

Let us say that the ring fiber plant is doubled, either with the installation or the appropriation of one additional fiber at each hop. As for the nodes, they may be either duplicated along with the fibers, with or without integration between their managing functionalities, or replaced at each node location by a routing node, i.e. a node with full routing capabilities. In the latter alternative, a path might exchange fibers when passing through a
node location, thus enhancing the routing ability to avoid blocking.

**IV.2.1 NODE DUPLICATION**

We consider now the duplication of the same four-wavelength ring already considered in the last Section. The resulting ring capacity upgrading depends on the way the two rings are managed to support the requested traffic. We have compared three levels of integrated management:

1. No Integration (NI). In this configuration, the traffic is randomly split between both rings. When blocked by its destination ring, a request is blocked forever, and no further attempt is made to accommodate it.

2. Sequential Request (SR). The request is initially addressed to a randomly chosen ring. If blocked by this ring, it is then submitted to the other ring.

3. Joint Management (JM). In this case, one single manager controls both rings. The manager applies the MaxSum algorithm to the set of \( 2L \) (fiber, wavelengths) pairs. Notice that this option is equivalent to the fiber load upgrading considered in Section IV.1. Under a first-fit algorithm based on a fixed priority wavelength list, joint management would have the same performance as sequential request.

**IV.2.2 ROUTING NODES (RN)**

When routing nodes are used to connect both rings, more alternatives are opened to the routing of physical paths. While duplicating the fiber plant is equivalent to double the number of wavelengths, routing nodes will effectively associate each of these wavelengths with another one to and from which it can be converted.

Strictly speaking, there are \( 2^H \) shortest routes for a request with \( H \) hops, which apparently raises a routing assignment problem to be solved prior to the wavelength assignment one. This is only apparent, however, since routing through any fiber is equivalent whenever a wavelength is available in both fibers at a hop. Therefore, MaxSum may keep being applied with the shortest path routing, but a proper meaning must be given to the concept of available routes for the purpose of counting them. We have considered a route to be available in a given wavelength if each of its links is available in at least one fiber.

Fig. 5 compares the single fiber ring performance with the two-fiber one for several levels of integration between both rings: no integration (NI), sequential request (SR), joint management (JM) and routing nodes (RN). With no integration, the serviced traffic is just doubled, thus yielding no trunking gains, as expected. All other cases yield similar trunking gains, suggesting that the added expense of the routing nodes may not be warranted. Joint management yields some gain over sequential request, but only if the assignment algorithm is sophisticated (MaxSum).

**Figure 5 – Influence of upgrading the fiber plant for several levels of integration between both rings in the blocking probability on a WDM ring.**

**V. CONCLUSIONS**

We have demonstrated a wavelength assignment algorithm that minimizes the blocking probability immediately after each assignment in linear topologies. This algorithm, however, has provided no significant improvement in the long-term blocking probability over the best known heuristics (MaxSum), at least for 16-node rings.

Nevertheless, the main contribution has been the derivation of simple metrics to implement these algorithms in distributed management environments, for two important spatial traffic distributions: uniform and exponential. In both cases, the assignment favors smaller hole sizes and asymmetric insertion into them, but the priority of these features depends on traffic spatial distribution. These metrics are based totally on local information.

Upgrading strategies for the ring have also been investigated, showing that increasing the wavelength pool size by about 25\% above the fiber load will yield almost the same performance improvement that may be obtained from full wavelength conversion capability in all nodes.

**VI. APPENDIX**

This Appendix derives Eq. 22.

Let \( q(n) = \sum_{i=1}^{n} (n-i+1) r^i \). Then: \[ q(n) = m + nq(n-1). \] (25)

Consider the following related series:

\[ \alpha(n) = q(n) - \frac{r}{1-r} n + \left( \frac{r}{1-r} \right)^2. \] (26)

In (25), let us express \( q(n) \) and \( q(n-1) \) in terms of \( \alpha(n) \) and \( \alpha(n-1) \), respectively, getting:

\[ \alpha(n) = r\alpha(n-1). \] (27)

Therefore:

\[ \alpha(n) = r^{n-1} \alpha(1) = \left( \frac{r}{1-r} \right)^2 r^n. \] (28)
Substituting (28) back into (26) and the resulting expression for $q(n)$ into (21) will then yield (22).

REFERENCES


ABOUT THE AUTHORS

**Helio Waldman** was born in São Paulo, Brazil, on June 20, 1944. He graduated as Engenheiro de Eletrônica from Instituto Tecnológico de Aeronáutica (ITA) at São José dos Campos, Brazil, in 1966, and received the M.S. and Ph.D. degrees from Stanford University at Stanford, California, USA, in 1968 and 1972, respectively, both in Electrical Engineering. He joined the State University of Campinas (UNICAMP), where he now works, in 1973, and has since then been active in research on digital communications systems. In 1982, he was appointed Director of the School of Engineering at Campinas. From 1986 to 1990, he was Research Vice-President of the University. Dr. Waldman is a Senior Member of IEEE and a Senior Member of SBrT, where he served as President between 1988 and 1990, and as Editor of the Brazilian Transactions on Telecommunications from 1992 to 1996. Since the eighties, his research interests have focused on the fiber optic channel. He has authored three books (all in Portuguese): “Digital Signal Processing” (1987), “Optical Fibers: Technology and System Design” (1991), and “Telecommunications: Principles and Trends” (1997). He has published twelve papers in international journals, and 34 papers in proceedings of scientific meetings. He has supervised 23 Master’s Theses and five doctoral Theses. His current research interests are in the areas of Optical Networking and Broadband Communications. He is active in the undergraduate Electrical Engineering course, teaching Electromagnetic Theory, Guided Waves, and Optical Communications. He is also interested in discussing the new communication technologies and their impact on labor, education and society.

**Divanilson Rodrigo de Sousa Campelo** was born in Recife, PE, on May 26, 1976. He received the B.S. degree from Federal University of Pernambuco (UFPE), Recife, PE, Brazil, in 1998 and the M.Sc. degree from the State University of Campinas (UNICAMP), Campinas, SP, Brazil, in 2001, both in Electrical Engineering. From April 2000 to October 2001 he was with Nortel Networks in Brazil, where he was a system and design engineer in SDH/DWDM optical networks for huge customers. Currently he is working towards the PhD degree in Electrical Engineering at UNICAMP. His current research interests include optical networking, network traffic, optical packet switching and wavelength routing algorithms, among others.

**Raul Camelo de Andrade Almeida Jr** was born on October 01, 1975, in Recife, Brazil. He received the B.S. degree in Electrical Engineering from the Federal University of Pernambuco (UFPE) in 1999. In the same year, he worked as Instructor at SENAI. In 2001, he received the M.S. degree in Electrical Engineering from the State University of Campinas (UNICAMP). Since then he has worked for the Optical Network Project (OptiNet), which is supported by Ericsson, and has been a Ph. D. student at UNICAMP. The main currently research interests are in the field of Wavelength Routing (WR), Optical Packet Switching (OPS) and Optical Burst Switching (OBS) Networks.