DIFFERENT NUMERICAL APPROACHES IN THE ANALYSIS OF DIELECTRIC OPTICAL WAVEGUIDES

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***Abstract* - The main purpose of this paper is to stablish a comparison between some finite element and finite difference techniques to solve the wave operator together with the boundary operator. We have applied numerical implementations to three dielectric structures, and the results are presented by proceeding normalization on the wavevector and the propagation constant.**

Index Terms - Optical Waveguides, Numerical Methods, Propagation Constant

I. INTRODUCTION

We can formulate the study of the propagation of a light optical signal in dielectric waveguides in terms of longitudinal components of the *H* field or trough the transverse components. In recent publications the main purpose of the authors has been to eliminate all spurious modes [1]-[5]. The Finite Element Method has been used in the analysis of a large number of optical waveguides with excellent results for the propagation constant [6]-[10].

Another recent results by using finite difference for rectangular dielectric structures have been adding substantial contributions to the broad area of optical waveguides problems [11]-[13]. In all the techniques presented in this paper, we are imposing boundary conditions on the interfaces between distinct dielectric materials. Some of these techniques impose continuity on the boundary interfaces [14]-[15], as well as longitudinal and transverse continuity conditions has boarded [17]-[23].

The sparsity property of the eigenvalues/vectors matrices involved in the numerical approaches is a fundamental step to solve the further generalized eigenvalue equations. The great contribution of this paper is the extension of the penalty function method for the complex case, which permit us to calculate the gain for the channel waveguide.

II. THEORETICAL FORMULATION

We assume H_t is the transverse component of the field, and ∇_t is the associated transverse operator.

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For the propagation of the light optical signal in the *z*-direction with exponential decay $e^{-i\beta z}$ and each region of the guide has a piecewise constant ε_r , the wave equation in Cartesian coordinates implies that:

$\nabla_t^2 H_x + \left(k^2 - \beta^2\right) H_x = 0 \qquad \nabla_t^2 H_y + \left(k^2 - \beta^2\right) H_y = 0$ where $k^2 = \omega(\varepsilon_r \varepsilon_o \mu_o)^{1/2}$, assuming μ_r = 1. After proceed integration over each region, we get:

$$\int_{c_{i,j}} H_a \cdot N dS + \iint_{r_{i,j}} \left(k^2 - \beta^2\right) dA = 0 \qquad (1)$$

where a = x or y, $c_{i,j}$ = boundary of the regions $r_{i,j}$, N = unit vector outward normal of $c_{i,j}$.

If we place the nodes of the mesh on the interfaces and by imposing the boundary conditions of H_t, the longitudinal continuity of the components E_z and H_z can be transformed in an equivalent system of the form:

$$\frac{1}{\varepsilon_1}\left(\frac{\partial H_{yi}}{\partial x} - \frac{\partial H_{xi}}{\partial y}\right) - \frac{1}{\varepsilon_2}\left(\frac{\partial H_{yj}}{\partial x} - \frac{\partial H_{xj}}{\partial y}\right) = 0$$

$$\left(\frac{\partial H_{xi}}{\partial x} - \frac{\partial H_{yi}}{\partial y}\right) - \left(\frac{\partial H_{xj}}{\partial x} - \frac{\partial H_{yj}}{\partial y}\right) = 0 \qquad (2)$$

for the i-j^{th} interface.

III. NUMERICAL DISCRETIZATIONS

We can proceed the analysis of the above system by using finite-difference or finite element solution, which are the more used numerical techniques to study this kind of problem. A brief resume of different numerical approaches to study this system we present bellow.

III.1. A FINITE DIFFERENCE APPROACH

A discretization for the wave equation for the H_x and the H_y components can be made by analyzing the system (1) and (2), simultaneously. In this case, the finite-difference technique results in the eigenvalue matrix equation

$$C.U = \lambda.B.U$$

where:

$$C=\begin{vmatrix} C_{xx} & C_{yx} \\ C_{xy} & C_{yy} \end{vmatrix}, \quad B=\begin{vmatrix} B_{xx} & 0 \\ 0 & B_{yy} \end{vmatrix}, \quad U=\begin{vmatrix} H_x \\ H_y \end{vmatrix}$$

The matrices C_{xx} and C_{yy} are five diagonal while C_{xy}, C_{yx}, B_{xx} and B_{yy} are three diagonals with B non-singular. Thus, we reduce our analysis to an equivalent eigenvalue problem in the form:

$$A.X = \lambda.X$$

where: $A = B^{-1}.C.B$, and $X = B.U$. In order to analyze only the guiding region, we can restrict our study on the last equation only for a few numbers of positive eingenvalues of the matrix *C*.

Basically, by using the Arnold algorithm and the Krylov basis, we can construct a sequence of extreme eigenvalues of an *H* Hessenberg matrix which approximates the eigenvalues of *A*. The Arnold method can detect the predictor error and stops the convergence of the sequence. In addition, by using the Schur-Wieland deflation process, we found an enough close approximation of the further positive eigenvalues after we get the first largest one. Finally, the *s-step* Orthomin method permits to reduce our problem and to get the initial step of the convergence by the inverse power numerical technique. We omit more details, which we can find in the reference [17].

III.2. A FINITE ELEMENT APPROACH

We can define the functional:

$$F(f_1, f_2) = \begin{pmatrix} F_1(f_1) \\ F_2(f_2) \end{pmatrix}$$

where:

$$F_i(f_i) = \int_D \left[\sum_{i=1}^{Nodes} \nabla f_i \cdot \nabla f_i + \left(k^2 - \beta^2\right) f_i \right] dxdy$$

The set *D* is the cross-section of the guide, and in our study: $f_1 = H_x$ and $f_2 = H_y$. If we subdivide the dielectric guide region in a right triangular mesh, our problem is transformed in to minimize the components of the functional *F* with respect the nodal values of H_x and H_y, and we get the following general complex eigenvalue equivalent problem:

$$[S].[H] = \beta^2[S][H] \qquad (3)$$

The continuity boundary condition for the operator in this case transforms the problem in another eigenvalue problem, like:

$$[R].[H] = \lambda[I].[H] \qquad (4)$$

where $[I]$ is the Identity Matrix and $[R]$ is singular.

At first, we search solutions of the last equation for $\lambda = 0$. The matrix of the eigenvectors $[Z]$ are related with $[H]$ and the new vectors $[C]$ of this equation according the relation:

$$[H] = [Z].[C] \qquad (5)$$

Now, if we define:

$$[S_1] = [Z^t].[S].[Z] \qquad [T_1] = [Z^t].[T].[Z]$$

we can derive an equivalent eigenvalue matrix equation:

$$[S_1].[C] = \beta^2[T_1][C] \qquad (6)$$

After, we solve this equation for the unknowns β and *C*. Thus, by using (5), we get the original vector $[H]$ which solves our initial problem. Details of this technique are available in the reference [14].

III.3. A FINITE ELEMENT PENALTY METHOD

We can use the same general complex eigenvalue equivalent problem presented in equation (3), where the eigenvalue problem related with the boundary condition (4) can be wrote in the form:

$$[B].[H] = 0$$

where $B = R - \lambda I$.

Our main purpose is to solve the coupled system composed by the first general eigenvalue complex system (3), and the second new boundary condition system (6) wrote above. Suppose that H_1 is a solution of the first problem, and H_2 is the solution of the second one. Thus, we search a solution of the main problem which depends linearly on H_1 and H_2, that is a solution: $\underline{H} = \alpha H_1 + \gamma H_2$.

The linear property of the $[S]$ operator permits to set a scaling process on the first eingenvalue problem. If we define: $H = (1/\alpha).\underline{H}$, our previous problem is transformed in: "Find a vector field $H = H_1 + \xi H_2$ of system (3), where the operator *S* is changed by $[S_1] = [S + \xi.B]$, with ξ a small positive parameter." The new operator $[S_1]$ is a small perturbation of the initial operators $[S]$ and $[B]$.

The great contribution of this technique is the fact that we do not require finding the kernel of the operator $[B]$, and so we have a reduction in the number of the full matrix equations. The operator $[S_1]$ retains the sparse property of $[S]$, and so the sparse matrix routines can be used. Although $[S_1]$ is no longer symmetric and positive definite, has relevant importance the fact that $[T]$ retains these properties. More details we can see in the reference [16] and [18-21].

IV. SOME RESULTS

We apply our previous studies for three dielectric waveguides. The results can be compared with the numerical approaches mentioned in [3]-[6], and [14]-[22]. We include a comparison between finite-difference and finite-element numerical techniques too, as was discussed previously.

At first, we consider the quantum well ridge waveguide optical structure (*MQW*). The dimensions for the guide are given in the insert of Fig. 1. The results for different width (*w*) at two different etch depth (*h*) and two different aluminum compositions are included in the Figure. We plot the refractive index (η) versus the width (w). As we can see, there is an excellent agreement between the curves from [4], and the results from the penalty function method [16], [18]-[21]. The penalty factor (*PF*) used for all simulations is 1.0, the wavelength is 0.86μm, and $\eta = \beta/k_o$, where $k_o = 2\pi/\lambda_o$.

In the second example, we would like to show the results for the square dielectric waveguide with weakly coupled modes. Since the guide is two folds symmetric, we solve the systems only for one quarter of the local guide with appropriate boundary condition along the vertical and horizontal axis of symmetry.

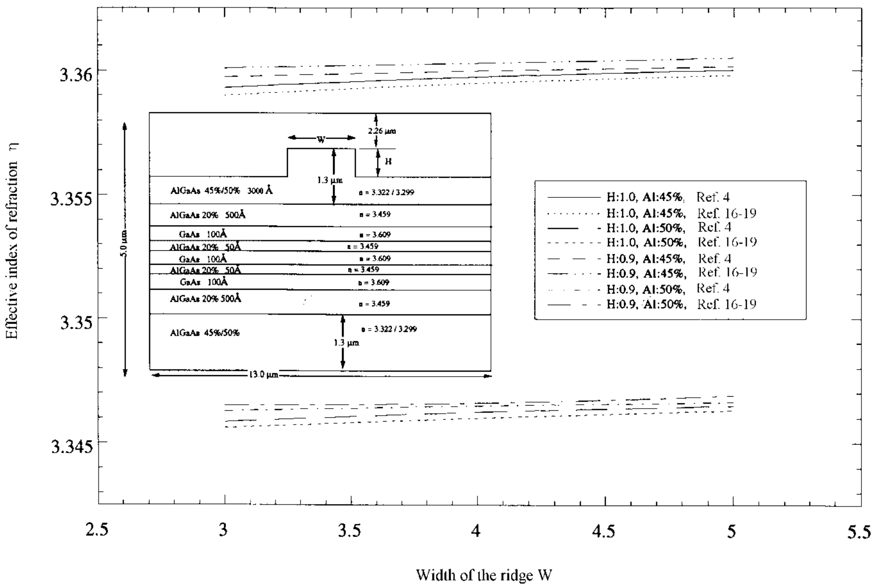

Fig. 1 – Effective index of refraction for different widths (w) of the ridge at different heights and different Aluminum compositions.

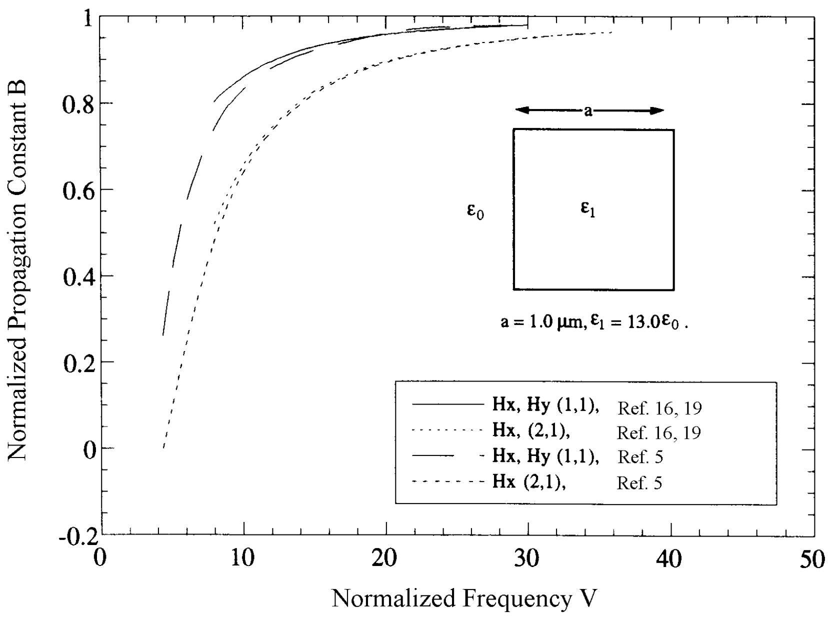

Fig. 2 – Plot of the normalized constant B versus the normalized frequency V for air clad square dielectric waveguide.

The results for the fundamental mode (H_{x11} and H_{y11}) and the first order mode (H_{x21}) are given in Fig. 2. The dimensions of the structure used in the simulation are inserts in it too. As we can see, the Penalty Function Method results matches the results from [5] very well. For a purely quasi-TE or TM modes, the *PF* used for all simulations, is 0.01. Decreasing the *PF* bellow 0.01 does not affect the results at all. The normalized frequency is $V = (2\pi/\lambda).a.(\varepsilon_1 - \varepsilon_2)^{1/2}$, while the normalized constant is calculated by $B = \{(\beta)^2 / .[(k_o)^2 - 1]\} / (\varepsilon_1 - \varepsilon_o)$.

Next, we consider the case of a circular optical dielectric waveguide. In this case, the guided step index optical fiber structure has the presence of highly coupled modes. Unlike, the square waveguide, the modes in a circular waveguide are hybrid due to his geometry. Again, since the guide is two folds symmetric, we solve only for a quarter of the total guide with an appropriate boundary condition along the vertical and horizontal axis of symmetry.

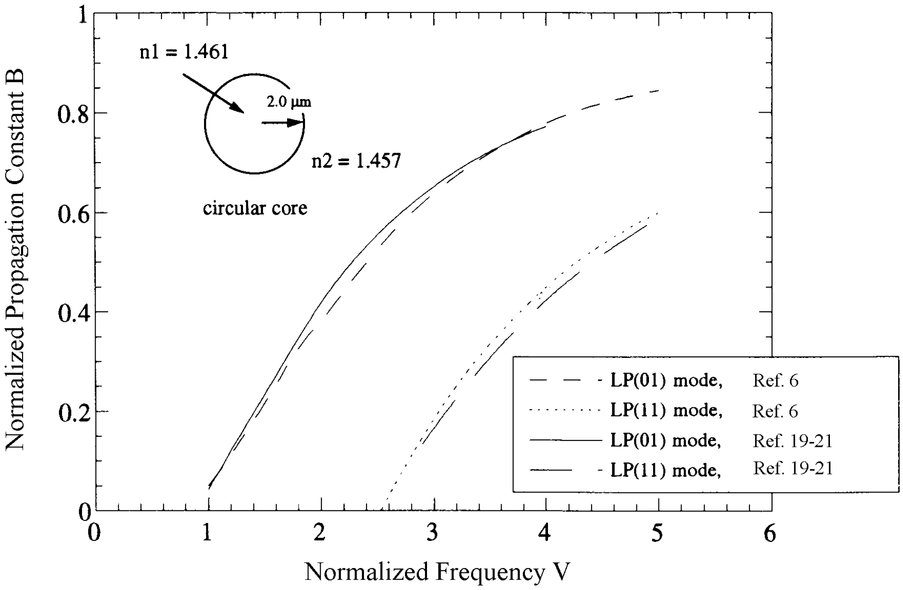

Fig. 3. Plot of the normalized constant B versus the normalized frequency V for a weakly guided step index fiber

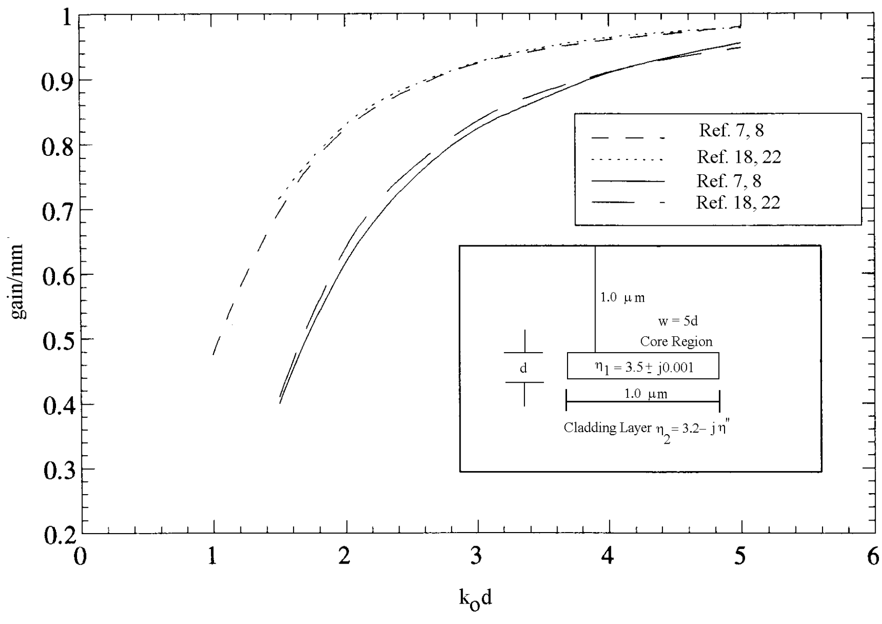

Fig. 4. Gain curve of the channel dielectric waveguide.

The results of the simulations for the fundamental mode (LP_{01}) and the first higher order mode (LP_{11}) are given in Fig. 3. As shown in this Figure there is complete agreement between the results using the Penalty Function Method [16], [18]-[20] and the analytical results of [6]. In this case, we adopt PF = 1.0 for all simulations. The expressions for the normalized frequency and the propagation constant are similar the second example. In this case, we need do use the radius r instead of the length a in the expression of the normalized frequency V.

Finally, we present the gain curve for the channel waveguide where the presence of loss can be take into account by using the Penalty Function Method. The gain is obtained when the imaginary part of the complex dielectric constant (ε'') is a positive value. On the other hand, we can get the attenuation when this constant has negative value (as show inside the cladding region of the guide). In Fig. 4 the dimensions of the structure and the results are presented. We analyze two different cases: in the first one there is no loss in the cladding, while in the second one it appears. There is an excellent agreement with the results showed in [7]-[8] and those presented in [18]-[20].

The gain is $g = Im(\gamma) / [2Real(\gamma)^{1/2} (k_o)]$. We assume that $\eta = \eta' + j\eta''$, with $\eta'' << \eta'$, and $\eta = (\varepsilon' \pm j\varepsilon '')^{1/2}$. As expected, the *PF* used is 0.01 because the modes are weakly coupled.

V. CONCLUSIONS

In all the theoretical arguments presented above the fundamental importance is the fact that all full matrices involved in the calculus of the eigenvalue/vectors have sparse form, which implies in a very fast convergence in the computations. The values of the propagation constant stay near of the product of the wavevector's amplitudes and the maximum dielectric constant, when the frequency oscillates in a neighborhood of the cut-off region.

We have obtained very close results for the MQW and the channel waveguide structures. Although, in the first case, we have a narrow guided region to get the convergence of the eigenvalues/vectors, and so we needed to use a very large number of nodes in the Mesh.

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