LETTER

Sum-Rate Evaluation of Multi-User MIMO-Relay Channel

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SUMMARY In this paper, the multi-user multiple-input multiple-output (MU-MIMO) relay channel is investigated, where the source node provides multi-beams to multi-users via a multi-antenna relay node. In this scenario, linear processing matrix at the relay node is designed around block diagonal (BD) scheme to improve the system sum-rate. Compared with the traditional linear processing matrix with zero-forcing (ZF) scheme at the relay node, the proposed matrix based on BD scheme can not only eliminate the multi-user interference to the same extent as the ZF scheme, but also realize optimal power allocation at the relay node. Numerical simulations demonstrate the BD scheme outperforms the ZF scheme and can significantly improve the sum-rate performance.
key words: *sum-rate, MU-MIMO, relay channel*

1. Introduction

Recently, multiple-input multiple-output relay channel is receiving significant attention. This can be attributed to two reasons, the first one is that exploiting relay in cellular networks can reduce the transmit power, and boost coverage area and link capacity in the regions with significant shadowing [1]. The other reason is that the well-known multiple-input multiple-output (MIMO) technology provides a wireless system with a large number of degrees of freedom [2], which can be used for increasing the capacity and/or reliability of the wireless link. These advantages make MIMO-relay as a key technology for future high data-rate wireless communication systems in cellular and ad-hoc networks. Due to MIMO regenerative relay scheme having high complexity and MIMO non-regenerative relay scheme having performance loss, linear relay scheme as a tradeoff between complexity and performance has been proposed in many newly published works [3]–[7]. In linear relay scheme, relay node does not decode the signals from the source node, but only simply process the received signals with a matrix multiplication. Based on different performance criteria, those prior works investigated optimal design of the linear process matrix at the relay node for single user. In practical system, due to the number of relay node is limited, one relay node usually supports multiple users, which constructing the MU-MIMO relay channel. How to design the linear processing matrix at relay node to improve the system performance is an essential problem to be solved, but few paper has been discussed this topic. In [8], the authors investigated the multi-user MIMO relay channel, but they assumed that each user only equipped with single antenna. In this paper, we study the MU-MIMO relay channel, and design the linear processing matrix based on BD scheme at relay node to improve the sum-rate of the system, which design can also pre-eliminate the inter user interference (IUI) or multi-user interference (MUI). This MU-MIMO relay system provides multiple data streams to each user at the same time. When there is only one user supporting by relay node, the proposed design is simplified to the same result appeared in [4]–[7], which indicates that our results are more generalized for MU-MIMO relay system design.

The rest of this paper is organized as follows. In Sect. 2, the system model of MU-MIMO relay channel is presented, and the sum-rate problems are formulated. Compare with the traditional ZF relaying scheme, the design of linear process matrix based on BD relaying scheme is proposed in Sect. 3, and Sect. 4 gives the numerical simulations to compare the sum-rate performance of system between BD relaying scheme and ZF relaying scheme. The conclusion is summarized in Sect. 5.

2. System Model

In this paper, we consider the MU-MIMO relay system, where one relay node supports service for multiple users, source node, relay node and each user equip with N, M and L antennas, respectively. System model is illustrated in Fig. 1. For briefly demonstration, we consider the system is performed in a half time-division duplex (TDD) mode. In the first timeslot, relay node receives the signal from the source node, and in the second timeslot, the received sig-

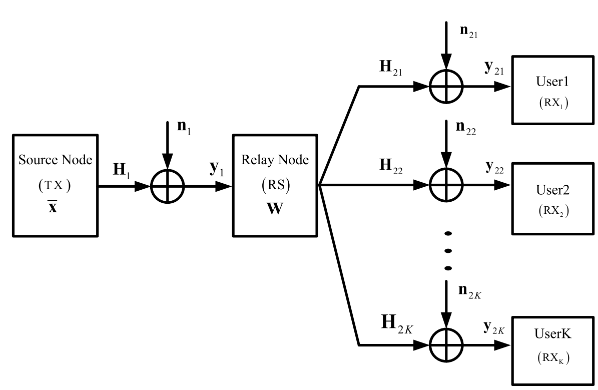

Fig. 1 Model of MU-MIMO relay system.

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nals at relay node are processed by a matrix multiplication and then are forwarded to the multi-users. The channel between the source node and the relay node is denoted by H_1, and channel between relay node and the i-th user is denoted as H_{2i}, we assume those channels are all slow, frequency-flat fading with a block fading model, the data transmission use two equal timeslot during which the channels all remain constant. In order to avoid the relay-induced signal-space collisions and ensure that each user can achieve multiple data streams, we assume $M \geq N \geq K \times L$, where K denotes the number of relay supporting users. During the transmission, the source node only know the CSI of H_1, the i-th user only knows its channel H_{2i}, and relay node knows CSI of all channels, those CSI assumptions can be easily achieved in TDD system. So the design of linear process matrix W can be only performed at relay node according to some performance criteria.

In the first time slot, source node transmits the signals to relay node, which denoted by a compositional data vector $\bar{\mathbf{x}}$ for K users simultaneously, and can be expressed as

$$\bar{\mathbf{x}} = \sum_{i=1}^{K} T_i \mathbf{x}_i \tag{1}$$

where T_i is the pre-coding matrix for the i-th user, $\mathbf{x}_i$ and $\mathbf{x}_j\,(j \neq i)$ are independent and $E\left(\mathbf{x}_i \mathbf{x}_i^H\right) = I$, where $E(A)$ denotes the expectation operation with respect to random matrix A. The total transmit power at source node is P_1, which is normalized with receiver noise power. With the equal power allocation at source node, the received signals at relay node during the first time slot can be written as

$$\mathbf{y}_1 = \sqrt{\frac{\mathrm{P}_1}{N}} H_1 \sum_{i=1}^{K} T_i \mathbf{x}_i + \mathbf{n}_1 \tag{2}$$

where $\mathbf{n}_1$ is the additive Gaussian white noise, and $E\left(\mathbf{n}_1 \mathbf{n}_1^H\right) = I$. In the second timeslot, the relay node linearly processes its received signals with matrix W, then broadcasts the proceeded signals to multi-users. The transmitted data from the relay node can be expressed as

$$\mathbf{r} = W\left(\sqrt{\frac{\mathrm{P}_1}{N}} H_1 \sum_{i=1}^{K} T_i \mathbf{x}_i + \mathbf{n}_1\right) \tag{3}$$

The normalized transmit power at relay node is assumed to P_2, so the power constraint condition at relay node can be formulated as: $trace\left(E\left(\mathbf{r}\mathbf{r}^H\right)\right) \leq P_2$, where $trace(B)$ denotes the operation of trace w.r.t the square matrix B. In the second timeslot, the received signals $\mathbf{y}_{2i}$ at the i-th user can be written as

$$\begin{aligned}\mathbf{y}_{2i} &= H_{2i} W\left(\sqrt{\frac{\mathrm{P}_1}{N}} H_1 \sum_{j=1}^{K} T_j \mathbf{x}_j + \mathbf{n}_1\right) + \mathbf{n}_{2i} \\ &= \underbrace{\sqrt{\frac{\mathrm{P}_1}{N}} H_{2i} W H_1 T_i \mathbf{x}_i}_{\textit{effective signals}} \\ &\quad + \underbrace{\sqrt{\frac{\mathrm{P}_1}{N}} H_{2i} W H_1 \sum_{j \neq i}^{K} T_j \mathbf{x}_j}_{MUI} + \underbrace{H_{2i} W \mathbf{n}_1 + \mathbf{n}_{2i}}_{Noise}\end{aligned} \tag{4}$$

where $\mathbf{n}_{2i}$ is the additive Gaussian white noise, and $E\left(\mathbf{n}_{2i} \mathbf{n}_{2i}^H\right) = I$. In (4), the first part is the effective signals for i-th user intended from source node, the second term is the multi-user interference (MUI), and the last two terms can cause the system performance degradation. According to (4), the achievable capacity C_i can be expressed as:

$$C_i = \log\left|\mathrm{I} + \frac{\frac{\mathrm{P}_1}{N}\left(H_{2i} W H_1 T_i\right)\left(H_{2i} W H_1 T_i\right)^H}{\mathrm{I} + \left(H_{2i} W\right)\left(H_{2i} W\right)^H + \frac{\mathrm{P}_1}{N} \Xi_{IUI}}\right| \tag{5}$$

where $\Xi_{IUI} = \left(\sum_{j \neq i}^{N} \left(H_{2i} W H_1 T_j\right)\left(H_{2i} W H_1 T_j\right)^H\right)$. And then sum-rate of the MU-MIMO relay channel can be achieved as

$$C = \sum_{i=1}^{K} \log\left|\mathrm{I} + \frac{\frac{\mathrm{P}_1}{N}\left(H_{2i} W H_1 T_i\right)\left(H_{2i} W H_1 T_i\right)^H}{\mathrm{I} + \left(H_{2i} W\right)\left(H_{2i} W\right)^H + \frac{\mathrm{P}_1}{N} \Xi_{IUI}}\right| \tag{6}$$

3. Linear Transceiver Matrix Design

In this section, we first briefly introduce the ZF scheme in MU-MIMO relay system, and then propose the BD scheme with optimal power allocation at the relay node to achieve the better sum-rate performance.

3.1 ZF Relaying Scheme

In MU-MIMO relay system, the ZF technique are adopted to decouple the user's data and to eliminate the multi-user interference [9]. Firstly, we define the matrix $H = \left[H_{21}^T, H_{22}^T, \cdots, H_{2K}^T\right]^T$, and its inverse matrix $H^\dagger = [Q_1, Q_2, \cdots, Q_K]$, where matrix Q_i with $M \times L$ dimension. And then the linear process matrix at relay node can be achieved as

$$W_{ZF} = \alpha \sum_{i=1}^{K} Q_i T_i^H H_1^\dagger \tag{7}$$

where α is a constant value to make sure the power constraint condition at relay node can be satisfied, and α can be decided by the following equation

$$\frac{P_1}{N} \sum_{i=1}^{K} \left\|W_{ZF} H_1 T_i\right\|_F^2 + \left\|W_{ZF}\right\|_F^2 = P_2 \tag{8}$$

where the entries of $N \times L$ matrix T_i are $(T_i)_{pq} = 1$ when $p = L(i-1)+q, q = 1, \cdots, L$, and all other entries are zeros. $\|A\|_F$ denotes the Frobenius norm of matrix A. Using W_{ZF} as in (7), relay node can decouple the multi-user's data and perform the pre-elimination of the multi-user interference.

3.2 BD Relaying Scheme

The merit of ZF technique used for W design is its low complexity, but this technique can cause some performance degradation. In the structure of matrix W_{ZF}, the first zero forcing matrix $H_1^{\dagger}$ will enhance the post noise power in the first timeslot at relay node [10], and the matrix Q_i achieved from the second zero-forcing matrix $H^{\dagger}$ will reduce the receive gain and then weak the system performance. Considering those shortcomings of the ZF technique design, we propose BD technique for matrix W design. The BD technique used for multiuser MIMO system have been extensively studied in the previous work [11], and in this paper, we extend this technique to multi-user MIMO relay system. Denote the singular value decomposition of one matrix by SVD, such as

$$\begin{aligned} SVD(H_1) &= U_1\Lambda_1 V_1^H \\ &= \left[U_{11},\cdots,U_{1K}\right]\begin{pmatrix}\Lambda_{11} & & 0\\ & \ddots & \\ 0 & & \Lambda_{1K}\end{pmatrix}\left[V_{11},\cdots,V_{1K}\right]^H \end{aligned} \tag{9}$$

where U_{1i} is $M\times L$ matrix, V_{1i} is $N\times L$ matrix, and Λ_{1i} is $L\times L$ matrix with diagonal entries $\left(\Lambda_{1i}\right)_{jj}\geq 0$ and other entries are all zeroes. Based on BD principle, we exploit the space orthogonal projection method [12] to construct the BD pre-coding matrix at the relay node to pre-eliminate the multi-user interference. Let

$$\tilde{H}_i = \left[(H_{21})^T,\cdots\left(H_{2(i-1)}\right)^T,\left(H_{2(i+1)}\right)^T,\cdots,(H_{2K})^T\right]^T \tag{10}$$

and the BD pre-code matrix for the i-th user can then be achieved as

$$F_i = I - \tilde{H}_i^H\left(\tilde{H}_i\tilde{H}_i^H\right)^{-1}\tilde{H}_i \tag{11}$$

Then the linear process matrix based the BD technique can be achieved as

$$W_{BD} = \sum_{i=1}^{K} F_i V_{2i} Z_{2i} U_{1i}^H \tag{12}$$

where $Z_{2i}$is $M\times L$ power allocation matrix for i-th user, and its entries $\left(Z_{2i}\right)_{jj}\geq 0, \left(Z_{2i}\right)_{jk} = 0$. The diagonal entries of $\left(Z_{2i}\right)_{jj}$ will be further optimized according to some performance criteria. And matrix V_{2i} is the right unitary matrix of $SVD(H_{2i}F_i) = U_{2i}\Lambda_{2i}V_{2i}^H$. In the BD relaying scheme, we set $T_i = V_{1i}$. Substitute (12) into (3) and (6), the sum-rate of system with the power constraint condition at relay node can be formulated as

$$\begin{aligned} &\max_{(\mathrm{Z}_{2i})_{kk}}\left(\sum_{i=1}^{K}\sum_{k=1}^{L}\log\left(1+\frac{\mathrm{P}_1}{N}\frac{\left((\Lambda_{2i})_{kk}(\mathrm{Z}_{2i})_{kk}(\Lambda_{1i})_{kk}\right)^2}{1+\left((\Lambda_{2i})_{kk}(\mathrm{Z}_{2i})_{kk}\right)^2}\right)\right)\\ &s.t.\quad \sum_{i=1}^{K}\sum_{k=1}^{L}\left(1+\frac{\mathrm{P}_1}{N}(\Lambda_{1i})_{kk}^2\right)\left(Z_{2i}\right)_{kk}^2 = P_2 \end{aligned} \tag{13}$$

It is clearly that (13) is a convex problem [13] and can be easily solved by the standard optimization algorithm. After got the parameter $\left(\mathrm{Z}_{2i}\right)_{kk}, i = 1,\cdots,K, k = 1,\cdots,L$, the linear process matrix W_{BD} can be achieved directly by(12). As a special case, setting $K = 1$, the linear process matrix W_{BD} in (12) can be designed for End to End MIMO communication via a MIMO relay node, and the same result of this single user scenario have been appeared in the works [7].

4. Simulation Results

In this section, numerical simulations are performed to demonstrate the BD relay scheme outperforms the ZF relay scheme. In all simulations, we assume that the entries of H_1 are independent and identically-distributed (i.i.d.) complex Gaussian variables with zero mean and variance κ_1^{-1}, and the entries of $H_{2i}, i = 1,\cdots,K$ are i.i.d. complex Gaussian variables with zero mean and variance κ_2^{-1}. The parameter κ_1 and κ_2 involve the effect of path loss for channel from source node to relay node and channel from relay node to users, respectively. Here, we define $SNR_1 = \kappa_1 P_1$, $SNR_2 = \kappa_2 P_2$, and in the simulations, we assume $SNR_1 = SNR_2$.

Figure 2 shows the performance of sum-rate of MU-MIMO relay system with the BD relaying scheme and the ZF relaying scheme, respectively. The source node with four antennas support two users with two data streams for each user, via a relay node with four antennas. Simulation result in Fig. 2 clearly shows that there is a big sum-rate gap between the BD relaying scheme and ZF relaying scheme in all SNR_2 range. This phenomenons can be attributed by the following reasons. One is that the BD relaying scheme does not enhance the post noise power at the relay node in the first timeslot, and the other is with the BD relay scheme, joint signal detection can be adopted at the designated user for detecting its own multi-data streams, the receiver gain can be achieved, and then system performance are improved.

Figure 3 still shows the sum-rate comparison of the BD relaying scheme and ZF relay scheme, while the relay node equips with six antennas. Simulation results also show that

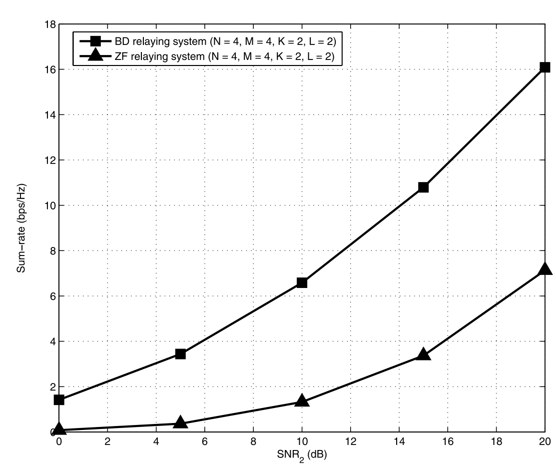

Fig. 2 Sum-rate v.s. SNR_2 at relay node $M = 4$.

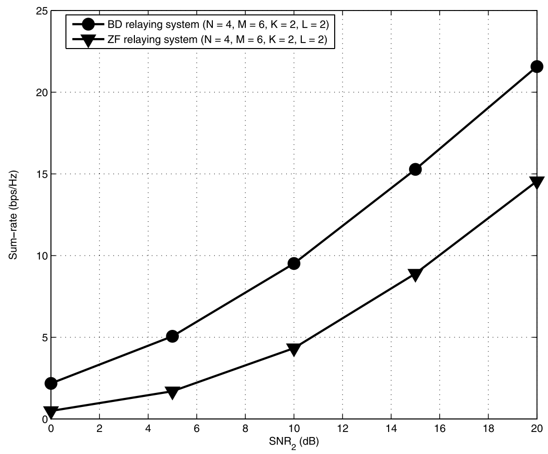

Fig. 3 Sum-rate v.s. SNR_2 at relay node $M = 6$.

the BD relay scheme outperforms the ZF relaying scheme in all SNR_2 regions. Furthermore, we can find some interesting things, one is sum-rate of system increase with the number of antennas at the relay node increase, the other is the sum-rate gap between the two relay scheme is reduced with the number of antennas at relay node increase. This is due to all the channel quality can be improved when the relay node equips more antennas. This simulation result also imply that the MU-MIMO relay system can flexible config the antennas at the relay node according to the real channel conditions and system available resource.

5. Conclusions

In MU-MIMO relay system, using the linear process matrix at the relay node not only eliminate the multi-user interference, but also improve the sum-rate performance of the system. However, when the number of active users in the system is larger than the supporting ability of the relay node, optimal multi-user scheduling should be further considered. Based on difference system performance criteria, how to allocate the resource of MU-MIMO relay system to the multi-active users is an interesting topic, and the solution of the optimal resource allocation in MU-MIMO relay system is being studied.

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