SPREAD SPECTRUM SIGNAL DETECTION IN OVERLAY SYSTEMS USING MAXIMUM ENTROPY PDF ESTIMATION BASED ON FRACTIONAL MOMENTS

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ABSTRACT

In this paper we introduce a new nonlinear detector to improve the performance of spread spectrum receiver in the presence of narrowband interference. The new detector is blind in the sense that no training data is required. In our scheme we use maximum likelihood (ML) detection rule in conjunction with maximum entropy method (MEM) for probability density function (PDF) estimation of the observation noise. We use MEM with a new approach based on fractional moments instead of integer moments. The estimated PDF based on fractional moments is quite close to the true PDF. The results indicate that the new nonlinear detector outperforms conventional matched filter, and well known locally optimal (LO) detector.

Index Terms— Maximum entropy method, maximum likelihood detection

1. INTRODUCTION

The idea of overlaying spread spectrum systems on narrowband networks offers a solution to obtain greater bandwidth efficiencies and better use of the overcrowded frequency spectrum [1, 2, 3, 4]. It is well known that direct sequence spread spectrum (DSSS) signal has an inherent immunity against narrowband signals. Even so, it has been shown that the capability of the spread spectrum system in rejecting narrowband interference can be significantly improved if an appropriate narrowband interference suppressing filter is employed in the receiver [1, 2, 3, 4, 5, 6]. Previously, the observation noise PDF in the spread spectrum receiver is obtained [3], which is a non-Gaussian PDF, hence, linear filters can not be optimal in this case. The approximate conditional mean (ACM) filter [1, 3] is used to suppress the effect of the narrowband signal. This filter is a nonlinear filter which is an approximation of optimal filter in non-Gaussian observation noise. [4, 5] developed some simpler nonlinear filters which have performance comparable to the ACM filter. The main drawback of these filters, ACM filter and one in [2] is their slow convergence rate using adaptive algorithms for implementation. These algorithms require long training sequences to work properly; therefore, they suffer a loss in the bandwidth efficiency. Some new methods such as one in [6] are proposed to improve the convergence rate of these methods. In this paper we propose a new nonlinear detector for the spread spectrum signal that does not require training data. When observation noise is non-Gaussian the LO detector has good performance in the weak signal case but its performance is poor in high signal to noise ratio (SNR) circumstances [7]. The proposed detector has better performance than LO detector. In our scheme we use two optimal methods, the first one is MEM which is used for PDF estimation of the observation noise. MEM is reasonable because the most likely PDF is one that includes more disorder and makes fewest assumptions about data which is more smoother and probable [8]. We use MEM method based on fractional moments instead of more familiar integer moments. The integer moments based method is developed for our problem to compare results with fractional case. The approximate density obtained resorting to MEM based on a few fractional moments is a more accurate estimate of a PDF because fractional moments can be expressed explicitly in terms of the infinite sequence of the integer moments [9, 10]. The second method is ML, this provides the test statistic for optimum Bayes' detection. This paper is organized as follows. In section 2, the problem formulation is presented. In section 3, PDF estimation based on integer and fractional moments is achieved. The proposed detector is derived in section 4. Section 5 presents the simulation results, and section 6 provides some concluding remarks.

2. PROBLEM FORMULATION

We assume the following model for the samples of the received spread spectrum signal in the presence of narrowband interference [1, 3, 4, 5, 6]

$$y[k] = s[k] + i[k] + \nu[k], \tag{1}$$

where $s[k]$, $i[k]$, and $\nu[k]$ are samples of the spread spectrum, narrowband signal, and white Gaussian noise respectively. We assume $s[k]$ is a sequence of independently identically distributed (IID) random variables, [3]. Previously three basic models have been used for the narrowband signal; tonal signal, narrowband autoregressive process and digital narrowband signal [1]. We use digital narrowband signal; a more realistic assumption, [2]. The sequences $s[k]$, $\nu[k]$ and $i[k]$ are assumed to be mutually independent [3]. In detecting the spread spectrum signal the observation noise is the sum of the narrowband interference and white Gaussian noise

$$w[k] = i[k] + \nu[k]. \tag{2}$$

Since $w[k]$ is the sum of two independent random variables, the PDF of $w[k]$ is the convolution of the PDF's of $i[k]$ and $\nu[k]$. $\nu[k]$ is a Gaussian random variable and $i[k]$ is a random variable taking on values $+I$ and $-I$ with equal probability. So the PDF of $w[k]$ is

$$f_w(w[k]) = \frac{1}{2}\left[\mathcal{N}_{\sigma^2}(w[k]-I) + \mathcal{N}_{\sigma^2}(w[k]+I)\right], \tag{3}$$

where $\mathcal{N}_{\sigma^2}(x)$ is defined as $\exp(-x^2/2\sigma^2)/\sqrt{2\pi\sigma^2}$ and σ^2 is the variance of each Gaussian shape component. Thus the observation noise is a non-Gaussian noise which can be unimodal or bimodal based on the values for σ^2 and I. The degree of non-Gaussianity of a symmetric PDF is measured by its 'Kurtosis Excess' which is the Kurtosis relative to Gaussian with following definition [7]

$$\gamma = \frac{E(w^4[k])}{E^2(w^2[k])} - 3,$$

where $E(\cdot)$ denotes the expectation operation. For the observation noise in (3), we have

$$\gamma = \frac{-2I^4}{(\sigma^2 + I^2)^2}.$$

Since this parameter is always negative, the PDF for the observation noise has tails that fall off faster than the Gaussian PDF [7]. Hence, the underlying binary spread spectrum signal detection is formulated in the following hypothesis testing problem

$$\begin{aligned} H_1 &: \quad y[k] = A + w[k], \quad k = 1, 2, \cdots, N \\ H_0 &: \quad y[k] = -A + w[k], \quad k = 1, 2, \cdots, N \end{aligned} \tag{4}$$

where $y[k]$ and $w[k]$ represent the samples of the received signal and observation non-Gaussian noise respectively. A represents the level of the binary spread spectrum signal and N is the number of chips per bit in the spread spectrum signal. When the observation noise is non-Gaussian there are always nonlinear detectors with better performance with respect to conventional optimum linear detectors [1, 3, 4, 5, 6, 7]. In the following sections we propose a new nonlinear detector for spread spectrum signal that requires no training data and has better performance than the matched filter and LO nonlinear detector.

3. MAXIMUM ENTROPY METHOD FOR PDF ESTIMATION

We assume the following moments of an unknown PDF, $f(\cdot)$, on the support set S are known; or estimated

$$\mu_{\alpha_i} = E(x^{\alpha_i}) = \int_S x^{\alpha_i} f(x)\, dx, \quad i = 0, \cdots, M \tag{5}$$

where $\alpha_0 = 0$ and other α_i's can be integer ($\alpha_i = i$) or fractional numbers. The traditional MEM is based on integer moments but we use this method based on both integer and the best set of fractional moments. Moments are attractive because their computation is algorithmically simple and uniquely defined for any random variable that meets the Carleman condition [11], and, since moments are global quantities, all available information is used making our integer or fractional moment-based methods less vulnerable to losses or changes of details than methods that use other criteria. But the most relevant information carried by the sequence of integer moments can be compacted in a few fractional moments [9, 10]. Hence, the estimated PDF with the appropriate fractional moments is a better approximation of the true PDF with respect to the same number of integer moments. Now, considering the moment constraints we estimate $f(\cdot)$. Clearly, this problem does not have a unique solution because there are many PDF's satisfying above constraints. Invoking maximum entropy principle we can find a unique solution. Maximizing the entropy functional $H[f] = -\int_S f(x) \ln f(x) dx$ subject to mentioned moment constraints yields the following functional form for PDF [8]

$$f_M(x) = \exp\left(\sum_{i=0}^{M} -\lambda_i x^{\alpha_i}\right), \tag{6}$$

where λ_i's are the Lagrangian multipliers that must be determined so that $f_M(\cdot)$ satisfies the moment constraints in (5). The entropy of $f_M(\cdot)$ is as follows

$$H[f_M] = -\int_S f_M(x) \ln f_M(x)\, dx = \sum_{i=0}^{M} \lambda_i \mu_{\alpha_i}. \tag{7}$$

Under the hypothesis of equivalent integer or fractional moments it is shown that $H(f_M)$ converges to $H(f)$ when $M \to \infty$ [9, 10]. Equivalent moments condition is also used to obtain the following equation for divergence measure of the two PDF's [9, 10],

$$\int_S f(x) \ln \frac{f(x)}{f_M(x)}\, dx = H[f_M] - H[f].$$

Hence divergence measure converges to zero by increasing M. A bound on the absolute difference between two PDF's is introduced in [9, 10]

$$\int_S |f_M(x) - f(x)| \le \sqrt{2(H(f_M) - H(f))}. \tag{8}$$

Therefore convergence in entropy is tantamount to convergence in distribution or PDF. In the other words, the absolute error obtained replacing $f(x)$ with $f_M(x)$ may be rendered arbitrarily small by increasing M. As an appraisal of the PDF approximation we compute the relative error defined as

$$\mathrm{RE} = \frac{|\text{True PDF} - \text{Approximated PDF}|}{\text{True PDF}} \tag{9}$$

3.1. Integer moments

Substituting (6) in (5) with $S = (-\infty, \infty)$ and using integration by parts for calculating this integral we reach the following set of equations for λ_i's

$$(j+1)\mu_j = \sum_{i=1}^{M} i\lambda_i \mu_{i+j}, \quad j = 1, 2, \cdots, M \tag{10}$$

We can express the above simultaneous equations in a matrix-vector form

$$\boldsymbol{A}\boldsymbol{z} = \boldsymbol{b} \tag{11}$$

where $\boldsymbol{A}$ is a $M \times M$ matrix with elements $a_{i,j} = \mu_{i+j-1}$, $\boldsymbol{b}$ and $\boldsymbol{z}$ are vectors defined as $\boldsymbol{b}$=$[2\mu_1,\, 3\mu_2,\, \cdots,\, (M+1)\mu_M]^T$ and $\boldsymbol{z} = [\lambda_1,\, 2\lambda_2,\, \cdots,\, M\lambda_M]^T$. The λ_i's for $i \neq 0$ are obtained by solving (11) and λ_0 can be obtained from PDF condition $\mu_0 = 1$. The characteristic function of the underlying noise is $\Phi(j\omega) = \cos(I\omega)\exp(-\sigma^2\omega^2/2)$, it is used to obtain the required moments from $\mu_i = E[x^i] = (-j)^i \psi^{(i)}(0)$. This method is used for estimating the observation noise PDF in (3) for $I = 3$ and $\sigma^2 = 1$ and the following PDF is estimated, depicted in Fig. 1,

$$f_M(x) = \exp(-2.4287 + 0.0959x^2 - 0.00597x^4).$$

Because the observation noise PDF in (3) is even symmetric, estimated PDF only includes terms with even powers in the exponent. The relative error also is depicted in Fig. 2. In Table. 1, the entropy difference between the estimated and true PDF is shown.

3.2. Fractional moments

For the fractional moments case the support set is assumed $S = [0, \infty)$, as in [9]. Since, the observation noise PDF is even symmetric and the sufficient information about PDF is available in the positive values, we initially estimate the PDF for positive values and then we will use $|\cdot|$ to obtain the PDF for all values. By investigating equation (8), we can deduce there is always an optimal choice of fractional moments and lagrangian multipliers in the sense that it accelerates the convergence of $H(f_M)$ to $H(f)$ and it can be obtained via following constrained optimization [9, 10]

$$\{(\alpha_i, \lambda_i)\}_{i=1}^{M} = \arg\min_{\alpha_i,\lambda_i} H[f_M] \tag{12}$$

with fractional moment constraints in (5). This is because $f_M(\cdot)$ is the PDF with maximum entropy among distributions with the same moments. Hence $H(f_M) > H(f)$ for $\forall\, \alpha_i, \lambda_i$. Therefore, the optimal set of parameters are obtained with above optimization which corresponds to the minimum distance between entropies and consequently PDF's due to (8). The above constraint optimization problem doesn't have an analytic solution and the solution must be obtained numerically. From (5) with $i = 0$, λ_0 is obtained as

$$\lambda_0 = \ln\left[\frac{1}{\mu_0}\int_0^{\infty} \exp\left(\sum_{i=1}^{M} -\lambda_i w^{\alpha_i}\right) dx\right]. \tag{13}$$

Using (7) and (13), the above optimization corresponds to the following one [9, 10]

$$\min_{\lambda_i,\alpha_i}\left\{\ln\int_0^{\infty}\frac{1}{\mu_0}\exp\left(\sum_{i=1}^{M} -\lambda_i w^{\alpha_i}\right) dw + \sum_{i=1}^{M}\lambda_i E(w^{\alpha_i})\right\},$$

where, $E(w^{\alpha_i})$'s are fractional moment constraints which must be used from (5) in the above optimization. This optimization is achieved for the case in the previous section and the following estimated PDF is obtained

$$f_M(x) = \exp(-5.4693 + 2.8656|x|^{1.2305} - 0.9177|x|^{1.879}).$$

In Fig. 1 and Fig. 2 the above estimated PDF is compared with the method based on integer moments. We notice that the PDF obtained based on fractional moments coincides with the true PDF and this method yields much better approximation of the true PDF. Hence, the information content of two optimal fractional moments is more than four integer moments for the underlying case. In Table 1 the entropy differences between the estimated PDF and true PDF for integer and fractional moments are shown. This table confirms the faster convergence of $H(f_M)$ to $H(f)$ for the fractional moments case.

4. NEW SPREAD SPECTRUM DETECTOR FOR OVERLAY SYSTEMS

In the previous section the estimated PDF based on integer and optimized fractional moments are obtained. We use the estimated PDF's in the log-likelihood ratio (LLR), to obtain the optimum Bayes' detector. Since the noise samples are IID variates, the test statistics for LLR is

$$T(\boldsymbol{y}) = \sum_{k=1}^{N}\ln\frac{f(y(k)|H_1)}{f(y(k)|H_0)},$$

where $\boldsymbol{y}$ shows the received vector of length N. As we see in the previous section the estimated PDF with fractional moments method has $|\cdot|$ in its structure. Hence, we use equation (6) with $|\cdot|$ for estimated PDF. Considering (4) and this model for noise samples, the log-likelihood test is obtained as

$$T(\boldsymbol{y}) = \sum_{k=1}^{N}\sum_{i=1}^{M} -\lambda_i\left(|y[k]-A|^{\alpha_i} - |y[k]+A|^{\alpha_i}\right) \underset{H_0}{\overset{H_1}{\gtrless}} 0.$$

Hence, the new detector includes an accumulator, threshold comparator and the following nonlinearity

$$g_{\mathrm{MEM,FM}}(x) = \sum_{i=1}^{M} -\lambda_i\left(|x-A|^{\alpha_i} - |x+A|^{\alpha_i}\right). \tag{14}$$

For integer moments, after some mathematical simplification, we observe that the even power terms in the log-likelihood ratio vanish and the nonlinearity is a polynomial consisting of odd power terms

$$g_{\mathrm{MEM,IM}}(x) = \textstyle\sum_{i=1, i\neq 2\ell}^{M-1} a_i x^i, \tag{15}$$

where a_i's are real numbers which can be obtained based on the values of A and λ_i's. The nonlinearities for integer and fractional cases are shown in Fig. 3 for $A = \sigma^2 = 1$ and $I = 3$.

5. SIMULATION RESULTS

In this section we examine the performance of the proposed detector. The performance measure is the probability of error obtained via Monte Carlo simulation. We compare the new proposed detector with conventional matched filter, the detector based on integer moments and the well known LO detector. The nonlinearity in LO detector is $g_{\mathrm{LO}}(x) = -f'_w(x)/f_w(x)$ [7]. For the observation noise PDF in the underlying spread spectrum problem, it is easy to show that LO test is

$$T(\boldsymbol{y}) = \sum_{k=1}^{N}\left(\frac{y(k)}{\sigma^2} - \frac{I}{\sigma^2}\tanh\left(I\frac{y(k)}{\sigma^2}\right)\right) \underset{H_0}{\overset{H_1}{\gtrless}} 0.$$

The nonlinearity in the above test statistics which is shown in the Fig. 3, is similar to the one in ACM filter [1, 3]. Simulations are achieved for $\sigma^2 = 1$, $I = 3$ and number of chips per bit $N = 17$. The results are depicted in Fig. 4. We note that the proposed detector based on fractional moments outperforms the conventional matched filter and LO detector and reach to the vicinity of lower bound. The lower bound is for the case where there is no interference, i.e. $I = 0$. In high SNR the performance of LO detector is worse than matched filter; due to the Taylor series approximation about zero which is used in deriving LO detector, that is valid only in weak signal condition [7].

6. CONCLUSION

In this paper we introduce a new blind nonlinear detector to improve the performance of spread spectrum receiver in the presence of narrowband interference. In our scheme we use ML detection rule coupled with MEM and a new approach based on fractional moments which yields a quite close approximation for the PDF of non-Gaussian observation noise. Simulation results indicate that our nonlinear detector outperforms conventional linear and LO nonlinear detector.

M integer/fractional moments	$H(f_M) - H(f)$
Four integer moments	0.20935
Two fractional moments	0.07095

Table 1. Entropy differences for integer and fractional moments.

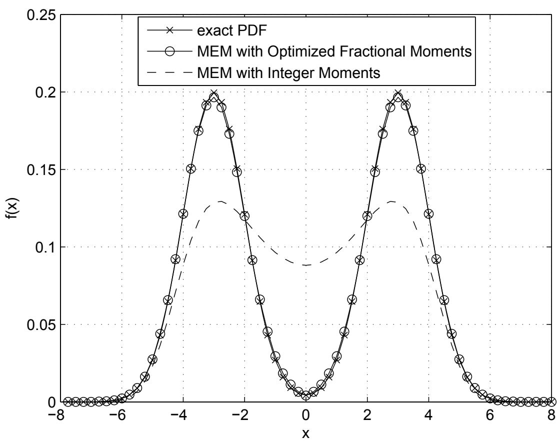

Fig. 1. Comparison of estimated PDF for $\sigma = 1$ and $I = 3$.

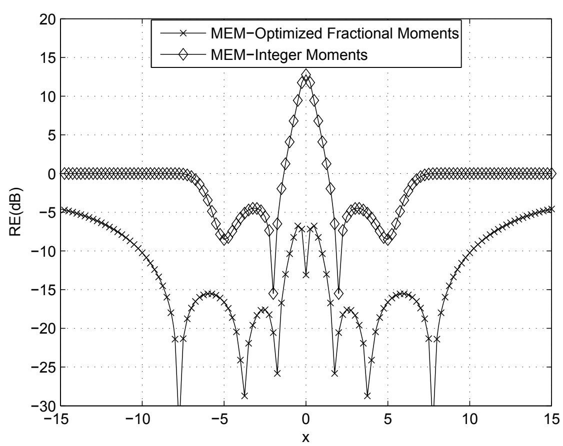

Fig. 2. Relative errors of MEM with fractional and integer moments.

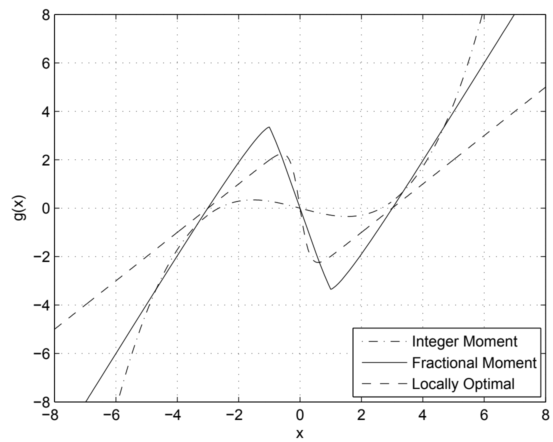

Fig. 3. Obtained nonlinearities for different detectors.

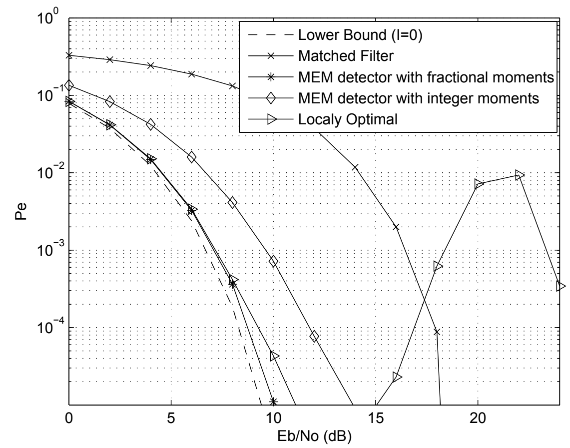

Fig. 4. BER Comparison of different detectors for $I = 3$.

7. REFERENCES

[1] H. V. Poor, "Active interference suppression in CDMA overlay systems," *IEEE Select. Areas Commun.*, vol. 19, no. 1, pp. 4-18, Jan. 2001.

[2] J. Wang and L. Milstein, "Adaptive LMS filters for cellular CDMA overlay situations," *IEEE J. Select. Areas Commun.*, vol. 14, no. 8, pp. 1548-1559, Oct. 1996.

[3] R. Vijayan and H. V. Poor, "Nonlinear techniques for interference suppression in spread-spectrum systems," *IEEE Trans. Commun.*, vol. 38, no. 7, pp. 1060-1065, July 1990.

[4] C. L. Wang and K. M. Wu, "A new narrowband interference suppression scheme for spread-spectrum CDMA communications," *IEEE Trans. Signal Processing*, vol. 49, no. 11, pp. 2832-2838, Nov. 2001.

[5] W. R. Wu and F. F. Yu, "New nonlinear algorithms for estimating and suppressing narrowband interference in DS spread spectrum systems," *IEEE Trans. Commun.*, vol. 44, no. 4, pp. 508-515, April 1996.

[6] J. T. Yuan and J. N. Lee, "Narrow-band interference rejection in DS/CDMA systems using adaptive (QRD-LSL)-based nonlinear ACM interpolator," *IEEE Trans. Vehicular Tech.*, vol. 52, no. 2, pp. 374-379, Mar. 2003.

[7] S. Kay, *Fundamentals of Statistical Signal Processing-Detection Theory*, Englewood Cliffs, NJ: PTR Prentice-Hall, 1998.

[8] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, NY: John Wiley, 1991.

[9] P. N. Inverardi, A. Petri, G. Pontuale, and A. Tagliani, "Stieltjes moment problem via fractional moments," *ELSEVIER Applied Mathematics and Computation*, vol. 166, pp. 664-677, 2005.

[10] P. N. Inverardi, G. Pontuale, A. Petri, and A. Tagliani, "Hausdroff moment problem via fractional moments," *ELSEVIER Applied Mathematics and Computation*, vol. 144, pp. 61-74, 2003.

[11] C. M. Bender, S. A. Orszag, "*Advanced Mathematical Methods for Scientists and Engineers*," McGraw–Hill, 1978.