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Sidelobe Level Reduction in Linear Array Pattern Synthesis Using Particle Swarm Optimization

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Abstract The design of nonuniformly spaced linear array antennas using Particle Swarm Optimization method is considered. The purpose is to match a desired radiation pattern and improve the performance of these arrays in terms of sidelobe level. This performance criterion determines how well the system is suitable for wireless communication applications and interference reduction. Two approaches are considered: in the first, the design of element placement with the constraint of array length being imposed is performed. The second is based on element position perturbation starting from a uniform element distribution. Many examples are treated to show the effectiveness of the designs and the effect some other parameters might have on the overall performance of the array.

Keywords Wireless communications · Nonuniform array antenna · Sidelobe level reduction · Particle swarm optimization

1 Introduction

Antenna arrays have been widely used in mobile and wireless communication systems to improve signal quality, hence to increase system coverage, capacity and link quality. The performance of these systems depends strongly on the antenna array design [1].

In many practical applications, the radiation pattern of the array is required to satisfy some basic criteria including, among others, directivity and sidelobe level

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[1, 2]. These two criteria constitute a trade-off that has to be optimized for real applications. One famous type of antenna arrays is the Dolph–Chebychev arrays that are uniformly spaced linear arrays fed by Dolph–Chebychev coefficients. These arrays have the important property that all side lobes in their radiation pattern have equal magnitude. Furthermore, the compromise between the directivity and sidelobe level for these arrays is optimal, meaning that for a specified sidelobe level, the directivity is the largest, and, alternatively, for a given directivity, the sidelobe level is the lowest [3, 4].

Works in the literature report many strategies for designing antenna arrays. Some works consider a uniform geometry with the element excitations being optimized for some desired characteristics [2]. Others assume a uniformly excited array with the physical dimensions being optimized [5]. A third approach considered by some authors consists in seeking both geometrical dimensions and element excitations [6]. In this work, varying the position-only alternative is considered with the technique of Particle Swarm Optimization (PSO) exploited to optimize the array layout.

Optimization techniques can be classified into two classes: local and global optimizers [2]. The local (also called classical) methods of optimization have been successfully used in finding the optimum solution of continuous and differentiable functions. These methods are analytical and make use of the techniques of differential calculus in locating the optimum points. Since some of the practical problems involve objective functions that are not continuous and/or differentiable, the classical optimization techniques have limited scope in practical applications [7]. In recent years, some optimization methods that are conceptually different from the traditional mathematical programming techniques have been developed. Most of these methods are based on certain characteristics and behavior of biological, molecular, swarm of insects, and neurobiological systems. Most of these methods have been developed only in recent years and require only the function values (and not the derivatives) [7]. The drawbacks of existing numerical methods have forced the researchers all over the world to rely on metaheuristic algorithms founded on simulations of some natural phenomena to solve antenna problems. These algorithms use an objective function of optimization which leads to the sidelobe suppression and null control [8]. Metaheuristic algorithms, such as Genetic Algorithms [9–11], Simulated Annealing [12], Tabu Search [13], Memetic Algorithms [14], Particle Swarm optimizers [15, 16], and Taguchi method [17] have been used in the design of antenna arrays.

The conventional methods of linear antenna array optimization use a set of linear or nonlinear design equations and solve them to get the optimal solution. Due to the complexity of the design problem, the solution lies in the use of evolutionary approaches like Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) of optimization for electromagnetic design [18]. PSO belongs to the class of evolutionary optimization techniques together with genetic algorithms and many other tools. PSO is similar in some ways to Genetic Algorithms (GA) and other evolutionary algorithms, but requires less computational bookkeeping and generally fewer lines of code. Furthermore, the basic algorithm is very easy to understand and implement. The method is inspired by the social behavior of swarms. The individuals evolve towards the global optimum in a competitive way based on a fitness function that involves the desired parameters to be optimized. The PSO algorithm has been applied for Electromagnetics and linear antenna array design problems [19–22]. Minimum sidelobe

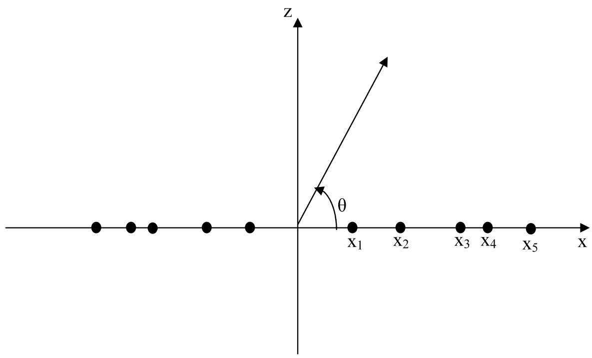

Fig. 1 Array placement for the optimization (shown here is a 10-element array)

level and control of null positions in case of a linear antenna array has been achieved by optimizing element positions using PSO [20]. Baskar et al. in [20] have applied the PSO and Comprehensive learning PSO (CLPSO) for the design of Yagi–Uda arrays, and have found that CLPSO gives superior performance than PSO for this design.

In this work, linear array antennas with uniform excitations are optimized for minimum possible sidelobe level reduction. Two approaches are considered. In the first, direct element position placement is performed with the total length of the array being imposed to be an additional constraint dictated by the fact that the array size should be within reasonable limit for practical feasibility of the designed array. In the second, element position perturbation is carried out starting from a uniformly spaced linear array for the purpose of having the lowest possible sidelobe level.

2 Problem Formulation

For a linear array of equally excited isotropic elements placed symmetrically along the x-axis as shown in Fig. 1, the array factor is given as [5]:

$$AF(\theta) := \sum_{k=1}^{N-1} \cos[2\pi x_k(\cos\theta - \cos\theta_0)] + \cos[2\pi x_{\max}(\cos\theta - \cos\theta_0)], \tag{1}$$

where

$2N$ is the number of elements;
θ is the scanning angle range as shown in Fig. 1 and varies from 0 to 180°;
θ_0 is the main beam direction (90° for broadside);
$x_{\max}$ is the outmost element position that dictates the total length of the array.

The problem of the first approach is to place the elements of a linear array in such a way that the minimum distance is 0.25λ and the maximum position is $x_{\max}$. The fitness function, denoted by f, is merely chosen to be the sidelobe level (abbreviated *SLL* hereafter) that is computed in the sidelobe region as:

$$f = SLL = \max[20\,\text{Log}(AF(\theta))]. \tag{2}$$

This is evaluated in the sidelobe region that includes all lobes different from the main lobe. The symmetry dictates that the optimization should be done on half the array (the right-hand side one in this work) with the other half constructed by symmetry.

Since the last element is fixed while the other elements are varying, the problem optimizing a $2N$ element array is reduced to only $N-1$ dimensions. The positions are constrained to be such that the minimum distance between the elements is never less that 0.25λ.

The problem can be rewritten as:

$$\begin{aligned} &\text{Minimize} \quad f \\ &\text{Subject to} \quad x_i \in [0, x_{\max}] \quad \text{and} \quad |x_i - x_j| > 0.25 \\ &\qquad\qquad \min(x_i) = 0.125 \quad \text{for } i, j = 1, \dots, N-1,\ i \neq j. \end{aligned} \tag{3}$$

The second approach starts from a uniform linear array with element spacing of 0.5λ, and then the positions are perturbed to reach the optimal positions that produce the lowest sidelobe level. The last element is now allowed to vary, and the array factor presented by (1) is rewritten to take account of this by entering the last term in the summation as:

$$AF(\theta) = \sum_{k=1}^{N} \cos[2\pi x_k(\cos\theta - \cos\theta_0)]. \tag{4}$$

The problem is then stated as:

$$\begin{aligned} &\text{Minimize} \quad f \\ &\text{Subject to} \quad -0.25\lambda < \Delta x_i < 0.25\lambda,\ i = 1, \dots, N, \\ &\qquad\qquad \text{with} \quad x_{i\text{new}} = x_{i0} + \Delta x_i. \end{aligned} \tag{5}$$

Again, the positions are constrained to be such that the minimal distance between the elements is never less that 0.25.

3 Particle Swarm Optimization

The PSO algorithm was developed by Eberhart and Kennedy in 1995, and was originated by imitating the behavior of a swarm of bees, a flock of birds, or a school of fish during their food-searching activities [5].

The PSO is a robust stochastic evolutionary computation based on the movement and intelligence of swarms. PSO has been shown to be effective in optimizing difficult multidimensional discontinuous problems in a variety of fields. Recently this technique has been successfully applied to antenna design [5]; the advantages of PSO include a simple structure, immediately accessible for practical applications, ease of implementation, speed to acquire solutions, and robustness [23].

3.1 PSO-Like Natural Behavior

PSO is an evolutionary algorithm based on the intelligence and cooperation of a group of birds or fish schooling. It maintains a swarm of particles where each particle represents a potential solution. In the PSO algorithm, particles are flown through a multi-dimensional search space, where the position of each particle is adjusted according to its own experience and that of its neighbors [24, 25]. Before presenting the algorithm, a glance on the natural analogous "algorithm" is given. Consider a swarm of bees in a field whose goal is to find the location with the highest density of flowers. Without any prior knowledge of the field, the bees begin with random locations at random velocities looking for flowers. Each bee can remember the locations where it found most flowers (called *pbest* in the algorithm after), and somehow, knows the locations where the other bees found an abundance of flowers (called *gbest* in the algorithm). Along the way, a bee might find a place with a higher concentration of flowers than it had found previously. It would then be drawn to this new location as well as the location of the most flowers found by the whole swarm. Occasionally, one bee may fly over a place with more flowers than encountered by any bee in the swarm. The whole swarm would then be drawn toward that location in addition to their own personal discovery. In this way, the bees explore the field and eventually, the bees' flight leads them to the one place in the field with the highest concentration of flowers. Soon, all the bees swarm around this point.

3.2 The PSO Algorithm

The natural behavior explained earlier is imitated as the following algorithm states:

1. Initialize randomly swarm locations and velocities: to begin searching for the optimal position in the solution space, each particle begins at its own random location with a velocity that is random both in direction and magnitude.

2. Systematically fly the particles through the solution space: each particle must then be moved through the solution space as if it were a bee in a swarm. The algorithm acts on each particle one by one, moving it by a small amount and cycling through the entire swarm. The following steps are performed on each particle individually:

(a) Evaluate the particle's fitness, compare to (global best) *gbest,* (particle best) *pbest*: the fitness function of a particle in the solution space returns a fitness value to be assigned to the current location if that value is greater than the value at respective *pbest* for that particle, or the gbest, then the appropriate location are replaced with the current location.

(b) Update the particle's velocity: the manipulation of a particle's velocity is the core element of the entire optimization. The velocity of the particle is changed according to the relative location of *pbest* and *gbest.* It is accelerated in the directions of these locations of greatest fitness according to the following equation:

$$V_n = W \times V_n + c_1 \times rand \times (p_{best,n} - x_n) + c_2 \times rand \times (g_{best,n} - x_n), \qquad (6)$$

where

v_v andx_n are the velocity and position coordinates of particles in the nth dimension, respectively.

W is the positive parameter called "inertia weight".
c_1 and c_2 are the scaling factors that determine the relative pull of *pbest* and *gbest*;
rand is a random function generating random numbers with uniform distribution on $]0, 1[$.

It is apparent from (6) that the new velocity is the old velocity scaled by W and increased in the direction of *gbest* and *pbest.*

(c) Move the particle: once the velocity has been determined it is simple to move to its next location. The velocity is applied for a given time step Δt, usually chosen to be one and the new coordinate x_n is computed for each of the N dimensions according to the following equation:

$$x_n = x_n + \Delta t \times v_n. \tag{7}$$

3. Repeat: after this process is carried out for each particle in the swarm, the process is repeated starting at step 4. An alternative representation of the velocity of (6) is:

$$v_n = k\{v_n + \varphi_1 \times rand \times (pbest, n - x_n) + \varphi_2 \times rand \times (gbest, n - x_n)\}, \tag{8}$$

where φ_1 and φ_2 have the same meaning as c_1 and c_2 and both of them are assigned the value of 2.05. The parameter k is the constriction factor determined from the following two equations:

$$\varphi = \varphi_1 + \varphi_2, \quad \varphi > 4, \tag{9}$$

$$k = \left| \frac{2}{2 - \varphi - \sqrt{\varphi^2 - 4\varphi}} \right|. \tag{10}$$

3.3 Boundary Condition of Particle Swarm Optimization

Experience has shown that *Vmax*, the constriction factor, and inertial weights do not always confine the particles within the solution space. To cope with this problem, three different boundary conditions are reported in the literature, namely:

1. Absorbing walls: when a particle hits the boundary of the solution space in one dimension, the velocity in that dimension zeroed, and the particle will eventually be pulled back toward to the allowed solution space. In this sense, the boundary "walls" absorb the energy of particles trying to escape the solution space.
2. Reflecting walls: when a particle hits a wall in one of the dimensions, the sign of the velocity of that dimension is changed and the particle is reflected back toward the solution space.
3. Invisible walls: the particles are allowed to fly without any physical restriction. However, particles that roam outside the allowed solution space are not evaluated for fitness.

4 Results and Discussions

4.1 Direct Element Placement with Array Length Constraint Approach

The first previously presented approach has been applied to some illustrative examples to show whether one can, through element position design, reach sidelobe levels that can be exploited in practical wireless applications. The idea is to have and increasing number of elements with the purpose of having an insight on how this parameter can affect the array performance in terms of sidelobe level. Also, the sidelobe level reduction along with the angle steering capability has been investigated to have a complete view of the success of the approach. Satisfactory results have been found that overtake the state-of-the-art designs and even the ones reported in the literature. The Dolph–Chebychev arrays are known to exhibit the least possible sidelobe level for a given imposed directivity and the best directivity for a given sidelobe level. For this reason, this type of array is considered to be a reference against which the resulting arrays are judged.

The first example that has been treated is a 10 element array which makes it a 4-dimensional problem. Figure 2 presents the resulting radiation pattern along with that of a Dolph–Chebychev array for the same sidelobe level. The resulting radiation pattern exhibits a sidelobe level of −20.45 dB which has been used as an input for the radiation pattern calculation of the Dolph–Chebychev array.

A look at the radiation patterns reveals that the optimal PSO array shows similar performance in terms of beamwidth compared to the Dolph–Chebychev array. Hence, the optimal PSO pattern with uniform excitation is similar to the pattern for equally spaced linear array with Dolph–Chebychev excitation. Therefore, a nonuniformly spaced linear array with a simple feed network can generate a pattern similar to the

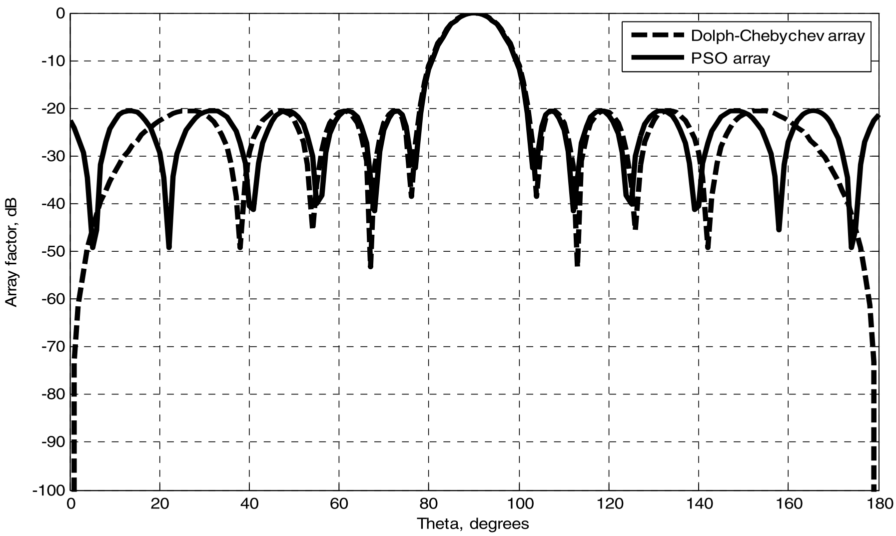

Fig. 2 The resulting PSO array pattern along with the Dolph–Chebychev array (10 elements)

Table 1 Summary of the resulting PSO array performance characteristics and their Dolph–Chebychev counterparts; first approach

Example	Array	N° of elements	Steering angle (°)	Array length (wavelengths)	SLL (dB)	Beamwidth (°)
1	PSO array	10	0	4.5	–20.45	11.2
	Dolph–Chebychev array	10	0	4.5	–20.45	11
2	PSO array	16	0	7.5	–23.10	7.17
	Dolph–Chebychev array	16	0	7.5	–23.10	7
3	PSO array	24	0	11.5	–24.97	4.88
	Dolph–Chebychev array	24	0	11.5	–24.97	4.4
4	PSO array	24	60	11.5	–17.68	4.8
5	PSO array	16	120	7.5	–18.94	10

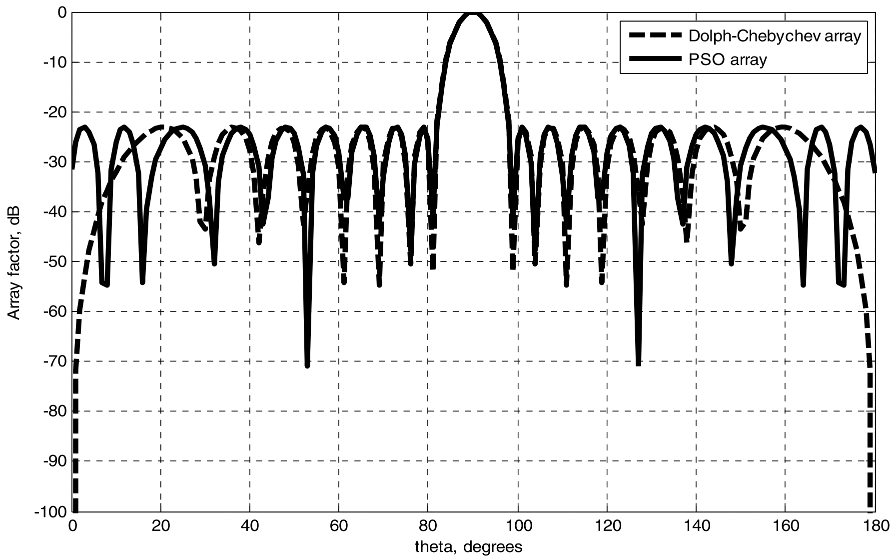

Fig. 3 The resulting array factor and the Dolph–Chebychev one for comparison (16 elements)

Dolph–Chebychev pattern. Another feature of the PSO array is that the total array length is the same as that of the Dolph–Chebychev array.

In a similar way, the next examples are treated to assess the performance of the approach. The results are summarized in Table 1. The second treated example is a 16 element array. The resulting PSO array has a pattern that exhibits a sidelobe level of −23.10 dB below the main lobe which is a very convenient value for modern wireless communication systems. Compared to the Dolph–Chebychev array as shown in Fig. 3, it can be noticed that though the two arrays show again practically similar behavior in terms of trade-off between sidelobe level and directivity, which consol-

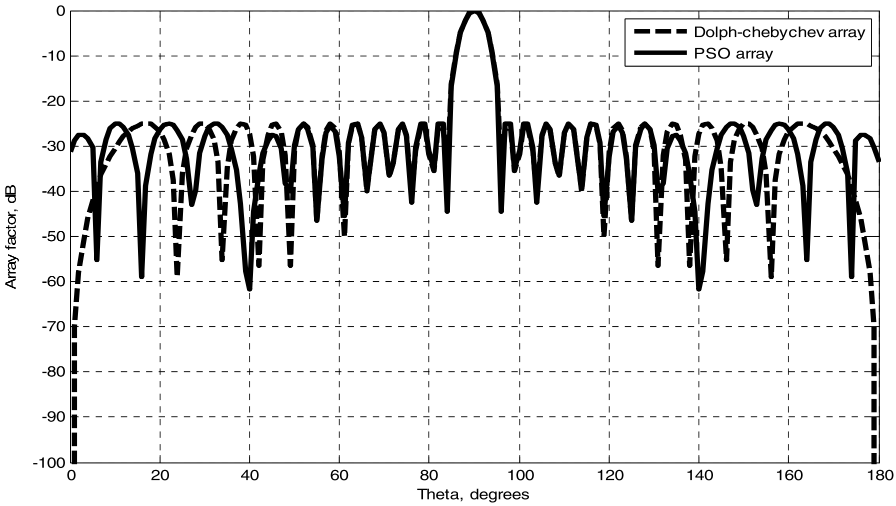

Fig. 4 The resulting PSO and Dolph–Chebychev array factors (24 elements)

idates the already obtained result concerning uniform and nonuniform arrays. Once more, the two arrays have the same length which is an additional advantage as the space occupied by the arrays is the same.

The third example that has been optimized in this study is a 24-element array. The resulting array shows a sidelobe level of −24.97 dB below the main lobe. The array factors for both the PSO and Dolph–Chebychev arrays are shown in Fig. 4. Again, the two arrays show similar characteristics in terms of directivity/sidelobe level trade-off with the same total array length.

As a conclusion to the previously carried out study, the increase in the number of elements had an impact on the sidelobe level reduction that goes hand in hand with the improvement in directivity. However, it can be noticed that in applications where sidelobe level is the major characteristic of interest, one can achieve acceptable values with fewer elements, which makes the design practically feasible. Another remark is that though the Dolph–Chebychev arrays are known to exhibit the best compromise between sidelobe level and directivity, designed PSO arrays demonstrate the same behavior with a simpler feeding network with an additional characteristic of the same array length.

To complete the picture of the design and to have practical use as in smart antenna applications, it is necessary to investigate whether the approach can work under an angle steering condition. For this, two examples of steering cases have been treated: a 24-element with steering angle of 60° and a 16-element with steering angle of 120°. Figures 5 and 6 show the resulting patterns for the two latter cases, respectively.

It can be noticed that the performance in terms of sidelobe level reduction is less than that obtained for a broadside array. Despite this, the sidelobe levels obtained are still exploitable in communication systems as other types of conventional arrays fail to reach such a level when the steering angle gets far from the broadside direction [6].

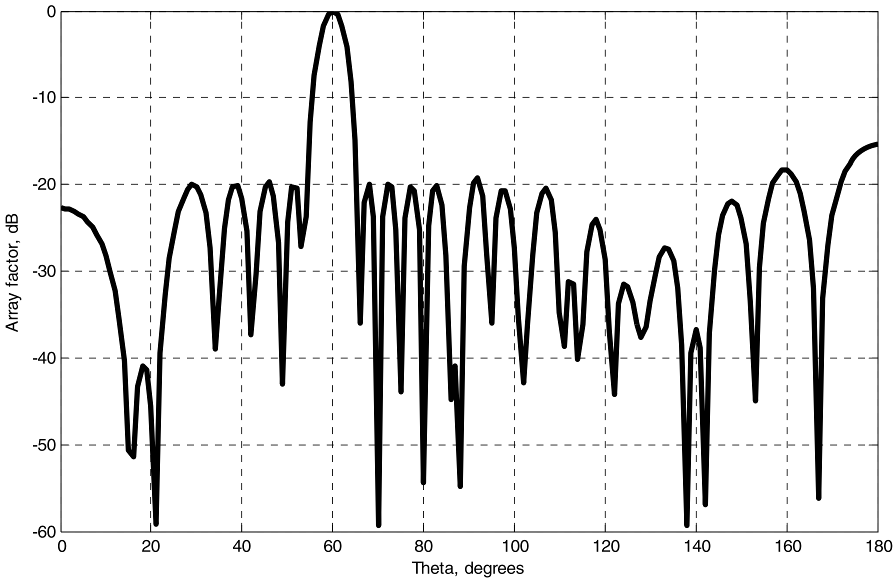

Fig. 5 The resulting array factor with the main beam steered towards 60° (24 elements)

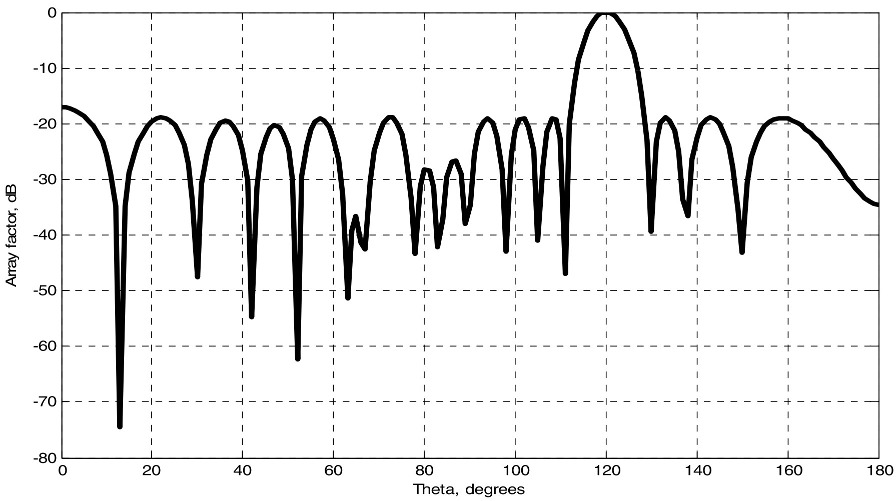

Fig. 6 The resulting array factor with the main beam steered towards 120° (16 elements)

The radiation pattern has a uniform sidelobe level over the range of angles far from the main lobe, which means a reduced amount of interference from neighboring users in a communication system. The sidelobe levels are at −17.68 dB and −18.94 dB down the main lobe for steering angles 60° and 120°, respectively.

Overall, the first approach has shown that it is possible to reach optimal performance in terms of the sidelobe level and directivity by direct element placement.

Furthermore, the designed arrays have the constraint that the space they occupy is the same as the Dolph–Chebychev arrays which are known to exhibit optimal performance in terms of these two parameters

4.2 Array Element Position Perturbation Approach

This part deals with the design of linear array antennas by element position perturbation starting from a uniform linear array. Unlike the first part, the outmost element positions of the array are allowed to vary, which adds a degree of freedom to the design problem. As it has been the case for the first approach, some illustrative examples on the second approach are presented. The resulting array factors are compared to those produced by the corresponding Dolph–Chebychev arrays in terms of the overall performance of the sidelobe level and directivity. At first, only broadside case is treated. Then, steering capability is investigated with the hope to achieve still good performance including this unique feature of antenna arrays.

In a similar way, the characteristics and the results of the treated examples are shown in Table 2. The first example is an array of 10 elements. The corresponding array factor pattern is illustrated in Fig. 7, which exhibits a side lobe level of −20.80 db and a 3-dB beamwidth equal to 10°. The corresponding Dolph–Chebychev array exhibits a 3-dB beamwidth of 11.35°. The results show that the obtained result is better than for the Dolph–Chebychev array because it improves the 3-dB beamwidth (directivity) for the same number of elements and sidelobe level. However, it turns out that the space occupied by the PSO array is larger than that of the Dolph–Chebychev array, which is a penalty paid for the improvement in the directivity/sidelobe level trade-off.

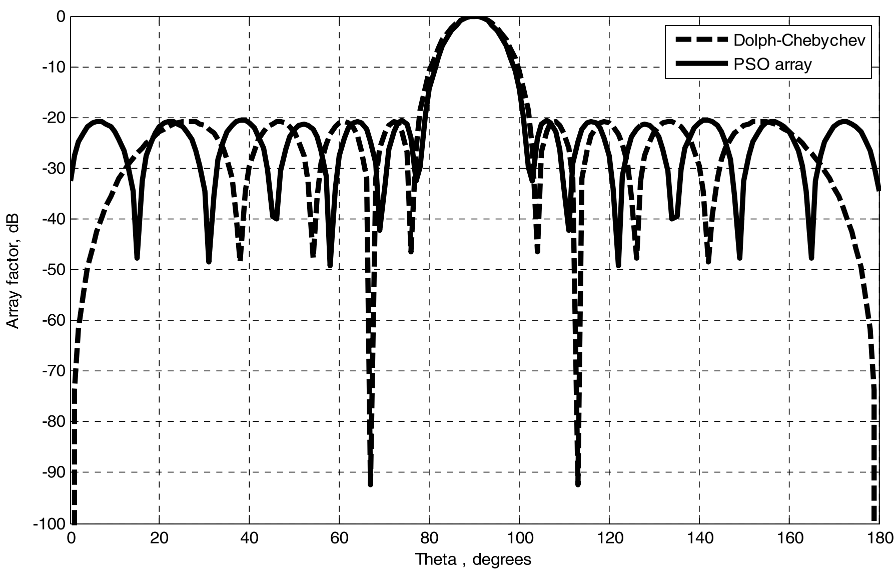

Fig. 7 The position perturbation resulting array factor for the broadside case (10 elements)

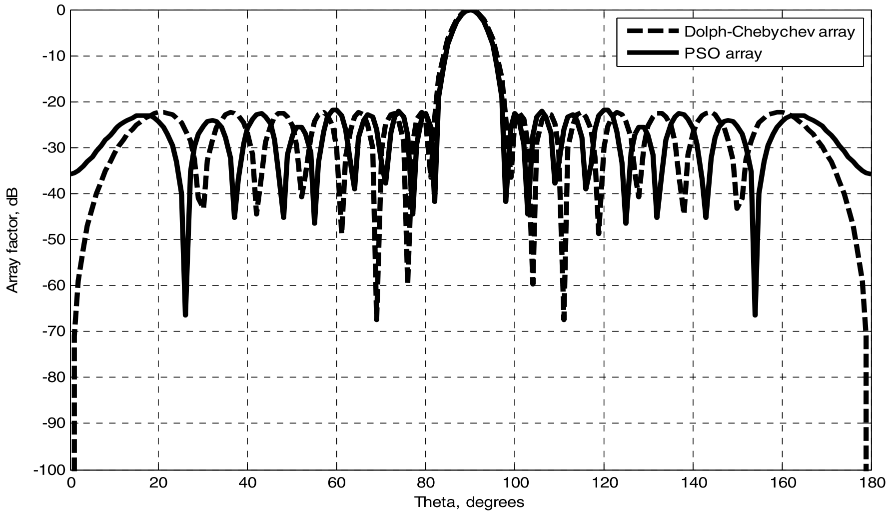

Fig. 8 The position perturbation resulting array factor for the broadside case (16 elements)

As a second example, a 16-element array is considered. The radiation pattern of the resulting PSO array is shown in Fig. 8. The optimal achieved sidelobe level is found to be −22.37 dB. The corresponding Dolph–Chebychev array pattern is shown in the same figure. It is noticed that for the same sidelobe level, the PSO array exhibits a narrower major lobe. Indeed, the PSO array has a 3-dB beamwidth of 6.4° while the Dolph–Chebychev array 3-dB beamwidth is 7.08°. However, once more, the PSO array length is larger than that of the Dolph–Chebychev array which is a drawback in the PSO design.

As a third example, a 24 element array is considered and the resulting pattern is shown in Fig. 9. The pattern exhibits an SLL and a 3-dB beamwidth of −24.38 dB and 4.88°, respectively. The radiation pattern of a uniformly spaced array (Dolph–Chebychev) with same number of elements and SLL has a 3-dB beamwidth of 5°, which is less than the result obtained for the PSO. The PSO total array length is again larger than the Dolph–Chebychev array which agrees with the two previously obtained results.

The previous examples have proved that the approach chosen is successful in terms of directivity/sidelobe level compromise improvement for the broadside case. To demonstrate the effectiveness of the design methodology, the next examples consider the case where the steering angle is not towards broadside direction.

The first example assumes a 10-element array with the main beam directed towards 80°. The radiation pattern corresponding to this angle is illustrated in Fig. 10 with the SLL found to be −19.07 dB and the 3-dB beamwidth equal to 11°. These values are slightly different from the broadside case (recall SLL = −20.80 dB and beamwidth = 10°). These results can be interpreted by the fact that the scanning is not far enough from broadside case.

As a second example of the angle scanning case, consider now the scanning angle to be far from broadside which is at 130°. The resulting pattern is characterized by

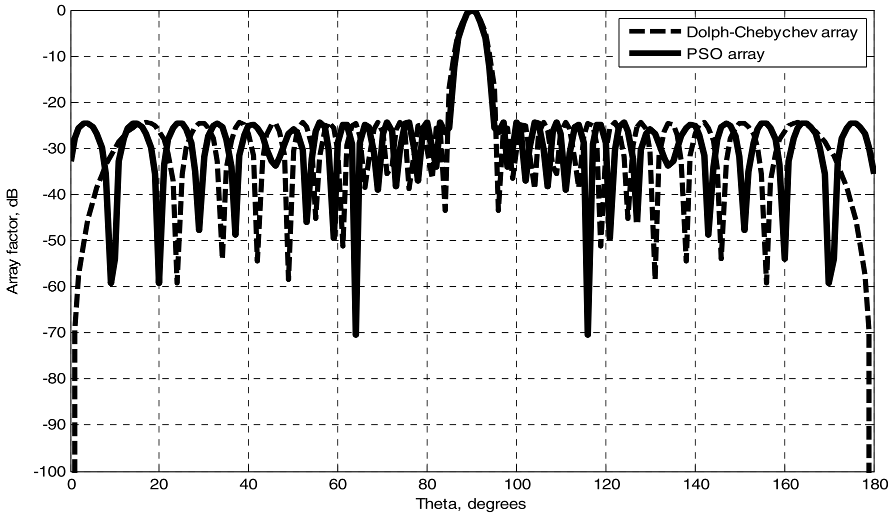

Fig. 9 The position perturbation resulting array factor for the broadside case (24 elements)

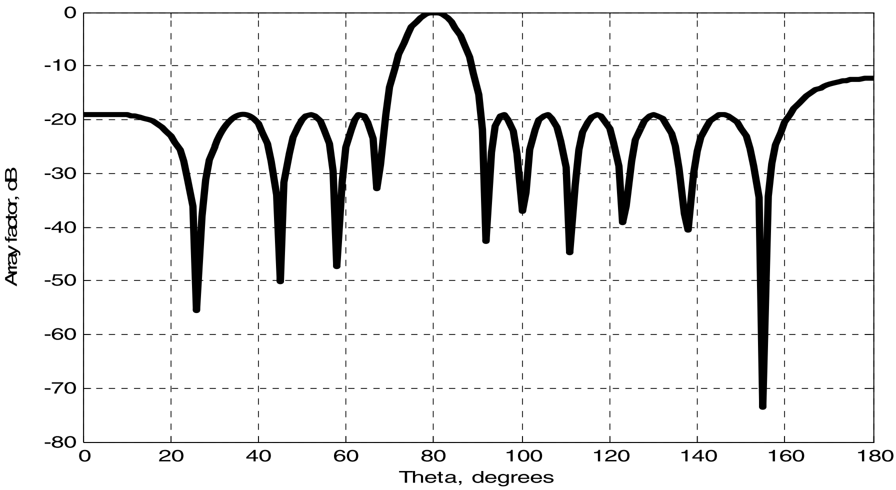

Fig. 10 The perturbation array factor with the main beam steered towards 80° (10 elements)

a SLL of −15.89 dB and 3-dB beam width of 14°. This degradation in performance is again expected as the steering angle is far from the broadside case. The radiation pattern corresponding to this example is shown in Fig. 11.

To sum up, the design by element position perturbation produces designs exhibiting better performance in terms of sidelobe level end directivity. However, one can notice that the array length increased compared to that of the conventional Dolph–Chebychev arrays which can be a problem if the spacing occupied by the antenna array is of great concern.

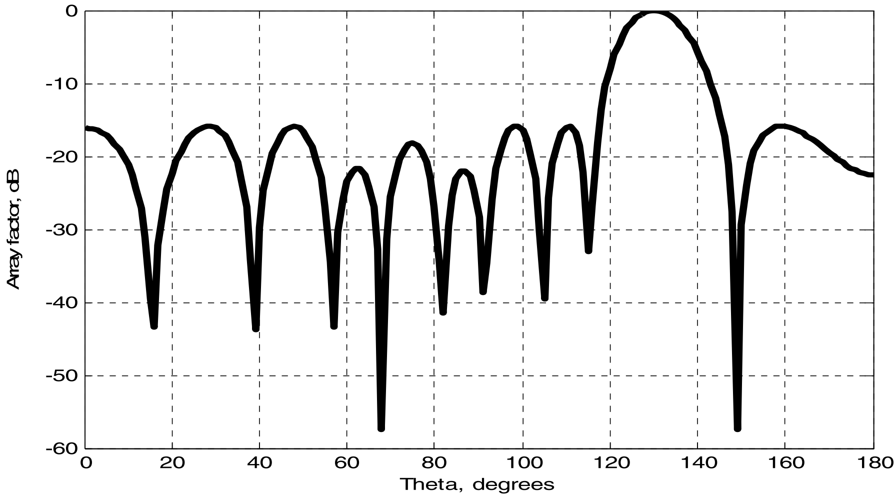

Fig. 11 The perturbation array factor with the main beam steered towards 130° (10 elements)

Table 2 Summary of the resulting PSO array performance characteristics and their Dolph–Chebychev counterparts; second approach

Example	Array	N° of elements	Steering angle (°)	Array length (wavelengths)	SLL (dB)	Beamwidth (°)
1	PSO array	10	0	4.83	–20.80	10
	Dolph–Chebychev array	10	0	4.5	–20.80	11.35
2	PSO array	16	0	8.06	–22.37	6.4
	Dolph–Chebychev array	16	0	7.5	–22.37	7.08
3	PSO array	24	0	12.8	–24.38	4.88
	Dolph–Chebychev array	24	0	11.5	–24.38	5
4	PSO array	10	80	4.82	–19.07	10
5	PSO array	10	130	4.42	–15.89	14

5 Conclusion

The problem of designing equally excited nonuniformly spaced linear array antennas using particle swarm optimization technique has been investigated. Two approaches were considered: one is based on direct element placement and the other on element position perturbation starting from a well-defined geometry. The purpose was to reach the lowest possible sidelobe level. The particle swarm optimization technique, which is a relatively new evolutionary optimization algorithm based on random search principle, has proved to be a powerful and an effective tool for designing the optimum antenna array.

Both design approaches illustrate almost identical results. However, the first approach puts a further constraint on the array length which is a practical choice. The designs satisfy the requirements of wireless communications systems and present an optimal geometry of antenna system that provides minimal signal-of-non-interest (interferers) reduction capability and enhances the signals-of-interest (very high directivity). Consequently, the results can be exploited in practical situations in wireless communication systems employing smart antennas and MIMO systems to enhance the capacity and reduce the level of interference in the communication system.

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