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PAPER

Second-Order Cone Programming Based Joint Design of OFDM Systems

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SUMMARY In this paper, a joint optimal design is investigated for orthogonal frequency division multiplexing (OFDM) systems to reduce peak interference-to-carrier ratio (PICR), out-of-band power (OBP) emissions, and peak-to-average power ratio (PAPR). Two approaches, namely, the phase rotation approach and the constellation extension approach, are proposed to convert this joint design problem into a second order cone programming (SOCP) problem, whose global optimal solution has been shown to exist and can be obtained efficiently. Simulation results are presented to demonstrate efficacy of the proposed algorithms in joint PICR, OBP, and PAPR reduction.

key words: *OFDM, second order cone programming, PICR, OBP, PAPR*

1. Introduction

Doppler frequency shift, mismatched oscillators, or timing synchronization error at receiver cause carrier frequency offset (CFO). A fast fading channel could be time-varying even within one OFDM symbol period, which, together with CFO, may destroy the orthogonality among subcarriers and lead to inter-carrier interference (ICI), thus impair detection performance at receiver. Peak interference-to-carrier ratio (PICR) is defined to represent the effect of ICI [1], and it is desirable to reduce the PICR of OFDM received signals. Due to sidelobes of modulated subcarriers, OFDM systems also suffer from high out-of-band power (OBP) emissions, which result in the need for wide guard band and thus inefficient usage of frequency band. The problem could become severe in some situations, e.g., in an OFDM based overlay cognitive radio system, which fills frequency gaps left by existing legacy systems, high OBP leads to significant interferences to the legacy systems [2]. Therefore, it is always desirable to reduce the OBP of OFDM signals. In addition, high peak-to-average power ratio (PAPR) is one of the major problems in orthogonal frequency division multiplexing (OFDM) systems, which forces nonlinear devices used in the system, such as power amplifiers (PAs), to leave a large backoff capacity to ensure that the devices always operate in their linear response regions to avoid signal distortion. However, this often leads to inefficient operation of these devices and degradation in system performance. PAPR reduction problem has been addressed extensively in literature [3].

Joint design of OFDM systems has been studied in literature in the past few years. The joint PAPR-OBP reduction was investigated in [4], and joint PAPR-PICR reduction was studied in [5]. However, most of previous studies consider PICR, OBP, or PAPR reduction problem separately, which leads that when only one of the three problems is considered, the performance of the other two problems of the same system can be quite bad. In this paper, we consider a joint OFDM design problem to reduce PICR, OBP, and PAPR simultaneously. Based on the phase rotation and the constellation extension approaches, the joint design problem is converted into a second order cone programming (SOCP) problem [6], which is a subclass of well-structured convex programming problems whose global optimal solution can be obtained using efficient interior-point algorithms in polynomial-time complexity [7]. Simulation results show that the proposed schemes are effective in joint reduction of PICR, OBP, and PAPR.

2. System Model

We consider an OFDM system of N subcarriers, each of which is independently modulated using either M-ary phase shift keying (MPSK) or M-ary quadrature amplitude modulation (MQAM) to transmit N data sub-streams in parallel. On both sides of the N subcarriers, $M/2$ cancellation subcarriers are inserted. Thus the subcarrier indices are denoted as $n = 0, \ldots, M/2-1, M/2, \ldots, M/2+N-1, M/2+N, \ldots, M+N-1$. Denoting the modulated symbol on the ith subcarrier by X_i ($i = M/2, \ldots, M/2+N-1$), we assume $\bar{\mathbf{X}} = [X_{M/2} \cdots X_{M/2+N-1}]^T$ as the information symbol vector and $\mathbf{X} = \left[\underbrace{0 \cdots 0}_{M/2}\ \bar{\mathbf{X}}^T\ \underbrace{0 \cdots 0}_{M/2}\right]^T$ as the *expanded information symbol vector*. Denoting $\mathbf{Y} = [Y_0 \cdots Y_{M/2-1}\ Y_{M/2} \cdots Y_{M/2+N-1}\ Y_{M/2+N} \cdots Y_{M+N-1}]^T$ as the *modification vector*, the actual transmitted symbol vector is thus $\mathbf{X} + \mathbf{Y}$. For the convenience of discussion, we define $M/2 = N_0$, $M/2 + N = N_1$, and $M + N = N_2$. By applying an inverse discrete Fourier transform (IDFT) upon $X_i + Y_i$ ($i = 0, \ldots, N_2 - 1$), we generate the time-domain transmitted signal as

$$\mathbf{x} + \mathbf{y} = \mathbf{F}(\mathbf{X} + \mathbf{Y}) \tag{1}$$

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where $\mathbf{x}+\mathbf{y} = [x_0+y_0 \ \cdots \ x_{N_2-1}+y_{N_2-1}]^T$ with x_n+y_n being the time-domain signal at the nth sampling instant, and $\mathbf{F}$ is the $N_2 \times N_2$ IDFT matrix with the $\{n,i\}$th element being $F(n,i) = \frac{1}{\sqrt{N_2}} e^{j2\pi ni/N_2}$ for $n, i = 0, \ldots, N_2-1$ and $j^2 = -1$.

A doubly frequency selective fading channel model is adopted in this paper [8]. Thus, a wide sense stationary uncorrelated scattering (WSSUS) channel is considered, whose impulse response is given as $h(t;\tau) = \sum_{d=0}^{\mu} h(t;\tau_d)\,\delta(\tau-\tau_d)$ where τ_d is the delay of the dth path with $\tau_0 < \tau_1 < \cdots < \tau_\mu$ and $\delta(\cdot)$ represents an impulse function. For a multipath fading channel, $h(t;\tau_d)$ is a complex random process. A discrete version of the WSSUS channel can be modeled as a tapped delay line filter with random taps $h(n,l) = h(nT_c; lT_c)$ where $h(n,l)$ denotes the channel coefficient for the lth tap at the nth sampling instant and the delay between two taps is T_c with $T_c = T_s/N$ and T_s being the OFDM symbol duration. For the sake of simplicity, inter-symbol interference (ISI) is assumed not in presence in the received signal, which can be achieved easily by inserting a cyclic prefix (CP) [9]. The discrete demodulated signal at the receiver can be expressed as

$$\mathbf{z} = \mathbf{D}_\varepsilon \mathbf{H}(\mathbf{x}+\mathbf{y}) + \mathbf{n} \tag{2}$$

where $\mathbf{z} = [z_0 \ z_1 \ \cdots \ z_{N_2-1}]^T$ denotes the time-domain received signal, $\mathbf{n} = [v_0 \ v_1 \ \cdots \ v_{N_2-1}]^T$ denotes the complex additive white Gaussian noise (AWGN) noise with zero mean and covariance matrix $2\sigma^2\mathbf{I}$ with $\mathbf{I}$ being the identity matrix, $\mathbf{D}_\varepsilon = \mathrm{diag}\{1, \ e^{j2\pi\varepsilon/N_2}, \ \ldots, e^{j2\pi(N_2-1)\varepsilon/N_2}\}$, ε is the normalized residual frequency offset which is defined as $\varepsilon = \Delta f \cdot T_s$ with Δf being the residual frequency offset of the local carrier frequency at the receiver above the correct carrier frequency [5], and $\mathbf{H}$ is the channel matrix. Note that when $h(t;\tau)$ remains constant within one OFDM symbol duration, we have $h(0,l) = h(1,l) = \cdots = h(N_2-1,l)$ $(l = 0,\ldots,\mu)$ and the channel is called *quasi-static (QS)*. Otherwise, when $h(t;\tau)$ changes within one OFDM symbol period, the channel becomes *fast time-varying (FTV)*. After removing the CP, the received signal $\mathbf{z}$ is passed through a discrete Fourier transform (DFT). Then we have

$$\mathbf{Z} = \mathbf{F}^H\mathbf{z} = \mathbf{A}_\varepsilon(\mathbf{X}+\mathbf{Y}) + \mathbf{N}_0 \tag{3}$$

where $\mathbf{A}_\varepsilon = \mathbf{F}_\varepsilon^H\mathbf{H}\mathbf{F}$ with $\mathbf{F}_\varepsilon^H = \mathbf{F}^H\mathbf{D}_\varepsilon$ and $\mathbf{N}_0 = \mathbf{F}^H\mathbf{n}$.

3. Problems for PICR, OBP, and PAPR Reduction

3.1 PICR Reduction

From (3), the received signal on the kth subcarrier can be expressed as

$$Z_k = (X_k+Y_k)\,A_\varepsilon(k,k) + \sum_{l=0,l\neq k}^{N_2-1} (X_l+Y_l)\,A_\varepsilon(k,l) + N_{0,k} \qquad k = 0,\ldots,N_2-1 \tag{4}$$

where $A_\varepsilon(k,l)$ denotes the $\{k,l\}$th elements of $\mathbf{A}_\varepsilon$ and $N_{0,k}$ denotes the kth element of $\mathbf{N}_0$. In (4), the first term represents the desired signal, the second term represents the ICI which will be denoted as I_k, and the last term is the noise. Note that only when $\varepsilon = 0$ and the channel is QS, $\mathbf{A}_\varepsilon$ is diagonal and $I_k = 0$. That is, ICI is caused by the frequency offset at the receiver or the FTV channel, or more likely, by both. PICR is defined in [1] as

$$\begin{aligned}\mathrm{PICR} &= \max_{N_0\le k\le N_1-1} \frac{|I_k|^2}{|(X_k+Y_k)A_\varepsilon(k,k)|^2} \\ &= \max_{N_0\le k\le N_1-1} \frac{\left|\mathbf{b}_k^T(\mathbf{X}+\mathbf{Y})\right|^2}{|(X_k+Y_k)A_\varepsilon(k,k)|^2}\end{aligned} \tag{5}$$

where $\mathbf{b}_k^T$ is the kth row of $\mathbf{A}_\varepsilon^0$ which is obtained by replacing all diagonal elements of $\mathbf{A}_\varepsilon$ in (3) by zeros. PICR specifies the worst-case ICI effect on any information subcarrier with $k = N_0,\ldots,N_1-1$ since only the symbols on these subcarriers need to be detected at receiver. It has been shown that we can improve bit-error rate (BER) performance of OFDM systems by reducing PICR or by minimizing (5) [1], [10].

3.2 OBP Reduction

By using the IDFT to transform OFDM signals from frequency domain to time domain, a rectangular pulse shaping filter is implicitly applied [2]. Therefore, the spectrum of the nth $(n = 0,\cdots,N_2-1)$ subcarrier at normalized frequency $\epsilon = fT_s$ is given by $s_n(\epsilon) = (X_n+Y_n)\,\mathrm{sinc}(\epsilon-\epsilon_n)$ where $\epsilon_n = f_nT_s$ is the normalized center frequency of the nth subcarrier. The spectrum of the transmitted OFDM symbol $s(\epsilon)$ is the superposition of the spectra of all individual subcarriers $s_n(\epsilon)$ $(n = 0,\ldots,N_2-1)$. Due to the characteristics of the sinc function, it is usually sufficient to represent $s(\epsilon)$ by a number of sampling values which span a frequency range over a few sidelobes for suppression of out-of-band power emissions. To reduce complexity, only one sample in the middle of each sidelobe is considered. In our discussion, K_1 and K_2 samples are considered on the left and right hand side sidelobes respectively, with $K_1+K_2 = K$. That is, $s_k = s(\epsilon)|_{\epsilon=w_k}$ with

$$w_k = \begin{cases} -(1.5+K_1-1-k) - f_{N_2}T_s/2 \\ \qquad \text{for } k = 0,\cdots,K_1-1 \\ f_{N_2/2-1}T_s + 1.5 - K_1 + k \\ \qquad \text{for } k = K_1,\cdots,K-1. \end{cases} \tag{6}$$

The parameter OBP is defined as

$$\mathrm{OBP} = \|\mathbf{s}\|_2^2/K = \|\mathbf{G}(\mathbf{X}+\mathbf{Y})\|_2^2/K \tag{7}$$

where $\mathbf{s} = [s_0 \ s_1 \ \cdots \ s_{K-1}]^T$ and $\mathbf{G}$ is a $K \times N_2$ real matrix with the (k,n)th element being $G(k,n) = \mathrm{sinc}(w_k-\epsilon_n)$. That is, OBP is the out-of-band power averaged over the sidelobes of interest. We can suppress out-of-band power emissions by minimizing (7).

3.3 PAPR Reduction

Due to the statistical independence of the modulated signal X_i+Y_i $(i = 0,\cdots,N_2-1)$, the corresponding complex-valued time-domain samples of OFDM signal x_n+y_n $(n =$

$0, \cdots, N_2 - 1$) are approximately complex Gaussian distributed if N_2 is large and the central limit theorem is applicable [9]. The magnitude of the OFDM signal is thus approximately Rayleigh distributed. This results that the PAPR of signal $\mathbf{x} + \mathbf{y}$, which is defined as

$$\mathrm{PAPR} = \frac{\|\mathbf{x} + \mathbf{y}\|_\infty^2}{E\left\{\|\mathbf{x} + \mathbf{y}\|_2^2 / N_2\right\}} \tag{8}$$

can be very high. In (8) $E\{\cdot\}$ denotes expectation, and $\|\cdot\|_2$ and $\|\cdot\|_\infty$ denote L_2 and L_∞ norms of a vector, respectively. We can achieve PAPR reduction by minimizing (8).

4. Joint Design Using SOCP

In most practical OFDM system design, the value of PICR is related to BER performance and needs to be reduced as much as it can be. Out-of-band emissions are usually restricted by spectrum mask such that the power of the sidelobe of OFDM signals generally needs to be lower than a threshold. PAPR of the OFDM signal is usually required to be lower than a specific threshold to ensure that nonlinear devices in the system can almost always operate in their linear response regions [11]. In addition, by adopting modification vector, part of the total transmission power is not used for information data transmission. In order to efficiently utilize the transmission power, the increment in the transmission power after adding the modification vector needs also to be limited. Based on the above discussion, a joint PICR-OBP-PAPR design problem can be formulated as

$$\text{minimize PICR} \tag{9a}$$
$$\text{subject to:} \quad \mathrm{OBP} \le \alpha \tag{9b}$$
$$\mathrm{PAPR} \le \tau \tag{9c}$$
$$\|\mathbf{X} + \mathbf{Y}\|_2^2 - \|\mathbf{X}\|_2^2 \le \beta \|\mathbf{X}\|_2^2, \quad 0 < \beta < 1 \tag{9d}$$

where α, τ, and β are constant parameters, with α being determined by out-of-band emission restriction, τ bing determined by the linear dynamic range of the nonlinear devices used in the system, and β being determined by power utilization efficiency requirement.

4.1 Algorithm Based on Phase Rotation Approach

As shown in the complex plane geometric representation in Fig. 1, in the phase rotation approach, the actual transmitted symbol on the kth ($k = N_0, \ldots, N_1 - 1$) subcarrier, which is $X_k + Y_k$, can be considered as obtained by rotating the original information symbol X_k by a rotation phasor $r_k = \frac{X_k + Y_k}{X_k} = 1 + \frac{Y_k}{X_k}$ with $\|r_k\|_2 = 1$ [3]. Thus, using the phase rotation approach, the joint design problem in (9) can be expressed as

$$\min_{\mathbf{Y}} \max_{k \in [N_0, N_1 - 1]} \frac{\left\|\mathbf{b}_k^T (\mathbf{X} + \mathbf{Y})\right\|_2}{\|A_\varepsilon(k,k)(X_k + Y_k)\|_2} \tag{10a}$$

subject to:

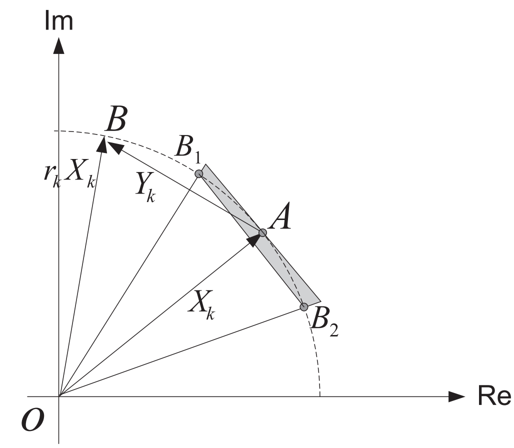

Fig. 1 Phase rotation approach and relaxed feasible region.

$$\left\|1 + \frac{Y_k}{X_k}\right\|_2 = 1 \qquad k = N_0, \cdots, N_1 - 1 \tag{10b}$$
$$\|\mathbf{G}(\mathbf{X} + \mathbf{Y})\|_2 \le \sqrt{K\alpha} \tag{10c}$$
$$\left\|\mathbf{f}_k^T (\mathbf{X} + \mathbf{Y})\right\|_2 \le \left[\frac{\tau}{N_2} \sum_{i=0}^{N_2 - 1} E\left\{\|X_i + Y_i\|_2^2\right\}\right]^{1/2} \quad k = 0, \ldots, N_2 - 1 \tag{10d}$$
$$\|\mathbf{X} + \mathbf{Y}\|_2 \le \sqrt{\beta + 1}\, \|\mathbf{X}\|_2 \tag{10e}$$

where $\mathbf{f}_k^T$ denotes the kth row of $\mathbf{F}$. Note that (10) is a nonconvex optimization problem whose solution is generally very difficult to obtain. Also note that r_k needs to be quantized to be transmitted to the receiver, and thus the number of quantization levels should in general be quite limited to reduce the associated overhead. Therefore, in order to solve the problem in (10), we restrict the quantization levels as $-\theta$, 0, and θ with $\theta \in (0, \pi/2)$ and constrain the rotation angle within the range of $[-\theta, \theta]$. As shown in Fig. 1, for small θ, (10b) can be relaxed as

$$\cos\theta \le \Re(1 + Y_k/X_k) \le 1 \tag{11a}$$
$$-\tan\theta \le \frac{\Im(1 + Y_k/X_k)}{\Re(1 + Y_k/X_k)} \le \tan\theta \quad k = N_0, \ldots, N_1 - 1. \tag{11b}$$

so that the tip of the vector corresponding to $X_k + Y_k$ locates in the shaded convex feasible region which is a trapezoid with hight $(1 - \cos\theta)$. In Fig. 1, $\angle B_1OA = \angle B_2OA = \theta$. In (11), $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary components of a complex number, respectively. In addition, as shown in Fig. 1, for small θ, we have $\|1 + Y_k/X_k\|_2 \approx 1$, i.e., $\|X_k + Y_k\|_2 \approx \|X_k\|_2$ for those Y_k with the tip of the vector associated with $X_k + Y_k$ locating in the shaded convex feasible region. Therefore, when θ is small, we can simplify (10a) and (10d) using this approximation.

Let θ be a small angle chosen from $(0, \pi/2)$, and denote $\mathbf{X} = \mathbf{X}_r + j\mathbf{X}_i$, $\bar{\mathbf{X}} = \bar{\mathbf{X}}_r + j\bar{\mathbf{X}}_i$, $\mathbf{Y} = \mathbf{Y}_r + j\mathbf{Y}_i$, $\mathbf{b}_k = \mathbf{b}_{r,k} + j\mathbf{b}_{i,k}$, $\mathbf{f}_k = \mathbf{f}_{r,k} + j\mathbf{f}_{i,k}$, and $\mathbf{g} = \mathbf{G}\mathbf{X} = \mathbf{g}_r + j\mathbf{g}_i$. Based on the above discussion and after some manipulations, the joint design problem in (10) can be relaxed into

$$\hat{\mathbf{D}}_R = \begin{bmatrix} \mathbf{0}_{N,\frac{M}{2}} & -\text{diag}\left\{\bar{\mathbf{X}}_r\right\} & \mathbf{0}_{N,M} & -\text{diag}\left\{\bar{\mathbf{X}}_i\right\} & \mathbf{0}_{N,\frac{M}{2}} \\ \mathbf{0}_{N,\frac{M}{2}} & \text{diag}\left\{\bar{\mathbf{X}}_r\right\} & \mathbf{0}_{N,M} & \text{diag}\left\{\bar{\mathbf{X}}_i\right\} & \mathbf{0}_{N,\frac{M}{2}} \\ \mathbf{0}_{N,\frac{M}{2}} & \text{diag}\left\{-\bar{\mathbf{X}}_i + \tan\theta\cdot\bar{\mathbf{X}}_r\right\} & \mathbf{0}_{N,M} & \text{diag}\left\{\bar{\mathbf{X}}_r + \tan\theta\cdot\bar{\mathbf{X}}_i\right\} & \mathbf{0}_{N,\frac{M}{2}} \\ \mathbf{0}_{N,\frac{M}{2}} & \text{diag}\left\{\bar{\mathbf{X}}_i + \tan\theta\cdot\bar{\mathbf{X}}_r\right\} & \mathbf{0}_{N,M} & \text{diag}\left\{-\bar{\mathbf{X}}_r + \tan\theta\cdot\bar{\mathbf{X}}_i\right\} & \mathbf{0}_{N,\frac{M}{2}} \end{bmatrix}$$

$$\text{minimize}_{\tilde{\mathbf{Y}}}\xi = \mathbf{d}^T\tilde{\mathbf{Y}} \quad (12a)$$

subject to:

$$\left\|\hat{\mathbf{s}}_k + \tilde{\mathbf{B}}_k^T\tilde{\mathbf{Y}}\right\|_2 \le \mathbf{c}_{Rk}^T\tilde{\mathbf{Y}} \quad k = N_0,\ldots,N_1-1 \quad (12b)$$

$$\left\|\hat{\mathbf{g}} + \tilde{\mathbf{G}}\tilde{\mathbf{Y}}\right\|_2 \le \sqrt{K\alpha} \quad (12c)$$

$$\left\|\hat{\mathbf{x}}_k + \tilde{\mathbf{P}}_k^T\tilde{\mathbf{Y}}\right\|_2 \le \sqrt{\tau\mathcal{E}} \quad k = 0,\ldots,N_2-1 \quad (12d)$$

$$\left\|\hat{\mathbf{X}} + \tilde{\mathbf{I}}\tilde{\mathbf{Y}}\right\|_2 \le \sqrt{1+\beta}\left\|\hat{\mathbf{X}}\right\|_2 \quad 0<\beta<1 \quad (12e)$$

$$\tilde{\mathbf{D}}_R\tilde{\mathbf{Y}} + \hat{\mathbf{f}} \ge \mathbf{0} \quad (12f)$$

where $\mathcal{E}$ denotes the average power of the original information symbols, $\mathbf{d} = [\mathbf{0}_{1,2N_2}\ 1]^T$, $\tilde{\mathbf{Y}} = \left[\hat{\mathbf{Y}}\ \xi\right]^T$, $\hat{\mathbf{Y}} = \left[\mathbf{Y}_r^T\ \mathbf{Y}_i^T\right]^T$, $\hat{\mathbf{s}}_k = \left[\Re\left\{\mathbf{b}_k^T\mathbf{X}\right\}\ \Im\left\{\mathbf{b}_k^T\mathbf{X}\right\}\right]^T$, $\tilde{\mathbf{B}}_k^T = \left[\hat{\mathbf{B}}_k^T\ \mathbf{0}_{2,1}\right]$, $\mathbf{c}_{Rk} = [\mathbf{0}_{1,2N_2}\ \|A_\varepsilon(k,k)X_k\|_2]^T$, $\hat{\mathbf{x}}_k = \left[\Re\left\{\mathbf{f}_k^T\mathbf{X}\right\}\ \Im\left\{\mathbf{f}_k^T\mathbf{X}\right\}\right]^T$, $\tilde{\mathbf{P}}_k^T = \left[\hat{\mathbf{P}}_k^T\ \mathbf{0}_{2,1}\right]$, $\hat{\mathbf{g}} = \left[\mathbf{g}_r^T\ \mathbf{g}_i^T\right]^T$, $\tilde{\mathbf{G}} = \left[\hat{\mathbf{G}}\ \mathbf{0}_{2K,1}\right]$, $\hat{\mathbf{X}} = \left[\mathbf{X}_r^T\ \mathbf{X}_i^T\right]^T$, $\tilde{\mathbf{I}} = [\mathbf{I}_{2N_2,2N_2}\ \mathbf{0}_{2N_2,1}]$, $\tilde{\mathbf{D}}_R = \left[\hat{\mathbf{D}}_R\ \mathbf{0}_{4N,1}\right]$,

$$\hat{\mathbf{B}}_k^T = \begin{bmatrix} \mathbf{b}_{rk}^T & -\mathbf{b}_{ik}^T \\ \mathbf{b}_{ik}^T & \mathbf{b}_{rk}^T \end{bmatrix},\ \hat{\mathbf{P}}_k^T = \begin{bmatrix} \mathbf{f}_{rk}^T & -\mathbf{f}_{ik}^T \\ \mathbf{f}_{ik}^T & \mathbf{f}_{rk}^T \end{bmatrix},$$

$$\hat{\mathbf{G}} = \begin{bmatrix} \mathbf{G} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} \end{bmatrix}$$

$$\hat{\mathbf{f}} = \begin{bmatrix} \mathbf{0}_{N,1} \\ \text{diag}\left\{\left\|\bar{X}_0\right\|_2^2,\cdots,\left\|\bar{X}_{N-1}\right\|_2^2\right\}(1-\cos\theta)\,\mathbf{E}_{N,1} \\ \text{diag}\left\{\left\|\bar{X}_0\right\|_2^2,\cdots,\left\|\bar{X}_{N-1}\right\|_2^2\right\}\tan\theta\cdot\mathbf{E}_{N,1} \\ \text{diag}\left\{\left\|\bar{X}_0\right\|_2^2,\cdots,\left\|\bar{X}_{N-1}\right\|_2^2\right\}\tan\theta\cdot\mathbf{E}_{N,1} \end{bmatrix}$$

and $\hat{\mathbf{D}}_R$ is put at the top of this page, with $\mathbf{0}_{n,m}$ and $\mathbf{E}_{n,m}$ being $n\times m$ matrixes with all elements being zeros and ones respectively.

The problem in (12) is a standard SOCP problem with a linear objective function and a set of second-order cone and linear equality constraints of $2N_2+1$ variables [6], which can be solved efficiently by the well established interior-point method in polynomial-time complexity [7]. Denoting the solution to the problem in (12) as Y_k^*, the actual transmitted symbol on the kth subcarrier is Y_k^* for $k = 0,\ldots,N_0-1,N_1,\ldots,N_2-1$ and $X_k e^{j\delta_k^*}$ for $k = N_0,\ldots,N_1-1$, with $\delta_k^* = \arg\text{minimize}_{\delta_k\in\{-\theta,0,\theta\}}\left|\arg\left(1+\frac{Y_k^*}{X_k}\right)-\delta_k\right|$ being the quantized rotation angle which needs to be sent to the receiver as side information.

It is noted that sometimes no feasible solution can be found for the optimization problem in (12) for some OFDM symbols under the given values of α, τ, and β. In this situation, for different applications, we may use different approaches to find a solution. For example, for the OFDM based overlay cognitive radio system, we can identify the solution as the one that minimizes OBP only. In this way, OBP is minimized while the values of PICR and PAPR may be large. From the simulation results obtained, since the phenomenon that no feasible solution can be found for the problem in (12) happens quite infrequently when the parameters of the system are set appropriately, the performance of PICR and PAPR reductions is not affected much. Thus, the above proposed phase rotation based algorithm can be used as a practical design tool for OFDM systems.

4.2 Algorithm Based on Constellation Extension Approach

In the constellation extension approach [3], the information symbol X_k is replaced by a new symbol corresponding to a new constellation, which is chosen to optimize the design objectives. In order to avoid transmitting side information and to maintain the BER performance of the original signal, each new constellation must be located in a certain feasible region so that the receiver is able to recover the original information symbol without any side information, and the smallest distance of any two new constellations is greater than or at least equal to the smallest distance of any two original constellations. For instance, the feasible region of

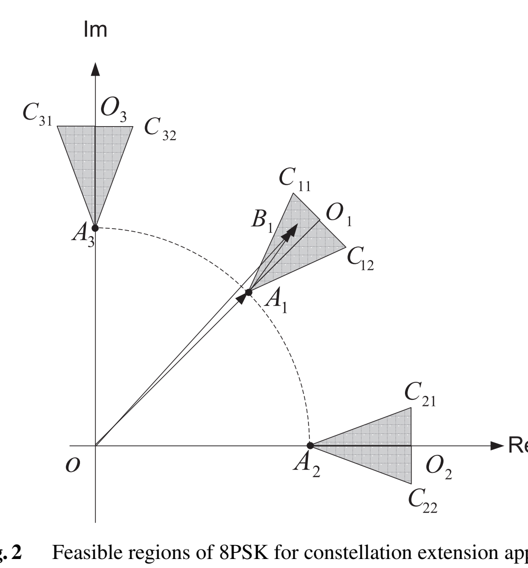

Fig. 2 Feasible regions of 8PSK for constellation extension approach.

$$\tilde{\mathbf{D}}_E^p = \begin{bmatrix} \mathbf{0}_{N,\frac{M}{2}} & \text{diag}\left\{\bar{\mathbf{X}}_r\right\} & \mathbf{0}_{N,M} & \text{diag}\left\{\bar{\mathbf{X}}_i\right\} & \mathbf{0}_{N,\frac{M}{2}} & \mathbf{0}_{N,1} \\ \mathbf{0}_{N,\frac{M}{2}} & \text{diag}\left\{\tan\theta\cdot\bar{\mathbf{X}}_r - \bar{\mathbf{X}}_i\right\} & \mathbf{0}_{N,M} & \text{diag}\left\{\tan\theta\cdot\bar{\mathbf{X}}_i + \bar{\mathbf{X}}_r\right\} & \mathbf{0}_{N,\frac{M}{2}} & \mathbf{0}_{N,1} \\ \mathbf{0}_{N,\frac{M}{2}} & \text{diag}\left\{\tan\theta\cdot\bar{\mathbf{X}}_r + \bar{\mathbf{X}}_i\right\} & \mathbf{0}_{N,M} & \text{diag}\left\{\tan\theta\cdot\bar{\mathbf{X}}_i - \bar{\mathbf{X}}_r\right\} & \mathbf{0}_{N,\frac{M}{2}} & \mathbf{0}_{N,1} \end{bmatrix}.$$

8PSK modulation is shown as the shaded areas in Fig. 2 with $\angle C_{11}A_1O_1 = \angle C_{12}A_1O_1 = \angle C_{21}A_2O_2 = \angle C_{22}A_2O_2 = \cdots = \theta$. Similar to the derivation in the phase rotation approach, the objective function of the joint design in (9a) can be formulated as

$$\text{minimize } \xi = \mathbf{d}^T\tilde{\mathbf{Y}} \tag{13a}$$

subject to:

$$\left\|\hat{\mathbf{s}}_k + \tilde{\mathbf{B}}_k^T\tilde{\mathbf{Y}}\right\|_2 \le \|A_\varepsilon(k,k)\|_2 \cdot \xi \|(X_k + Y_k)\|_2 \quad k = N_0,\ldots,N_1 - 1 \tag{13b}$$

Note that the constraint (13b) is non-convex, and the constellation extension based joint design problem is a non-convex optimization problem. In order to solve this problem, by using Taylor extension, $z = \xi\|(X_k + Y_k)\|_2 = \xi\sqrt{(X_{rk} + Y_{rk})^2 + (X_{ik} + Y_{ik})^2}$ can be linearized at $(\xi_0, Y_{k,0} = Y_{rk,0} + jY_{ik,0})$ as

$$\begin{aligned} z \approx & -\frac{\xi_0}{\left\|X_k + Y_{k,0}\right\|_2}\left[(X_{rk} + Y_{rk,0})Y_{rk,0}\right. \\ & \left. + (X_{ik} + Y_{ik,0})Y_{ik,0}\right] + \frac{\xi_0(X_{rk} + Y_{rk,0})}{\left\|X_k + Y_{k,0}\right\|_2}Y_{rk} \\ & + \frac{\xi_0(X_{ik} + Y_{ik,0})}{\left\|X_k + Y_{k,0}\right\|_2}Y_{ik} + \left\|X_k + Y_{k,0}\right\|_2\xi \end{aligned} \tag{14}$$

Then the nonconvex constraint in (13b) can be relaxed into a convex one as

$$\left\|\hat{\mathbf{s}}_k + \tilde{\mathbf{B}}_k^T\tilde{\mathbf{Y}}\right\|_2 \le \mathbf{c}_{Ek}^T\tilde{\mathbf{Y}} + d_{Ek} \quad k = N_0,\ldots,N_1 - 1 \tag{15}$$

where $d_{Ek} = -\xi_0\left\|\frac{A_\varepsilon(k,k)}{X_k + Y_{k,0}}\right\|_2\left[(X_{rk} + Y_{rk,0})Y_{rk,0} + (X_{ik} + Y_{ik,0})Y_{ik,0}\right]$ and $\mathbf{c}_{Ek}^T$ is the kth row of the matrix

$$\mathbf{R} = \left[\mathbf{0}_{N,\frac{M}{2}} \quad \xi_0\cdot\text{diag}\left\{\bar{\mathbf{X}}_r^L\right\} \quad \mathbf{0}_{N,M} \quad \xi_0\cdot\text{diag}\left\{\bar{\mathbf{X}}_i^L\right\} \quad \mathbf{0}_{N,\frac{M}{2}} \quad \bar{\mathbf{L}}\right] \tag{16}$$

with $\bar{\mathbf{X}}_r^L = \text{diag}\{\mathbf{L}\}\left(\bar{\mathbf{X}}_r + \bar{\mathbf{Y}}_{r,0}\right)$, $\bar{\mathbf{X}}_i^L = \text{diag}\{\mathbf{L}\}\cdot\left(\bar{\mathbf{X}}_i + \bar{\mathbf{Y}}_{i,0}\right)$, the kth ($k = N_0,\ldots,N_1 - 1$) elements of vectors $\bar{\mathbf{L}}$ and $\mathbf{L}$ being $\left\|A_\varepsilon(k,k)(X_k + Y_{k,0})\right\|_2$ and $\left\|\frac{A_\varepsilon(k,k)}{X_k + Y_{k,0}}\right\|_2$ respectively, and $\bar{\mathbf{Y}}_0 = \bar{\mathbf{Y}}_{r,0} + j\bar{\mathbf{Y}}_{i,0} = \left[Y_{N_0,0} \cdots Y_{N_1-1,0}\right]^T$.

In addition, as indicated in literature, the increase of average power in the optimally modified symbol using the constellation extension approach is fairly moderate. So we will replace the average power by $\mathcal{E}$. Therefore, using the constellation extension approach, the joint design problem in (9) can be relaxed as

$$\text{minimize}_{\tilde{\mathbf{Y}}} \ \xi = \mathbf{d}^T\tilde{\mathbf{Y}} \tag{17a}$$

subject to:

$$\left\|\hat{\mathbf{s}}_k + \tilde{\mathbf{B}}_k^T\tilde{\mathbf{Y}}\right\|_2 \le \mathbf{c}_{Ek}^T\tilde{\mathbf{Y}} + d_{Ek} \quad k = N_0,\ldots,N_1 - 1 \tag{17b}$$

(17c)–(17e): same as (12c)–(12e)

$$X_k + Y_k \text{ be feasible} \quad k = N_0,\ldots,N_1 - 1 \tag{17f}$$

We have shown that for both MPSK and MQAM cases, the feasible region constraint in (17f) can be expressed in the format of linear constraint. Thus, similar to (12), the problem in (17) is also a standard SOCP problem which can be solved in polynomial-time complexity and the proposed constellation extension based algorithm can also be used as a practical design tool for OFDM systems. In the following, we only present the MPSK case for conciseness. From Fig. 2, the feasible region of MPSK modulation for the constellation extension approach can be expressed as $-\theta \le \arg(Y_k/X_k) \le \theta$ and $\Re(1 + Y_k/X_k) \ge 1$ ($k = N_0,\ldots,N_1 - 1$), where $\theta = \pi/M$. Similar to the derivation of (12f), we get the matrix form of the feasible region constraint in (17f) as

$$\tilde{\mathbf{D}}_E^p\tilde{\mathbf{Y}} \ge \mathbf{0} \tag{18}$$

where $\tilde{\mathbf{D}}_E^p$ is put at the top of previous page.

5. Simulation Results

Computer simulations were conducted to evaluate the efficacy of the proposed algorithms in joint PICR, OBP, and PAPR design. We considered an OFDM system in an FTV frequency-selective Rayleigh fading channel with $\mu = 10$ and normalized total power in multipaths. $N = 64$, $M = 4$, $K_1 = K_2 = 16$, $\varepsilon = 0.1$, $\tau = 9$ dB and 16QAM were adopted in our simulations. θ was set as $\pi/8$ for the phase rotation approach based algorithm, and 10^4 OFDM symbols were used to obtain the results shown in this paper.

Figures 3, 4, and 5 show the complimentary cumulative density function (CCDFs) of PICR, OBP, and PAPR of OFDM symbols using the proposed phase rotation approach based joint design algorithm with $\beta = 1/30$ and $\alpha = 0.6/32$. For comparison, the CCDFs of PICR, OBP, and PAPR of OFDM symbols obtained by minimizing these three parameters alone with power utilization efficiency constraint, as well as the results with quantized solutions which are labeled as "with Q" on the legends, are also included in these figures. From these results, it can be seen that the effect of quantization on the results is quite minor. Similar results were obtained for the cases using the proposed constellation extension approach based joint design algorithm with

Table 1 Comparison of joint design algorithms.

	Original	Phase Rotation (PR)		Reduction by	Constellation	Reduction
		Optimal	Quantized	Quantized PR	Extension (CE)	by CE
PICR	69.8 dB	52.9 dB	53.3 dB	16.5 dB	62.0 dB	7.8 dB
OBP	−26.0 dBW	−42.7 dBW	−42.5 dBW	16.5 dB	−36.1 dBW	10.1 dB
PAPR	10.3 dB	9.5 dB	9.6 dB	0.7 dB	9.6 dB	0.7 dB

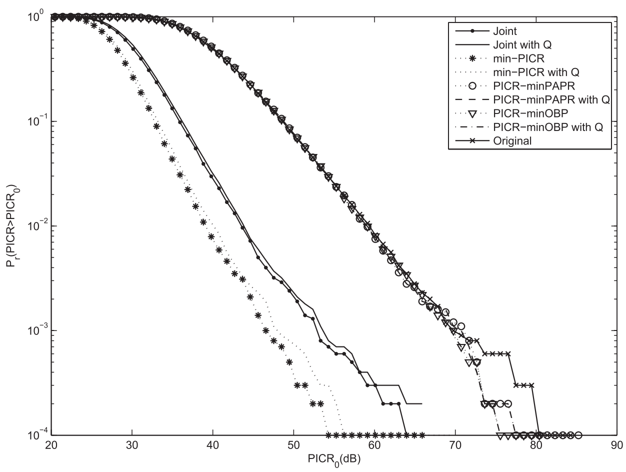

Fig. 3 PICR performance of phase rotation based algorithms for OFDM systems with 16QAM.

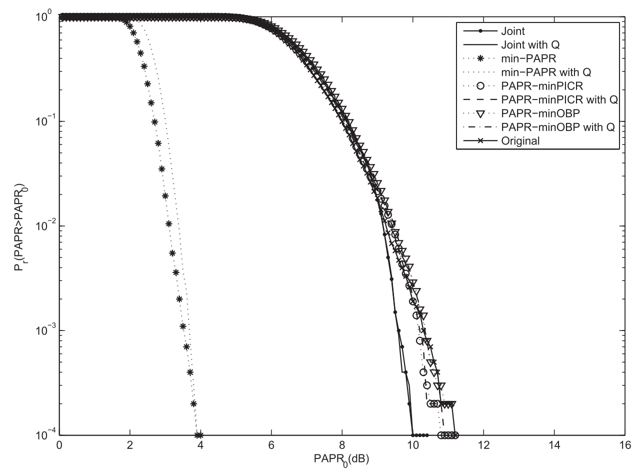

Fig. 5 PAPR performance of phase rotation based algorithms for OFDM systems with 16QAM.

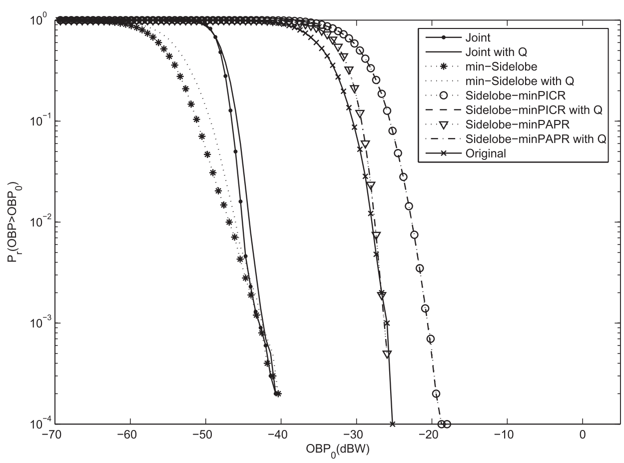

Fig. 4 OBP emission performance of phase rotation based algorithms for OFDM systems with 16QAM.

$\beta = 1/4$ and $\alpha = 0.6/32$. Table 1 on next page lists the PICR, OBP, and PAPR thresholds which only one out of 10^3 transmitted OFDM symbols has PICR, OBP, and PAPR exceeding respectively. From the discussion in Sect. 4, it can be seen that the proposed algorithms can be easily extended to the cases where other modulation schemes are used. We also simulated the cases where other MPSK or MQAM modulation schemes were adopted, and obtained similar results as in the 16QAM cases. Nevertheless, as the size of modulation alphabet set increases, the performance improvement brought by the proposed constellation extension based algorithm degrades due to smaller region for constellation selection.

From the simulation results, it is seen that both the proposed phase rotation and constellation extension approaches based joint design algorithms are effective in jointly reducing PICR, OBP, and PAPR of OFDM signals. It is also seen that except for the PAPR reduction in which the performance of both algorithms is quite close, generally the phase rotation approach based algorithm achieves better performance than the constellation extension approach based algorithm, but at the expense of the need of sending side information from transmitter to receiver.

6. Conclusion

A new joint PICR, OBP, and PAPR design problem has been formulated to improve the performance of an OFDM system. Algorithms based on the phase rotation and the constellation extension approaches have been proposed to solve the joint design problem. In the proposed algorithms, the joint design problem has been relaxed to SOCP problems, whose global optimal solutions can be obtained efficiently. Simulation results have been presented to show that the proposed algorithms are effective in jointly reducing PICR, OBP, and PAPR of OFDM signals.

Note that the formulation in (9) provides only one possibility for joint design problem formulation in OFDM systems. Based on different practical system requirements, different problem formulations may be adopted for the joint design problem.

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