Receive Antenna Selection in MIMO Systems using Convex Optimization

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***Abstract*— A critical factor in the deployment of multiple-input multiple-output (MIMO) systems is the cost of multiple analog transmit/receive chains. This problem can be mitigated by antenna subset selection at the transmitter/receiver. With antenna selection, a small number of analog chains are multiplexed between a much larger number of transmit/receive antenna elements. In this paper, we present a low complexity approach to receive antenna selection for capacity maximization, based on the theory of convex optimization. We show via extensive Monte-Carlo simulations that the proposed algorithm provides performance very close to that of optimal selection based on exhaustive search. We also extend this approach to receive antenna selection for the JMMSE and OSIC V-BLAST architectures.**

***Index Terms*— Receive antenna selection, MIMO, JMMSE, OSIC, V-BLAST, convex optimization.**

I. Introduction

MULTIPLE input multiple output (MIMO) systems have received increased attention because they significantly improve wireless link performance through capacity and diversity gains [1]. A major limiting factor in the deployment of MIMO systems is the cost of multiple analog chains (such as low noise amplifiers, mixers and analog-to-digital converters) at the receiver end. Antenna selection at the transmitter/receiver is a powerful technique that reduces the number of analog chains required, yet preserving the diversity benefits obtained from the full MIMO system. With antenna selection, a limited number of transmit/receive chains are dynamically multiplexed between several transmit/receive antennas.

Prior work on antenna subset selection can be classified into two categories - antenna selection for MIMO channel capacity maximization [2] and antenna selection for practical signaling schemes [3], [4]. See [5], [6] for a review of various transmit and receive antenna selection schemes. In [2], the authors present sub-optimal schemes for receive antenna selection that offer a performance comparable to optimal selection based on exhaustive search, however at a lower complexity. The computational complexity of these schemes might still be prohibitive from an implementation perspective, especially for a large MIMO system. Recently, the authors in [7] have proposed a near-optimal selection algorithm with complexity much lower than the schemes in [2].

In this paper, we offer an alternative approach to receive antenna selection for capacity maximization that offers near-optimal performance at a complexity significantly lower than the schemes in [2] but marginally greater than the schemes in [7]. Our approach is based on formulating the selection problem as a combinatorial optimization problem and relaxing it to obtain a problem with a convex objective function and constraints. Several low complexity techniques exist for solving such problems. A key feature of our work is to extend the convex formulation to antenna selection for capacity maximization for two practical MIMO spatial multiplexing (SM) schemes - minimum mean squared error receiver with joint encoding of data streams (JMMSE) and ordered successive interference cancellation (OSIC) with independently encoded layers (V-BLAST architecture) [8]. Another distinguishing feature of our work is the fact that the complexity of the proposed algorithm is independent of the number of transmit antennas, which offers a distinct advantage in some scenarios.

Manuscript received November 20, 2004; revised May 9, 2005 and August 22, 2005; accepted July 29, 2005. The associate editor coordinating the review of this letter and approving it for publication was D. Gesbert.

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Digital Object Identifier 10.1109/TWC.2006.04769.

II. System Model

We consider a MIMO system with N transmit and M receive antennas. The channel is assumed to have frequency-flat Rayleigh fading with additive white Gaussian noise (AWGN) at the receiver. The received signal can thus be represented as

$$\mathbf{x}(k) = \sqrt{E_s}\mathbf{H}\mathbf{s}(k) + \mathbf{n}(k), \tag{1}$$

where the $M \times 1$ vector $\mathbf{x}(k) = [x_1(k), \ldots, x_M(k)]^T$ represents the k^{th} sample of the signals collected at the M receive antennas, sampled at symbol rate. The $N \times 1$ vector $\mathbf{s}(k) = [s_1(k), \ldots, s_N(k)]^T$ is the k^{th} sample of the signal transmitted from the N transmit antennas. E_s is the average energy per receive antenna and per channel use, $\mathbf{n}(k) = [n_1(k), \ldots, n_M(k)]^T$ is AWGN with energy $N_0/2$ per complex dimension and $\mathbf{H}$ is the $M \times N$ channel matrix, where $H_{p,q}$ ($p = 1, \ldots, M$, $q = 1, \ldots, N$) is a scalar channel between the p^{th} receive antenna and q^{th} transmit antenna. The entries of $\mathbf{H}$ are assumed to be zero-mean circularly symmetric complex Gaussian (ZMCSCG), such that the covariance matrix of any two columns of $\mathbf{H}$ is a scaled identity matrix. Perfect channel state information (CSI) is assumed at the receiver while performing antenna subset selection. No CSI is assumed at the transmitter.

III. Receive Antenna Selection in MIMO Systems

The earliest works on antenna selection have been in the context of single-input multiple-output (SIMO) systems. For example, selection diversity, where the receiver only selects the strongest antenna signal has long been used in SIMO systems [9]. Receive antenna selection in a MIMO system offers more degrees of freedom than in a SIMO system. We focus here on receive antenna selection for capacity maximization. The capacity of the MIMO system described in Section II is given by the well known formula

$$C(\mathbf{H}) = \log_2 \det\left(\mathbf{I}_N + \gamma \mathbf{R}_{ss}\mathbf{H}^H\mathbf{H}\right), \tag{2}$$

where $\gamma = E_s/N_0$, $\mathbf{R}_{ss} = E\left(\mathbf{s}(k)\mathbf{s}(k)^H\right)$ is the covariance matrix of the transmitted signals with trace $(\mathbf{R}_{ss}) = 1$, det$(\cdot)$ denotes the determinant and $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. However, when only $M' < M$ receive antennas are used, the capacity becomes a function of the antennas chosen. If we represent the indices of the selected antennas by $\mathbf{r} = [r_1, \ldots, r_{M'}]$, the effective channel matrix is $\mathbf{H}$ with only rows corresponding to these indices. Denoting the resulting $M' \times N$ matrix by $\mathbf{H}_r$, the channel capacity with antenna selection is given by

$$C_r(\mathbf{H}_r) = \log_2 \det\left(\mathbf{I}_N + \gamma \mathbf{R}_{ss} {\mathbf{H}_r}^H \mathbf{H}_r\right). \quad (3)$$

In the absence of CSI at the transmitter, $\mathbf{R}_{ss}$ is chosen as $\mathbf{I}_N/N$. Our goal is to chose the index set $\mathbf{r}$ such that the capacity in (3) is maximized. A closed form characterization of the optimal solution is difficult. We propose one possible selection scheme in the next section.

IV. Antenna selection as an optimization problem

In this section, we formulate the problem of receive antenna selection as a constrained convex optimization problem [10] that can be solved efficiently using numerical methods such as interior-point algorithms [11].

Define Δ_i $(i = 1, \ldots, M)$ such that

$$\Delta_i = \begin{cases} 1, & i^{th} \text{ receive antenna selected} \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

By definition, $\Delta_i = 1$ if $r_i \in \mathbf{r}$, and 0 else. Now, consider an $M \times M$ diagonal matrix $\mathbf{\Delta}$ that has Δ_i as its diagonal entries. Let us denote $\mathbf{F} = \mathbf{\Delta H}$. Based on notation introduced earlier, $\mathbf{F}$ can be written as $\mathbf{H}_r$ with $(M - M')$ zero rows appended to it and left multiplied by a $M \times M$ row-permutation matrix $\mathbf{P}$. Thus,

$$\mathbf{F} = \mathbf{P}\begin{bmatrix} \mathbf{H}_r \\ \mathbf{0}_{(M-M')\times N} \end{bmatrix} = \mathbf{P}\widetilde{\mathbf{H}_r}.$$

Since $\mathbf{P}$ is a permutation matrix, $\mathbf{P}^H\mathbf{P} = \mathbf{I}_M$. This gives, $\mathbf{F}^H\mathbf{F} = \widetilde{\mathbf{H}_r}^H \mathbf{P}^H\mathbf{P}\widetilde{\mathbf{H}_r} = \widetilde{\mathbf{H}_r}^H \widetilde{\mathbf{H}_r} = {\mathbf{H}_r}^H\mathbf{H}_r$. Now, using (3) we can write the channel capacity as a function of $\mathbf{\Delta}$:

$$\begin{aligned} C_r(\mathbf{\Delta}) &= \log_2 \det\left(\mathbf{I}_N + \gamma \mathbf{F}^H\mathbf{F}\right) \\ &= \log_2 \det\left(\mathbf{I}_N + \gamma \mathbf{H}^H \mathbf{\Delta}^H \mathbf{\Delta H}\right). \end{aligned}$$

However, $\mathbf{\Delta}$ is a diagonal matrix with diagonal entries either 0 or 1, so $\mathbf{\Delta}^H\mathbf{\Delta} = \mathbf{\Delta}$. Thus, the MIMO channel capacity with antenna selection can be re-written as

$$\begin{aligned} C_r(\mathbf{\Delta}) &= \log_2 \det\left(\mathbf{I}_N + \gamma \mathbf{H}^{\mathbf{H}} \mathbf{\Delta H}\right) \\ &= \log_2 \det\left(\mathbf{I}_M + \gamma \mathbf{\Delta H H}^H\right). \end{aligned} \quad (5)$$

The second equality in (5) follows from the matrix identity

$$\det(\mathbf{I}_m + \mathbf{AB}) = \det(\mathbf{I}_n + \mathbf{BA}).$$

Lemma 1: The capacity expression given by $C_r(\mathbf{\Delta})$ is concave in Δ_i $(i = 1, \ldots, M)$.

Proof: See the Appendix. ■

The variables Δ_i are binary valued (0 or 1) integer variables, thereby rendering the selection problem NP-hard. We seek a simplification by relaxing the binary integer constraints and allowing $\Delta_i \in [0, 1]$. Thus, the problem of receive antenna subset selection for capacity maximization is approximated by the following constrained convex relaxation plus rounding scheme:

$$\begin{gathered} \text{maximize: } \log_2 \det\left(\mathbf{I}_M + \gamma \mathbf{\Delta H H}^H\right) \\ \text{subject to} \\ 0 \le \Delta_i \le 1, \quad i = 1, \ldots, M \\ \text{trace}(\mathbf{\Delta}) = \sum_{i=1}^{M} \Delta_i = M'. \end{gathered} \quad (6)$$

From the (possibly) fractional solution obtained by solving the above problem, the M' largest Δ_i's are chosen and the corresponding indices represent the receive antennas to be selected.

V. Practical MIMO-SM Schemes

We now turn our attention to the problem of receive antenna selection for two practical MIMO-SM architectures, namely JMMSE and OSIC.

A. JMMSE

In this scheme, a sequence of encoded symbols is interleaved over N streams and transmitted by the respective antennas. At the receiver, the streams are extracted using the MMSE receiver and are then multiplexed into a single stream, de-interleaved and decoded. The capacity $C_J(\mathbf{H})$ achievable with the JMMSE architecture is given by [2]:

$$C_J(\mathbf{H}) = \sum_{k=1}^{N} \log_2(1 + \rho_k^2), \quad (7)$$

where

$$\rho_k^2 = \left[\left(\mathbf{I}_N + \gamma \mathbf{H}^H\mathbf{H}\right)^{-1}\right]_{k,k}^{-1} - 1, \quad k = 1, \ldots, N, \quad (8)$$

is the SINR of the k^{th} output stream of the MMSE receiver. With receive antenna selection, the channel matrix is given by $\mathbf{H}_r$. As shown in Section IV, ${\mathbf{H}_r}^H\mathbf{H}_r = \mathbf{F}^H\mathbf{F} = \mathbf{H}^H\mathbf{\Delta H}$. Thus, the capacity of the JMMSE architecture with receive antenna selection is given by

$$C_{Jr}(\mathbf{\Delta}) = \sum_{k=1}^{N} \log_2(1 + \tilde{\rho}_k^2), \quad (9)$$

where $\tilde{\rho}_k^2$ are obtained from (8) by replacing $\mathbf{H}^H\mathbf{H}$ with $\mathbf{H}^H\mathbf{\Delta H}$.

Lemma 2: The capacity expression given by C_{Jr} is concave in Δ_i $(i = 1, \ldots, M)$.

Proof: See the Appendix. ■

Based on the foregoing discussion, we approximate the problem of receive antenna selection for capacity maximization for a JMMSE receiver structure by a constrained convex relaxation plus a rounding scheme, with the objective function given by (9) and constraints given by (6).

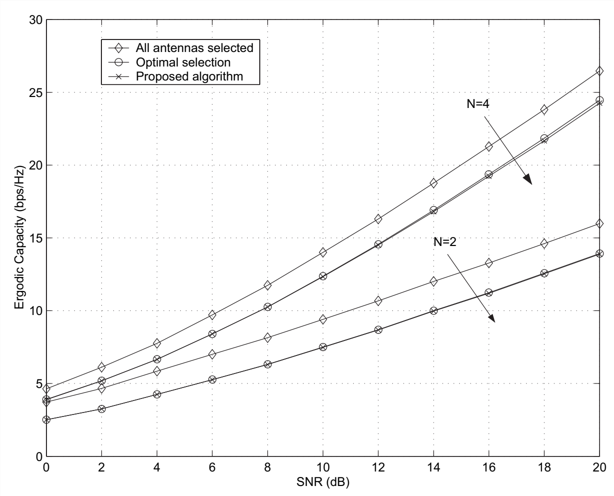

Fig. 1. Ergodic capacity v/s SNR ($N\gamma$), $M = 6$, $N = 2, 4$, $M' = N$.

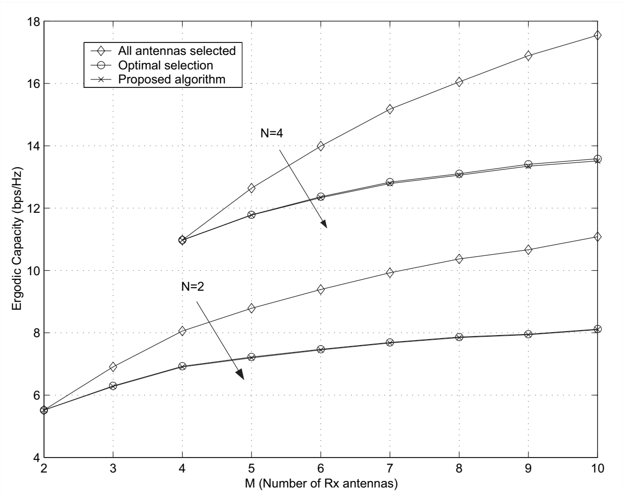

Fig. 2. Ergodic capacity v/s M, SNR=10dB, $N = 2, 4$, $M' = N$.

B. OSIC

In this scheme, the transmitter forms N parallel data streams that are encoded and modulated independently and then transmitted over N antennas. At the receiver, a linear MMSE filter is used to extract the transmitted data streams successively by peeling out the contribution of previously decoded streams (layers). As shown in [8], it is optimal to peel the layer with the highest SINR at each stage. Define $\pi : \{1, \ldots, N\} \mapsto \{1, \ldots, N\}$ as a permutation that specifies the ordering. With linear MMSE decoding, the SINR of the k^{th} surviving stream at the i^{th} stage is given by

$$\rho_{k,i}^2 = \left[\left(\mathbf{I}_{N-i+1} + \gamma {\mathbf{H}^{(i)}}^H \mathbf{H}^{(i)} \right)^{-1} \right]_{k,k}^{-1} - 1, \quad k = 1, \ldots, (N - i + 1), \quad (10)$$

where $\mathbf{H}^{(i)}$ is the channel matrix with columns with indices $[\pi(1), \ldots, \pi(i-1)]$ removed. The SINR of the stream decoded at the i^{th} stage is given by

$$\omega_i^2 = \max_{k=1,\ldots,N-i+1} \rho_{k,i}^2.$$

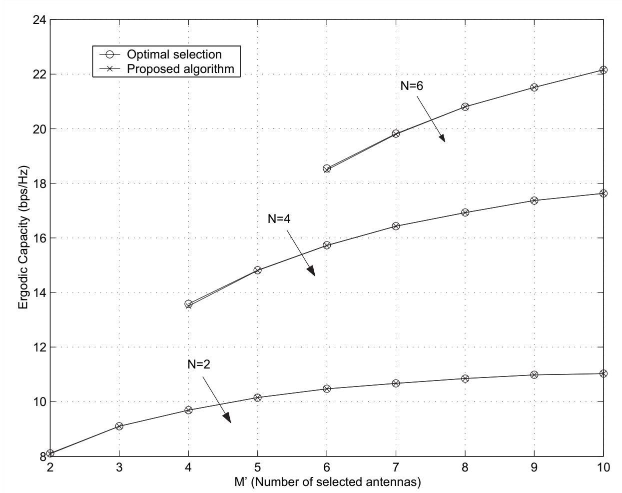

Fig. 3. Ergodic capacity v/s M', SNR=10dB, $N = 2, 4, 6$, $M = 10$.

Since the performance of the vertically layered system is limited by that of the worst layer, the capacity is lower bounded as

$$C_O(\mathbf{H}) \geq N \log_2 \left(1 + \min_{i=1,\ldots,N} \omega_i^2 \right).$$

Noting that the max is always greater than or equal to the average, a simple lower bound on the OSIC capacity (with selection) is given by

$$C_O(\mathbf{H}) \geq N \log_2 \left(1 + \min_{i=1,\ldots,N} \tilde{\omega}_i^2 \right) = \widetilde{C_O}(\mathbf{H}), \quad (11)$$

where

$$\tilde{\omega}_i^2 = \frac{1}{N-i+1} \sum_{k=1}^{N-i+1} \tilde{\rho}_{k,i}^2.$$

As discussed for the JMMSE case, with antenna selection, $\tilde{\rho}_{k,i}^2$ are given by (10), with ${\mathbf{H}^{(i)}}^H \mathbf{H}^{(i)}$ replaced by ${\mathbf{H}^{(i)}}^H \mathbf{\Delta} \mathbf{H}^{(i)}$.

Lemma 3: The lower bound on OSIC capacity given by (11) is concave in Δ_i $(i = 1, \ldots, M)$.

Proof: See the Appendix. ■

We propose to perform antenna selection to maximize this lower bound on OSIC capacity. Thus, we approximate the problem of antenna selection for capacity maximization by a constrained convex relaxation plus a rounding scheme, with the objective function given by (11) and constraints given by (6).

VI. Results

In this section, we evaluate the performance of the proposed antenna selection algorithm via Monte-Carlo simulations. We use ergodic capacity[1] as a metric for performance evaluation, which is obtained by averaging over results obtained from 2000 independent realizations of the channel matrix $\mathbf{H}$. For each realization, the entries of the channel matrix are uncorrelated ZMCSCG random variables. Figures 1, 2 and 3 depict the

[1] Since the capacity expression is concave for each channel realization, so is the ergodic capacity, since it is a non-negative weighted sum of concave functions.

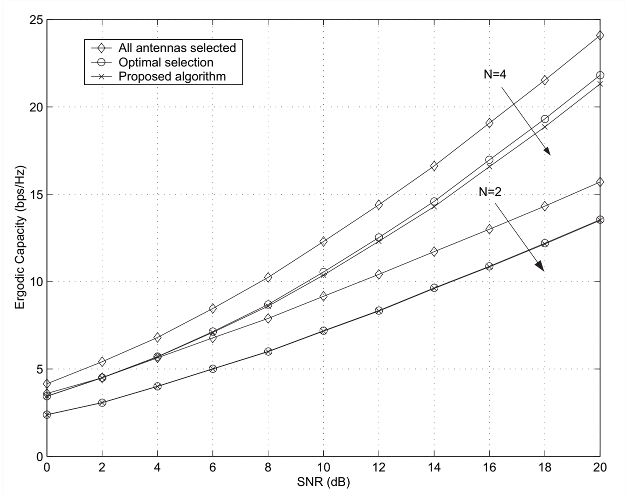

Fig. 4. Ergodic capacity v/s SNR ($N\gamma$) for JMMSE, $M = 6$, $N = 2, 4$, $M' = N$.

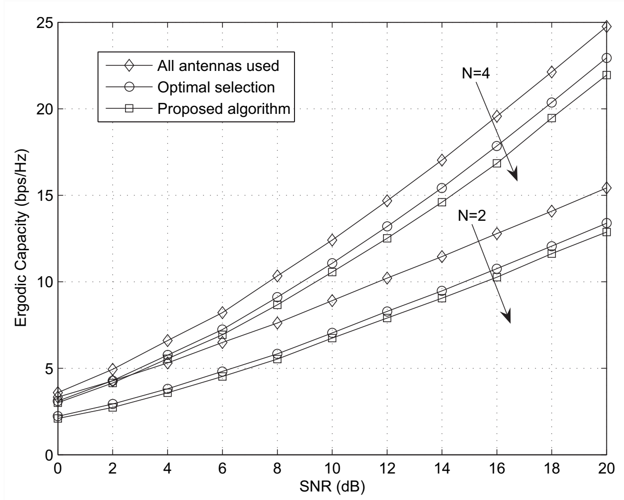

Fig. 5. Ergodic capacity v/s SNR ($N\gamma$) for OSIC, $M = 6$, $N = 2, 4$, $M' = N$.

ergodic capacity as a function of the received SNR per antenna ($N\gamma$), number of receiver antennas and number of antennas selected at the receiver, respectively. Figure 4 and Figure 5 depict the ergodic capacity as a function of received SNR for the JMMSE and OSIC receiver architectures respectively. The efficacy of the proposed selection algorithm is evident from the plots.

Optimal selection involves an exhaustive search over all possible $\binom{M}{M'}$ subsets of receive antennas, requiring around $\binom{M}{M'}M'^3$ complex additions/multiplications, which grows exponentially with M for $M' \approx M/2$. This can be seen using Stirling's approximation for the factorial. The selection algorithms proposed in [2] have complexity $O(M^5)$. We use the *barrier method* to solve the convex relaxation in (6). A brief description is provided in the Appendix. For the barrier method, the number of Newton steps is upper bounded by $\sqrt{M}$. Each Newton step has $O(M^3)$ complexity. Thus, the total complexity is $O(M^{3.5})$, a significant improvement over $O(M^5)$. However, we re-iterate a remark from [10] that the number of Newton steps do not typically increase significantly as M increases. As a consequence, the proposed selection scheme can be expected to have $O(M^3)$ complexity for practical purposes. The authors in [7] have also proposed an $O(M^3)$ complexity selection algorithm. Finally, we remark that the complexity of the proposed algorithm is independent of the number of transmit antennas, which has potential advantages in some scenarios. Other selection algorithms in literature do not exhibit this property.

VII. Conclusions

The problem of receive antenna subset selection has been approximated by a constrained convex relaxation (along with a rounding scheme) that can be solved using standard low complexity techniques from optimization theory. Simulation results show that the performance of the scheme is very close to optimal. The convex formulation has also been extended to two practical MIMO-SM schemes, namely JMMSE and OSIC (V-BLAST architecture).

Appendix

A. *Proof of Lemma 1*

The proof follows from the following facts: The function $f(\mathbf{X}) = \log_2\det(\mathbf{X})$ is concave in the entries of $\mathbf{X}$ if $\mathbf{X}$ is a positive definite matrix, and the concavity of a function is preserved under an affine transformation [10].

B. *Proof of Lemma 2*

We have,

$$\begin{aligned} C_{Jr}(\boldsymbol{\Delta}) &= \sum_{k=1}^{N}\log_2(1+\tilde{\rho}_k^2) \\ &= -\sum_{k=1}^{N}\log_2\left(\left[\left[\mathbf{I}_N+\gamma\mathbf{H}^H\boldsymbol{\Delta}\mathbf{H}\right]^{-1}\right]_{k,k}\right). \end{aligned} \tag{12}$$

Now, consider

$$f(\mathbf{X}) = -\sum_{k=1}^{N}\log_2\left(\left[\mathbf{X}^{-1}\right]_{k,k}\right).$$

We prove concavity by considering an arbitrary line given by $\mathbf{X} = \mathbf{Z} + t\mathbf{V}$, where $\mathbf{Z}$ and $\mathbf{V}$ are symmetric matrices and t is restricted to a domain such that $\mathbf{X}$ is positive semi-definite. We have,

$$\begin{aligned} g(t) &= -\sum_{k=1}^{N}\log_2\left(\left[(\mathbf{Z}+t\mathbf{V})^{-1}\right]_{k,k}\right) \\ &= -\sum_{k=1}^{N}\log_2\left(\left[\mathbf{Z}^{-1/2}\mathbf{P}(\mathbf{I}_N+t\boldsymbol{\Lambda})^{-1}\mathbf{P}^H\mathbf{Z}^{-1/2}\right]_{k,k}\right), \end{aligned}$$

where $\mathbf{Z}^{-1/2}\mathbf{V}\mathbf{Z}^{-1/2}$ has an eigen-value decomposition given by $\mathbf{P}\boldsymbol{\Lambda}\mathbf{P}^H$, and $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_k)$. If we denote $\boldsymbol{\Phi} = \mathbf{P}^H\mathbf{Z}^{-1/2}$, we get

$$g(t) = -\sum_{k=1}^{N}\log_2\left(\left[\boldsymbol{\Phi}^H(\mathbf{I}_N+t\boldsymbol{\Lambda})^{-1}\boldsymbol{\Phi}\right]_{k,k}\right). \tag{13}$$

The k^{th} term in the summation in (13) is given by

$$g_k(t) = -\log_2 \left(\sum_{i=1}^{N} \frac{\|\Phi_{i,k}\|^2}{1+t\lambda_i} \right).$$

Let $\tau_i = \frac{1+t\lambda_i}{\|\Phi_{i,k}\|^2}$. Then, $g_k(t) = -\log_2 \left(\sum_{i=1}^{N} \frac{1}{\tau_i} \right)$, which is concave in the τ_is, from the log-concavity of the harmonic mean [10]. Since the τ_is are affine functions of t, $g_k(t)$ is concave in t. Finally, since $g(t)$ is a sum of $g_k(t)$s, $g(t)$ is concave in t, from which we conclude concavity of $f(\mathbf{X})$. From (12), and the invariance of concavity under an affine transformation [10], we conclude that $C_{Jr}(\boldsymbol{\Delta})$ is a concave function of Δ_i $(i = 1, \ldots, M)$.

C. Proof of Lemma 3

Let us denote the lower bound on OSIC capacity with antenna selection given by (11) by $\widetilde{C_{O_r}}(\boldsymbol{\Delta})$. We have

$$\widetilde{C_{O_r}}(\boldsymbol{\Delta}) = N\log_2 \left(1 + \min_{1 \le i \le N} \frac{1}{N-i+1} \sum_{k=1}^{N-i+1} \tilde{\rho}_{k,i}^2(\boldsymbol{\Delta}) \right). \quad (14)$$

Using the log-concavity of the harmonic mean, it was shown in the proof of Lemma 2 that the output SINRs of the linear MMSE filter for each stream are concave functions of Δ_j $(j = 1, \ldots, M)$. Since $\tilde{\rho}_{k,i}^2$s are the output SINRs of the MMSE filter at the i^{th} decoding stage of OSIC, we conclude that they are concave functions of Δ_j $(j = 1, \ldots, M)$. A non-negative weighted sum of concave functions is concave, and so is a pointwise minimum of non-negative concave functions [10].

Hence, the argument of the $\log_2(\cdot)$ function in (14) is concave. Since the logarithm of a concave and non-negative function is concave [10], we conclude that the lower bound on OSIC capacity is a concave function of Δ_j $(j = 1, \ldots, M)$.

D. The Barrier Method

Given strictly feasible Δ_i $(i = 1, \ldots, M)$, $t = t^{(0)} > 0$, $\mu > 1$ (update parameter), $\epsilon > 0$ (tolerance), **repeat** the following steps:

1) Compute $\Delta_i^\star(t)$ by minimizing $tf_0(\boldsymbol{\Delta}) + \phi(\boldsymbol{\Delta})$, subject to $\sum_{i=1}^{M} \Delta_i = M'$, starting at Δ_i using Newton's method.
2) Update $\Delta_i := \Delta_i^\star(t)$ $(i = 1, \ldots, M)$.
3) If $M/t < \epsilon$, **stop**, else update $t := \mu t$.

Here,

$$\begin{aligned} f_0(\boldsymbol{\Delta}) &= -C_r(\boldsymbol{\Delta}) \\ \phi(\boldsymbol{\Delta}) &= -\sum_{i=1}^{M} \log\left(\Delta_i(1-\Delta_i)\right). \end{aligned}$$

We refer the reader to [10] for further details.

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