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LETTER

Overall Resource Efficiency Measure of Digital Modulation Methods

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SUMMARY A coordinate plane representation of the resource requirements of digital modulation methods is presented, and an overall resource efficiency measure is proposed. This measure can be used for the comparison of digital modulation methods and the evaluation of an emerging modulation technique. Several typical digital modulation methods are compared based on this measure to show its validity.

key words: *digital modulation, bandwidth efficiency, bit error rate (BER), channel capacity, overall resource efficiency*

1. Introduction

Bit error rate (BER) and bandwidth efficiency are two fundamental performance measures of digital modulation methods. There has been considerable literature on them with regard to assessing various digital modulation methods or communication systems. As is well known, for digital modulation methods, the BER performance and bandwidth efficiency are inter-constraint, and tradeoff between them can be made, that is, the increase of bandwidth efficiency can be obtained at the expense of BER performance loss, and vice versa. In [1] and [2], this tradeoff is actually referred to, although the authors did not treat it as a subject intentionally. Of course it is desired that the tradeoff is made in a most efficient way, for example, a certain increase of bandwidth efficiency can be achieved with minimum BER performance loss. However, for a specific implementation of such a tradeoff, one can not tell if this is accomplished. In the design of a communication system and in the exploration of a novel digital modulation method, the comparison of digital modulation methods is always needed. Up to the present, this comparison has been made essentially by comparing their bandwidth efficiencies and BER performances separately. However, from such a comparison one can not necessarily decide as to which is the best on the whole.

The problem described above may be summarized as follows: suppose two modulation systems, system A and system B, achieve the same bit rate R_b with the same BER P_b, while they require different bandwidths, W_A and W_B respectively, and different signal-to-noise ratio (SNR), γ_{bA} (dB) and γ_{bB} (dB) respectively. Assume $W_A > W_B$ and $\gamma_{bA} < \gamma_{bB}$. Naturally the bandwidth efficiency of system B is larger than that of system A, and for system B, the bandwidth efficiency gain is at the cost of the increase of the SNR requirement, $\Delta\gamma_b = \gamma_{bB} - \gamma_{bA}$, but we do not know which system is more efficient on the whole, and whether the cost $\Delta\gamma_b$ is reasonable or not. To solve this problem analytically, we must find a way to represent the overall resource efficiency of digital modulation methods, based on the analysis of their resource requirements which include bandwidth requirement and energy requirement.

2. Resource Requirement Plane of Digital Modulation Methods

Consider a digital modulation system which requires bandwidth W and average power S of received signal to achieve bit rate R_b and BER P_b in an additive white Gaussian noise (AWGN) channel. Its normalized bandwidth requirement μ is defined as

$$\mu = W/R_b \tag{1}$$

its normalized power requirement E_b (energy per bit) as

$$E_b = S/R_b \tag{2}$$

and its normalized SNR requirement γ_b as

$$\gamma_b = (S/N_0)/R_b = E_b/N_0 \tag{3}$$

where N_0 is the power spectrum density of AWGN. Assume that the relationship between γ_b and P_b has been determined as

$$P_b = f(\gamma_b) \tag{4}$$

For a given BER demand P_b, the normalized SNR requirement of this modulation system is thus

$$\gamma_b = f^{-1}(P_b) \tag{5}$$

Let us imagine a system that the channel capacity C is achieved with its BER tending toward zero and that the same bandwidth W and the same average power S are required. We call it the corresponding ideal system of this modulation system. As is well known,

$$C = W\log_2[1 + S/(WN_0)] \tag{6}$$

The normalized bandwidth requirement of this ideal system is

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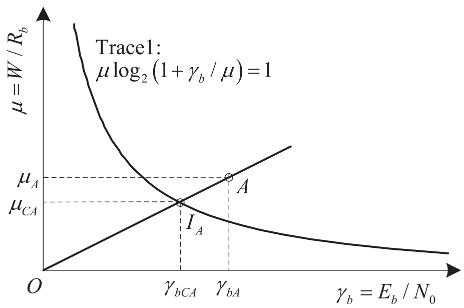

Fig. 1 Requirement plane of digital modulation methods.

$$\mu_C = W/C \tag{7}$$

and its normalized SNR requirement is

$$\gamma_{bC} = (S/N_0)/C \tag{8}$$

From (7) and (8), we can rewrite (6) as

$$\mu_C \log_2 (1 + \gamma_{bC}/\mu_C) - 1 = 0 \tag{9}$$

Figure 1 shows the requirement plane of digital modulation methods constructed in terms of the normalized bandwidth μ versus the normalized SNR γ_b. Trace 1 in Fig. 1 corresponds to (9), which demonstrates the theoretical limit of minimum resource requirements of any type of digital modulation method. For a common BER demand P_b, each digital modulation method occupies a particular point in the requirement plane, termed requirement point. The position of the requirement point of a digital modulation method indicates its resource requirements, its characteristic of power-limited or bandwidth-limited, and the distance between its resource requirements and the theoretical limit.

It is important to note that the normalized bandwidth μ is the reciprocal of the bandwidth efficiency $\eta = R_b/W$, but in nature it means a sort of resource requirement rather than efficiency. In [3], a graph of the bandwidth efficiency η versus the normalized SNR γ_b is presented to show the characteristics of various digital modulation methods. In fact it is difficult to reveal the further implications behind this graph because of the inappropriate variable selection for the vertical axis.

3. Overall Resource Efficiency Measure of Digital Modulation Methods

If the value of a given BER is small enough, consequentially we have $R_b < C$. From (1), (3), (7) and (8), we get $\mu_C/\mu = R_b/C < 1$ and $\gamma_{bC}/\gamma_b = R_b/C < 1$. Let β denotes R_b/C, then

$$\beta = R_b/C = \mu_C/\mu = \gamma_{bC}/\gamma_b < 1 \tag{10}$$

Substituting in (6) into (10) and using (1), (2) and (3), we obtain

$$\beta = \frac{R_b}{W \log_2 [1 + S/(WN_0)]} = \frac{1}{\mu \log_2 (1 + \gamma_b/\mu)} \tag{11}$$

Suppose there is a digital modulation system, denoted by SYS A, whose parameters are denoted by W_A, S_A, R_{bA}, P_{bA}, μ_A, γ_{bA} and β_A respectively. The requirement point of SYS A is $A(\gamma_{bA}, \mu_A)$ in Fig. 1. The corresponding ideal system of SYS A, denoted by IS A, whose parameters are denoted by C_A, μ_{CA} and γ_{bCA} respectively, corresponds to point $I_A(\gamma_{bCA}, \mu_{CA})$ in Fig. 1. With a subscript 'A' appended, all parameter notations for SYS A and IS A here are of the same meanings as aforementioned. Since μ_{CA} and γ_{bCA} satisfy (9) and (10), they must be the solutions of the following system of equations:

$$\begin{cases} \mu \log_2 (1 + \gamma_b/\mu) - 1 = 0 \\ \mu = (\mu_A/\gamma_{bA})\,\gamma_b \end{cases} \tag{12}$$

The equations in (12) correspond to Trace 1 and line OA in Fig. 1 respectively, thus, $I_A(\gamma_{bCA}, \mu_{CA})$, namely the requirement point of IS A, is the point of intersect of Trace 1 and line OA.

From Fig. 1, we observe the following facts:

OA represents the root mean square (RMS) resource requirement of SYS A since

$$OA = \sqrt{\mu_A^2 + \gamma_{bA}^2} \tag{13}$$

OI_A represents the RMS resource requirement of IS A because

$$OI_A = \sqrt{\mu_{CA}^2 + \gamma_{bCA}^2} \tag{14}$$

This parameter reflects the inherent characteristic of the resource requirements of a digital modulation method. For example, the value of this parameter of a wideband system is large relatively because it may trade tremendous increase in bandwidth for a very small reduction in SNR.

Since $\beta_A = \mu_{CA}/\mu_A = \gamma_{bCA}/\gamma_{bA} = R_{bA}/C_A$ according to (10), the ratio of OI_A to OA is β_A, that is,

$$\beta_A = OI_A/OA = \sqrt{\mu_{CA}^2 + \gamma_{bCA}^2} \Big/ \sqrt{\mu_A^2 + \gamma_{bA}^2} \tag{15}$$

In essence, β_A indicates the extent that the RMS resource requirement of SYS A approaches that of IS A (and that R_{bA} approaches C_A also). Hence, it is reasonable to define β as a measure of the overall resource efficiency of digital modulation methods.

AI_A is the absolute difference between the RMS resource requirement of SYS A and that of its ideal system IS A,

$$AI_A = OA - OI_A = [(1 - \beta_A)/\beta_A] \cdot \sqrt{\mu_{CA}^2 + \gamma_{bCA}^2} \tag{16}$$

where $(1 - \beta_A)/\beta_A$ is the relative difference between OA and OI_A, which depends on β_A.

The slope of line OA is

$$K_A = \mu_A/\gamma_{bA} \tag{17}$$

It characterizes quantitatively SYS A as power-limited or bandwidth-limited.

4. Comparison of Digital Modulation Methods

Generally, the normalized bandwidth requirement μ and the BER performance expression of a digital modulation method are determinable. Therefore, for a common BER demand, $P_b = 10^{-5}$ for example, its normalized SNR requirement γ_b can be determined according to (5). Its overall resource efficiency β can be calculated by using (11). Hence the comparison of various digital modulation methods can be made in terms of the parameters μ, γ_b and β.

The requirement points of several typical digital modulation methods are shown in Fig. 2 and their parameters μ, γ_b and β in Table 1. In the calculations, we assume that the type of the baseband pulse of all modulation methods involved is a rectangular waveform whose bandwidth is approximately the width of its main spectral lobe [4]; the BER performances of them refer to the ones of the optimum receiver case. The number of subcarriers of all orthogonal frequency division multiplexing (OFDM) modulation here is 1000. It is necessary to note that in our analysis, the pulse shaping filter for baseband pulse is not used in 2PSK, 4PSK, 8PSK, 16PSK, 16QAM, 64QAM and 256QAM for conciseness, and the guard interval as well as the guard frequency band are not used in OFDM for the same reason.

Among the digital modulation methods listed in Table 1, 2PSK has the least overall resource efficiency and OFDM/256QAM (OFDM that each subcarrier of it uses 256QAM) the largest one. Here we compare 2PSK with OFDM/256QAM as an example. On the whole, OFDM/256QAM is far more efficient than 2PSK because its overall resource efficiency (0.7630) is a lot larger than that of 2PSK (0.2023). In other words, the resource requirement of OFDM/256QAM is more close to its minimum theoretical limit, and its bit rate more close to its channel capacity for a common BER demand. The bandwidth efficiency of OFDM/256QAM (1/0.1251=7.9936 bits/s/Hz) is larger than that of 2PSK (1/2 bits/s/Hz) at the cost of the SNR increase, $\Delta\gamma_b = 22.5032 - 9.5879 = 12.9153$ (dB). The cost is very little because, even if the SNR requirement of OFDM/256QAM was 109.9204 dB, its overall requirement efficiency could achieve the same as that of 2PSK (0.2023), which is calculated according to (11).

In addition, the following results can be concluded from Fig. 2 and Table 1: The overall resource efficiency of an M-ary QAM is larger than that of an M-ary PSK for the same M. The overall resource efficiency of an OFDM/M-ary QAM is larger than that of an M-ary QAM for the same M. In the case of M-ary QAM or OFDM/M-ary QAM, the overall resource efficiency increases as M increases.

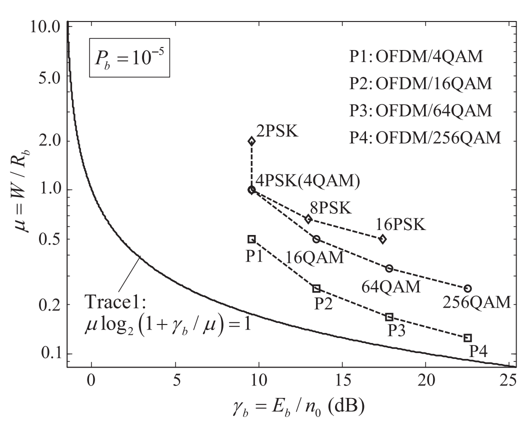

Fig. 2 Requirement points of several digital modulation methods.

Table 1 Resource requirements and overall resource efficiencies of several digital modulation methods.

Modulation Type	μ	γ_b(dB)	β	P_b
2PSK	2.0000	9.5879	0.2023	
4PSK	1.0000	9.5879	0.2998	
8PSK	0.6667	12.9716	0.3035	
16PSK	0.5000	17.4359	0.2939	
16QAM	0.5000	13.4345	0.3640	
64QAM	0.3333	17.7869	0.3999	10^{-5}
256QAM	0.2500	22.5032	0.4221	
OFDM/4QAM	0.5005	9.5879	0.4689	
OFDM/16QAM	0.2502	13.4345	0.6169	
OFDM/64QAM	0.1668	17.7869	0.7055	
OFDM/256QAM	0.1251	22.5032	0.7630	

5. Conclusion

We have proposed the overall resource efficiency as a performance measure of digital modulation methods. Based on this measure, together with the normalized bandwidth requirement and the normalized SNR requirement, a compact and meaningful comparison of various digital modulation methods can be made. The overall resource efficiency measure can also be used to evaluate an emerging modulation technique objectively. For example, when referring to the multiple input multiple output (MIMO) system, which is viewed as the spatial multiplexing modulation, it would not be sufficient to focus only on its high bandwidth efficiency. It is necessary to obtain the accurate expression of its BER performance and then compute its overall resource efficiency.

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References

[1] S. Ohno and K.A.D. Teo, “Universal BER performance ordering of MIMO systems over flat channels,” IEEE Trans. Wirel. Commun., vol.6, no.10, pp.3678–3687, Oct. 2007.
[2] S. Catreux, L.J. Greenstein, and V. Erceg, “Some results and insights on the performance gains of MIMO systems,” IEEE J. Sel. Areas Commun., vol.21, no.5, pp.839–847, June 2003.
[3] J.G. Proakis, Digital communications, fourth ed., McGraw-Hill, New York, 2001.
[4] F. Amoroso, “The bandwidth of digital signals,” IEEE Commun. Mag., vol.18, no.6, pp.13–24, Nov. 1980.