

Outage Capacity and Optimal Transmission for Dying Channels

Meng Zeng, Rui Zhang, and Shuguang Cui

Abstract

In wireless networks, communication links may be subject to random fatal impacts: for example, sensor networks under sudden power losses or cognitive radio networks with unpredictable primary user spectrum occupancy. Under such circumstances, it is critical to quantify how fast and reliably the information can be collected over attacked links. For a single point-to-point channel subject to a random attack, named as a *dying channel*, we model it as a block-fading (BF) channel with a finite and random delay constraint. First, we define the outage capacity as the performance measure, followed by studying the optimal coding length K such that the outage probability is minimized when uniform power allocation is assumed. For a given rate target and a coding length K , we then minimize the outage probability over the power allocation vector P_K , and show that this optimization problem can be cast into a convex optimization problem under some conditions. The optimal solutions for several special cases are discussed.

Furthermore, we extend the single point-to-point dying channel result to the parallel multi-channel case where each sub-channel is a dying channel, and investigate the corresponding asymptotic behavior of the overall outage probability with two different attack models: the independent-attack case and the m -dependent-attack case. It can be shown that the overall outage probability diminishes to zero for both cases as the number of sub-channels increases if the *rate per unit cost* is less than a certain threshold. The outage exponents are also studied to reveal how fast the outage probability improves over the number of sub-channels.

Index Terms

Asymptotic Outage Probability, Convex Optimization, Dying Channel, Fading Channel, Outage Capacity, Optimal Power Allocation, Parallel Channel, Random Delay Constraint.

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Meng Zeng and Shuguang Cui are with the Department of Electrical and Computer Engineering, Texas A&M University, College Station, TX, 77843. Emails: {zengm321, cui}@tamu.edu.

Rui Zhang is with the Institute for Infocomm Research, A*STAR, Singapore. Email: rzhang@i2r.a-star.edu.sg.

I. INTRODUCTION

Information-theoretic limits of fading channels have been thoroughly studied in the literature and to date many important results are known (see [1] and references therein). Generally speaking, if the transmission delay is not of concern, the classic Shannon capacity for a deterministic additive white Gaussian noise (AWGN) channel can be extended to the ergodic capacity for a fading AWGN channel, which is achievable by a random Gaussian codebook with infinite-length codewords spanning over many fading blocks such that the randomness induced by fading can be averaged out [2][3]. With the transmitter and receiver channel state information (CSI) perfectly known, the adaptive power allocation serves as an effective method to increase the ergodic capacity. This allocation has the well-known “water-filling” structure [2], where power is allocated over the channel state space. With such an allocation scheme, a user transmits at high power when the channel is good and at low or zero power when the channel is poor. When the CSI is only known at the receiver, the capacity is achievable with special “single-codebook, constant-power” schemes [4].

The validity of the ergodic capacity is based on the fundamental assumption that the delay limit is infinite. However, many wireless communication applications have certain delay constraints, which limit the practical codeword length to be finite. Thus, the ergodic capacity is no longer a meaningful performance measure. Such situations give rise to the notions of *outage capacity*, *delay-limited capacity*, and *average capacity* [5][6], each of which provides a more meaningful performance measure than the ergodic capacity. In particular, there usually exists a capacity-versus-outage tradeoff for transmissions over fading channels with finite delay constraints [7], where an outage event occurs when the “instantaneous” mutual information of the fading channel falls below the transmitted code rate, and a higher target rate results in a larger outage probability. The maximum transmit rate that can be reliably communicated under some prescribed transmit power budget and outage probability constraint is known as the outage capacity. In the extreme case of requiring zero outage probability, the outage capacity then becomes the zero-outage or delay-limited capacity [8]. To study the delay-limited system, the authors in [5] adopt a K -block block-fading (BF) AWGN channel model, where K indicates the constraint on transmission delay or the maximum codeword length in blocks. Such a channel model is briefly described as follows. Suppose a codeword is required to transmit within KB symbols, with the integer K being the number of blocks spanned by a codeword, which is also referred to as the interleaving depth (we call it coding length to emphasize how many blocks over which a codeword spans); it is also a measure of the overall transmission delay.

The parameter B is the number of channel uses in each block, which is called block length. A codeword of length KB is also referred to as a frame, where the fading gain within each block remains the same (over B symbols) and changes independently from block to block. The number of channel uses B in each block is assumed to be large enough for reliable communication, but still small compared to the channel coherence time. If the CSI for each K -block transmission is known non-causally at the transmitter, transmit power control can significantly improve the outage capacity of the K -block BF channel [4]. When the CSI can be only revealed to the transmitter in a causal manner, a dynamic programming algorithm is developed to achieve the outage capacity of the K -block BF channel in [9].

In the above existing works, the delay limit is either infinite or finite but deterministic. However, there are indeed some practical scenarios where the delay constraint is both finite and random. For example, in a wireless sensor network operating in a hostile environment, sensors may die due to sudden physical attacks such as fire or power losses. Another example may be a cognitive radio network with opportunistic spectrum sharing between the secondary and primary users, where an active secondary link can be corrupted unpredictably when the channel is reoccupied by a primary transmission. How fast and reliably can a piece of information be transmitted over such a channel? This question motivates us to formally define the maximum achievable information rate over a channel with a random and finite delay constraint, named as a *dying channel*. This type of dying channels has never been thoroughly studied in the traditional information theory, and important theorems are missing to address the fundamental capacity limits. In this paper, we start investigating such channels by focusing on a point-to-point dying link and model it by a K -block BF channel subject to a fatal attack that may happen at a random moment within any of the K transmission blocks, or may not happen at all over K blocks. Note that the delay limit in the case of a dying channel is a random variable due to the random attack, instead of being deterministically equal to K as in a traditional delay-limited BF channel. Since the successfully transmitted number of blocks is random and up to K , a dying channel is delay-limited and hence non-ergodic in nature. Thus its information-theoretic limit can be measured by the outage capacity. It is well known that coding over only one block of a fading channel may lead to a poor performance due to the lack of diversity. However, when we code over multiple blocks to achieve more diversity in a dying channel, we must bear the larger possibility that the random attack happens in the middle of the transmission and renders the rest of the codeword useless. Therefore, it is neither wise to span a codeword over too many blocks nor just over one block. We need to consider the tradeoff between the potential diversity and the attack avoidance for

the selection of the codeword length over such a dying channel. In other words, given a distribution of the random attack, we need to seek an optimal K that “matches” the number of surviving blocks in a probabilistic sense such that the achievable diversity is maximized and the outage probability is minimized.

In a system with multiple parallel sub-channels (e.g., in a OFDM-based system), each sub-channel may be under a potential random attack. In such a scenario, we are interested in the overall system outage probability and how the outage probability behaves as the number of sub-channels increases. This leads us to examine the asymptotic outage behavior for the case of a parallel dying channel. We will consider two models of random attacks over the sub-channels: 1) the case of independent random attacks, where the attacks across the sub-channels are independently and identically distributed (i.i.d.) ; and 2) the case of m -dependent random attacks, where the attacks over m adjacent sub-channels are correlated and the attacks on sub-channels that are m -sub-channel away from each other are independent.

In the following, we briefly summarize the main results in this paper:

- 1) We introduce the notion of a dying channel and formally define its outage capacity. Suppose we code over K blocks, and the number of surviving blocks is random and up to K . An outage occurs if the total mutual information over the surviving blocks normalized by K is less than a predefined rate R . Correspondingly, the outage capacity is the largest rate that satisfies an outage probability requirement.
- 2) We study the optimal coding length K that “matches” the attack time in a probabilistic sense such that the outage probability is minimized when uniform power allocation is assumed. We then investigate the optimal power allocation over these K blocks, where we obtain the general properties for the optimal power vector \mathbf{P}_K . We find that, for some cases, the optimization problem over \mathbf{P}_K can be cast into a convex problem.
- 3) We further extend the single dying channel result to the parallel dying channel case where each sub-channel is an individual dying channel. In this case, we investigate the outage behavior with two different random attack models: the independent-attack case and the m -dependent-attack case. Specifically, we characterize the asymptotic behavior of the outage probabilities for the above two cases with a given target rate. By the central limit theorems for independent and m -dependent sequences, we show that the outage probability diminishes to zero for both cases as the number of sub-channels increases if the target rate per unit cost is below a threshold. The outage exponents for both cases are studied to reveal how fast the outage probability improves.

The rest of this paper is organized as follows. Section II presents the system model for a single dying channel, as well as the definition of the corresponding outage capacity. In Section III, we study the optimal coding length by considering uniform power allocation and derive the lower and upper bounds of the outage probability. Moreover, we obtain the closed-form expression of outage probability for the high signal-to-noise ratio (SNR) Rayleigh fading case. In Section IV, we optimize over the power vector to minimize the outage probability. In Section V, we extend the single dying channel model to the parallel dying channel case. In particular, we examine the corresponding asymptotic outage probability with two setups: the independent-attack case and the m -dependent-attack case, in Sections VI and VII, respectively. In Section VIII, the outage exponents for both cases are examined to reveal how fast the outage probability improves. Section IX concludes the paper.

Notation: we define the notations used throughout this paper as follows.

- \mathbb{R} indicates the set of real numbers, \mathbb{R}_+ is the set of nonnegative real numbers, and \mathbb{R}_+^N is the set of N -dimensional nonnegative real vectors.
- The error function: $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$.
- The normalized cumulative normal distribution function: $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$.
- The Q -function: $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$.
- $\log(x)$ is the natural logarithm.
- $\lceil \cdot \rceil$ is the ceiling operator and $\lfloor \cdot \rfloor$ is the flooring operator.

II. OUTAGE CAPACITY DEFINITION OF A SINGLE DYING CHANNEL

We consider a point-to-point delay-limited fading channel subject to a random fatal attack, while the exact timing of the attack is unknown to neither the transmitter nor the receiver. Only the distribution of the random attack time is known to both the transmitter and the receiver. We further assume that there is no channel state information at the transmitter (CSIT) while there is perfect channel state information at the receiver (CSIR). The transmitter transmits a codeword over K blocks within the delay constraint; and when the fatal attack occurs, the communication link is cut off immediately with the current and rest of the blocks lost. We build our model of such a dying link based on the K -block BF-AWGN channel [5], which is described as follows.

Let \mathbf{x} , \mathbf{y} , and \mathbf{z} be vectors in \mathbb{R}^{KB} representing the channel input, output, and noise sequences, respectively, where \mathbf{z} is the Gaussian random vector with zero mean and covariance matrix $\sigma^2 \mathbf{I}_{KB}$. Rearrange the components of \mathbf{x} , \mathbf{y} , and \mathbf{z} as $K \times B$ matrices, denoted as \mathbf{X} , \mathbf{Y} , and \mathbf{Z} , respectively

(Each row is associated with B symbols from a particular block.). A codeword with length KB spans K blocks and the input-output relation over the channel can be written as follows:

$$\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{Z},$$

where $\mathbf{A} = \text{diag}(|h_1|, \dots, |h_K|)$ is a $K \times K$ matrix with the diagonal elements being the fading amplitudes. Let $\hat{\mathbf{X}}_i$ be the i -th column of \mathbf{X} for $i \in \{1, \dots, B\}$. Similarly, let $\hat{\mathbf{Y}}_i$ and $\hat{\mathbf{Z}}_i$ be the i -th columns of \mathbf{Y} and \mathbf{Z} , respectively. These are related as:

$$\hat{\mathbf{Y}}_i = \mathbf{A}\hat{\mathbf{X}}_i + \hat{\mathbf{Z}}_i, \quad i \in \{1, \dots, B\}, \quad (1)$$

which implies that the input symbols on the same row of \mathbf{X} experience the same fading gain, i.e., they are transmitted over the same block. Since $\hat{\mathbf{Z}}_i$'s are i.i.d random vectors, we can view this channel as K independent parallel channels with each channel corresponding to a block. Hence, KB uses of the original channel corresponds to B uses of the K parallel channels in (1). The parallel channels over which a codeword is transmitted are determined by the channel state h_1, h_2, \dots, h_K , which can also be viewed as a composite channel [5][10]¹ that consists of a family of channels $\{\Gamma(\theta), \theta \in \Theta\}$ indexed by a particular set of Θ . For a block fading channel with delay constraint K , it can be modeled as a composite channel $\{\Gamma(\theta) : \theta \in \Theta_K\}$ as follows: Let $\Theta_K \subset \mathbb{R}^K$ be the set of all length- K sequences of channel gains $\boldsymbol{\theta}_K = \{h_1, h_2, \dots, h_K\}$, which occurs with probability π_θ under the joint distribution of $\{H_1, H_2, \dots, H_K\}$. For each $\boldsymbol{\theta}_K = \{h_1, h_2, \dots, h_K\} \in \Theta_K$, we associate a channel $\Gamma(\boldsymbol{\theta}_K)$, where $\Gamma(\boldsymbol{\theta}_K)$ consists of K parallel Gaussian channels. Let $\boldsymbol{\alpha}_K = \{\alpha_1, \alpha_2, \dots, \alpha_K\}$ be the fading power gain vector, i.e., $\alpha_i = |h_i|^2, i = 1, \dots, K$, and $\mathbf{P}_K = \{P_1, P_2, \dots, P_K\}$ be the transmit power allocation vector. For a given set of $\boldsymbol{\theta}_K$ and \mathbf{P}_K , the maximum average mutual information rate over channel $\Gamma(\boldsymbol{\theta}_K)$ is [5]:

$$C_{\text{BF}}(\boldsymbol{\theta}_K, \mathbf{P}_K, K) = \frac{1}{K} \sum_{i=1}^K \log(1 + \alpha_i P_i), \quad (2)$$

where we assume a unit noise variance throughout this paper.

In our model of the dying channel, the delay constraint is random rather than deterministically equal to K due to the fact that a random attack may happen within any block out of the K blocks or may not happen at all within the K blocks. If the fatal attack happens during the transmission, the current block and the blocks after the attack moment will be discarded. An outage occurs whenever the total mutual

¹A composite channel is a compound channel with prior probabilities.

information of the surviving blocks normalized by K is less than the transmitted code rate. Therefore, the dying channel is non-ergodic and an appropriately defined outage capacity serves as the reasonable performance measure.

Let T be the random attack time that is normalized by the block length. As we know from the results of parallel Gaussian channels [11], with random coding schemes, we can decode the codeword even if the attack happens within the K blocks as long as the average mutual information of surviving blocks is greater than the code rate R of the transmission, i.e., if we have

$$\frac{1}{K} \sum_{i=1}^L \log(1 + \alpha_i P_i) \geq R,$$

then the codeword is decodable, where the random integer $L = \min(K, \lfloor T \rfloor)$ with $\lfloor \cdot \rfloor$ being the flooring operator.

Hence the outage capacity of a dying channel can be formally defined as follows:

Definition 1: The outage capacity of a K -block BF-AWGN dying channel with an average transmit power constraint P and a required outage probability η is expressed as

$$C_{\text{out}}(P, \eta) = \max_K \sup_{\mathbf{P}_K: \sum_{i=1}^K P_k \leq KP} \left\{ R : \Pr\left\{ \frac{1}{K} \sum_{i=1}^L \log(1 + \alpha_i P_i) < R \right\} < \eta \right\}. \quad (3)$$

Note that the outage probability above is defined over the distributions of the α_i 's and T , where we assume that the α_i 's and T are independent of each other and the transmitter does not know the values of the α_i 's and T *a priori*, but knows their distributions. As we see from (3), there are two sets of variables to be optimized: One is the number of coding blocks K , and the other is the power allocation vector \mathbf{P}_K . From the perspective of optimal transmission schemes, the outage capacity maximization problem is equivalent to the outage probability minimization problem [4]. In the next section, we first study the optimal coding length K to “match” the attack time in a probabilistic sense such that the outage probability is minimized.

III. OPTIMAL CODING LENGTH WITH UNIFORM POWER ALLOCATION

As discussed before, we can optimize over the coding length K and the power vector \mathbf{P}_K to achieve the maximum outage capacity (or equivalently the minimum outage probability). If a uniform power allocation strategy is adopted, the only thing left for optimization is the coding length K . On one hand, we can have

a larger $L = \min(K, \lfloor T \rfloor)$ by increasing K , meaning that we potentially have higher diversity to achieve a lower outage probability. On the other hand, a larger K incurs a higher percentage of blocks being lost after the attack such that the average achievable mutual information per block is lower, and hence results in a larger outage probability. Since the random attack determines the number of surviving blocks and K determines the average base, we are interested in finding a proper value of K to “match” the random attack property in the sense that the outage probability is minimized.

With uniform power allocation, according to the law of total probability, the outage probability can be rewritten as a summation of the probabilities conditioned on different numbers of surviving blocks, i.e.:

$$\begin{aligned} & \Pr \left\{ \frac{1}{K} \sum_{i=1}^L \log(1 + \alpha_i P) < R \right\} \\ = & w_0 + \Pr\{A_1\}w_1 + \Pr\{A_2\}w_2 + \cdots \\ & + \Pr\{A_{K-1}\}w_{K-1} + \Pr\{A_K\}w_K^*, \end{aligned} \quad (4)$$

where $A_j = \{\sum_{i=1}^j \log(1 + \alpha_i P) < KR\}$ for $j = 1, \dots, K$, $w_i = \Pr(i < T \leq i+1)$ for $i = 0, \dots, K-1$, and $w_K^* = \Pr(T > K)$. Given the distributions of α_i and T , in general, there are no tractable closed-form expressions for $\Pr\{A_j\}$'s. Alternatively, we could first seek the bounds of the outage probability and then study more exact forms for some special cases where we show how to find the optimal K .

A. Outage Probability Lower Bound

Notice that the following relationship holds:

$$\begin{aligned} & \Pr \left\{ \sum_{i=1}^j \log(1 + \alpha_i P) < KR \right\} \\ \geq & \prod_{i=1}^j \Pr \left\{ \log(1 + \alpha_i P) < \frac{KR}{j} \right\}. \end{aligned}$$

Since the fading gains α_i 's of different blocks are i.i.d, we have

$$\begin{aligned} & \prod_{i=1}^j \Pr \left\{ \log(1 + \alpha_i P) < \frac{KR}{j} \right\} \\ = & \left\{ F \left(\frac{e^{\frac{KR}{j}} - 1}{P} \right) \right\}^j, \end{aligned} \quad (5)$$

where $F(x)$ is the cumulative distribution function (CDF) of the random variable α_i .

Therefore, with the relationship in (5), we have a lower bound for the outage probability in (4) as

$$\begin{aligned}
& \Pr \left\{ \frac{1}{K} \sum_{i=1}^L \log(1 + \alpha_i P) < R \right\} \\
& \geq w_0 + \sum_{i=1}^{K-1} \left\{ F \left(\frac{e^{KR/i} - 1}{P} \right) \right\}^i w_i \\
& \quad + \left\{ F \left(\frac{e^R - 1}{P} \right) \right\}^K w_K^*.
\end{aligned} \tag{6}$$

B. Outage Probability Upper Bound

On the other hand, there exists a simple upper bound for the outage probability:

$$\Pr \left\{ \frac{1}{K} \sum_{i=1}^j \log(1 + \alpha_i P) < R \right\} \leq \prod_{i=1}^j \Pr \{ \log(1 + \alpha_i P) < KR \}, \quad j = 1, \dots, K,$$

hence yielding

$$\Pr \left\{ \frac{1}{K} \sum_{i=1}^j \log(1 + \alpha_i P) < R \right\} \leq \left\{ F \left(\frac{e^{KR} - 1}{P} \right) \right\}^j, \quad j = 1, \dots, K.$$

Therefore, an upper bound for the outage probability in (4) is given as

$$\begin{aligned}
& \Pr \left\{ \frac{1}{K} \sum_{i=1}^L \log(1 + \alpha_i P) < R \right\} \\
& \leq w_0 + \sum_{i=1}^{K-1} \left\{ F \left(\frac{e^{KR} - 1}{P} \right) \right\}^i w_i \\
& \quad + \left\{ F \left(\frac{e^{KR} - 1}{P} \right) \right\}^K w_K^*.
\end{aligned} \tag{7}$$

C. High SNR Rayleigh Fading Case

From the previous discussion, we know how to bound the outage probability in terms of K with the general SNR values. However, there usually exists a significant gap between the lower and upper bounds. Fortunately, with appropriate approximations in the high SNR regime² for Rayleigh fading, we can obtain a tractable expression for the outage probability and hence further derive a closed-form solution for the optimal K .

For our K -block fading channel model with high SNR values, outage typically occurs when each sub-channel cannot support an evenly-divided rate budget (see Exercise 5.18 in [13]). Thus, conditioned on

²Here by high SNR, we mean that P is large, i.e., $P \gg 1$.

the attack time T , the outage probability can be written as :

$$\begin{aligned} p_{out|T} &= \Pr \left\{ \frac{1}{K} \sum_{i=1}^L \log(1 + \alpha_i P) < R \right\} \\ &\approx \left(\Pr \left\{ \log(1 + \alpha_i P) < \frac{K}{L} R \right\} \right)^L. \end{aligned} \quad (8)$$

For Rayleigh fading, we have $\Pr(\alpha_i < 1/x) \approx 1/x$ when x is large. Thus, when SNR is high, we can simplify (8) as

$$p_{out|T} \approx \frac{e^{KR}}{P^L}. \quad (9)$$

With the conditional outage probability given by (9), the overall outage probability is

$$\begin{aligned} p_{out}(K) &= w_0 + \sum_{i=1}^K p_{out|T} \cdot p(L = i) \\ &= w_0 + \sum_{i=1}^{K-1} \frac{e^{KR}}{P^i} w_i + \frac{e^{KR}}{P^K} w_K^*. \end{aligned} \quad (10)$$

Let $G(t)$ be the CDF of the attack time, which is assumed to be exponentially distributed with parameter λ . Let $w_K^* = 1 - G(K)$, $w_i = G(i+1) - G(i) = e^{-\lambda i}(1 - e^{-\lambda}) = \beta^i c$ (for $\forall i < K$) with $c = 1 - e^{-\lambda}$ and $\beta = e^{-\lambda}$. We can rewrite (10) as

$$\begin{aligned} p_{out}(K) &= e^{KR} \sum_{i=1}^{K-1} \frac{\beta^i c}{P^i} + \frac{e^{KR}}{P^K} [1 - G(K)] + w_0 \\ &= e^{KR} c \frac{\frac{\beta}{P} - (\frac{\beta}{P})^K}{1 - \frac{\beta}{P}} + \frac{1 - G(K)}{P^K e^{-KR}} + w_0. \end{aligned} \quad (11)$$

For high SNR, with $0 < \beta < 1$, $\frac{\beta}{P}$ is small. Hence, $\frac{\beta/P - (\beta/P)^K}{1 - \beta/P} \approx \frac{\beta/P}{1 - \beta/P}$ when $K \geq 2$, and (11) can be approximated to:

$$p_{out}(K) \approx \xi e^{KR} + \frac{1}{P^K e^{(\lambda-R)K}} + w_0, \quad (12)$$

where $\xi = (1 - e^{-\lambda}) \frac{\beta/P}{1 - \beta/P}$. In order to obtain the optimal K by minimizing $p_{out}(K)$, we first treat (12) as a continuous function of K , although K is an integer.

Let us first consider the convexity of (12) over a real-valued K . By taking the second-order derivative of (12) over K , we have the following:

$$\frac{\partial^2 p_{out}(K)}{\partial K^2} = \xi R^2 e^{KR} + \frac{[\lambda + \log P - R]^2}{(P e^{\lambda-R})^K}. \quad (13)$$

Since we have $\lambda > 0$ and $1 - \beta/P > 0$ in the high SNR regime, it holds that $\xi > 0$. Therefore, (13) is non-negative in the high SNR regime, which means that (12) is convex over real-valued K .

Given the convexity of (12), the optimal K can be derived by setting its first-order derivative to zero and finding the root. Consequently, the optimal solution K^* is obtained as follows:

$$K^* = \log \left[\frac{\lambda + \log P - R}{\xi R} \right] \frac{1}{\lambda + \log P}.$$

Obviously, K^* is unique given a set of ξ, P, R , and λ . Since a feasible K for the original problem should be an integer, we need to choose the optimal integer solution from $\lfloor K^* \rfloor$ and $\lceil K^* \rceil$, whichever gives a smaller value of (12).

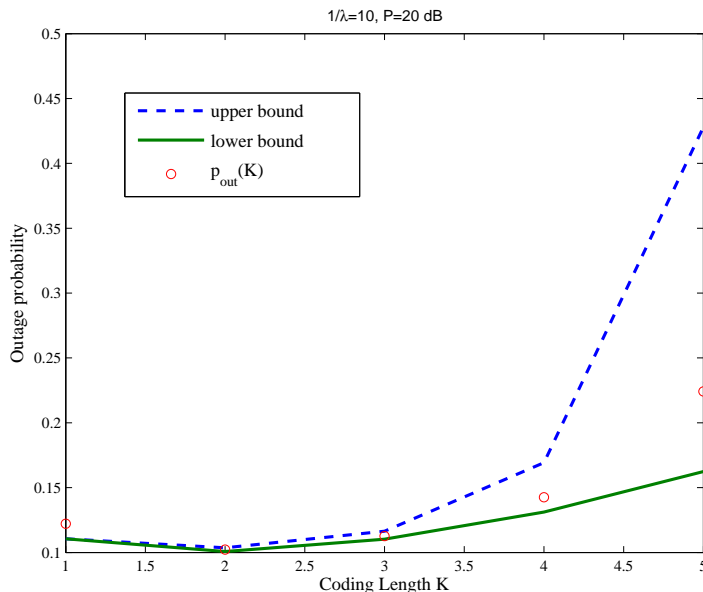


Fig. 1. Outage probability vs. coding length K , $P=20\text{dB}$.

In Fig. 1 and Fig. 2, we plot the coding length K versus the outage probability. We assume that the fading is Rayleigh, the random attack time is exponentially distributed with parameter $1/\lambda = 10$ (normalized by the transmission block length), the target rate is $R = 1$ nats/s/Hz, and the transmit power is set as 20 dB and 30 dB, respectively. As shown in Fig. 1, the dashed curve and the solid curve are the lower and upper bounds given by (6) and (7), respectively. The circles are obtained by using (12). As can be seen, firstly, the high-SNR approximation in (12) is quite accurate. The circles are located between the upper and the lower bounds except for $K = 1$. This is due to the fact that when $K = 1$, $\frac{\beta/P - (\beta/P)^K}{1 - \beta/P} \approx \frac{\beta/P}{1 - \beta/P}$ does not hold. Secondly, we see that there exists a minimum outage probability over K as shown in Fig. 2. At last, comparing Fig. 1 and Fig. 2, we see that the upper and lower bounds

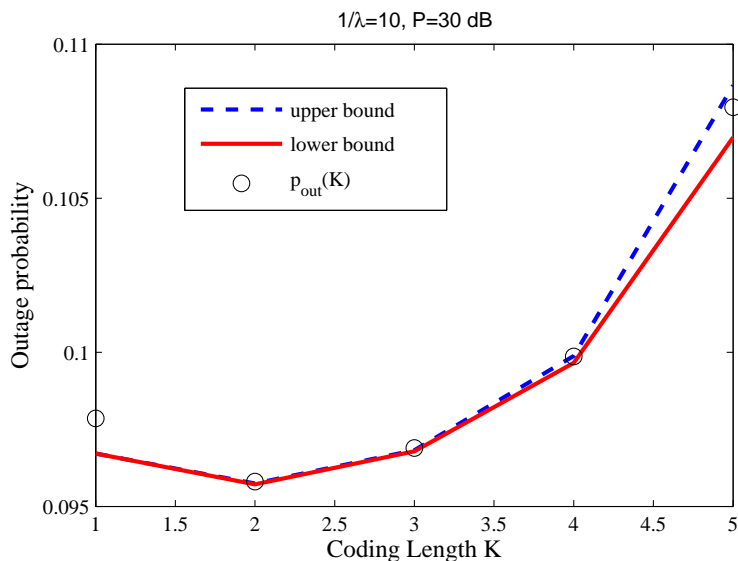


Fig. 2. Outage probability vs. coding length K , $P=30$ dB.

get closer as the SNR increases with the values from (12) are in between; hence the approximation in (12) becomes more accurate.

D. Low SNR Regime with Arbitrary Fading

When SNR is low, we have $\log(1 + \alpha_i P) \approx \alpha_i P$. Thus, when we span a codeword over K blocks, the outage probability conditioned on T is given as

$$\begin{aligned}
 p_{out|T}^{par} &= \Pr \left\{ \sum_{i=1}^L \log(1 + \alpha_i P) < KR \right\} \\
 &\approx \Pr \left\{ \sum_{i=1}^L \alpha_i < KR/P \right\}.
 \end{aligned} \tag{14}$$

When using a repetition transmission (over blocks), the outage probability is given as

$$\begin{aligned}
 p_{out|T}^{rep} &= \Pr \left\{ \log(1 + \sum_{i=1}^L \alpha_i P) < KR \right\} \\
 &\approx \Pr \left\{ \sum_{i=1}^L \alpha_i < KR/P \right\}.
 \end{aligned} \tag{15}$$

Comparing (14) and (15), we see that the outage performances of these two schemes are the same in the low-SNR regime. This is due to fact that in low SNR regime it is SNR-limited rather than degree-of-freedom-limited such that coding over different blocks does not help with decreasing the outage probability. Hence, repetition transmission is approximately optimal for a dying channel in the low SNR regime.

IV. JOINT OPTIMIZATION OVER CODING LENGTH AND POWER ALLOCATION

In the previous section, we investigated the optimal coding length K that minimizes the outage probability by assuming uniform power allocation. We now consider optimizing over both the coding length K and the power vector \mathbf{P}_K to minimize the outage probability. We note that optimizing over K is in general a 1-D search over integers, which is not complex. Since the main complexity of solving (3) lies in the optimization over \mathbf{P}_K , we first focus on the outage probability minimization problem over \mathbf{P}_K for a given fixed K , which is expressed as:

$$\begin{aligned} \min_{\mathbf{P}_K} \quad & \Pr \left\{ \frac{1}{K} \sum_{i=1}^L \log(1 + \alpha_i P_i) < R \right\} \\ \text{s.t.} \quad & \frac{1}{K} \sum_{i=1}^K P_i \leq P. \end{aligned} \quad (16)$$

After obtaining the optimal outage probabilities conditioned on a range of K values, we choose the minimum one as the global optimal value.

A. Properties of Optimal Power Allocation

We start solving the above optimization problem by investigating the general properties of the optimal power allocation over a dying channel for a given K .

Let E_j be the event that

$$E_j = \left\{ \frac{1}{K} \sum_{i=1}^j \log(1 + \alpha_i P_i) < R \right\}, \quad j = 1, \dots, K.$$

It is obvious that the events E_j 's are decreasing events, which means $E_1 \supseteq E_2 \supseteq \dots \supseteq E_K$. With the law of total probability, we can expand the outage probability in the objective of (16) as follows,

$$\begin{aligned} p_{out}(K) &= \Pr \left\{ \frac{1}{K} \sum_{i=1}^L \log(1 + \alpha_i P_i) < R \right\} \\ &= w_0 + \Pr\{E_1\}w_1 + \Pr\{E_2\}w_2 + \dots \\ &\quad + \Pr\{E_{K-1}\}w_{K-1} + \Pr\{E_K\}w_K^*, \end{aligned} \quad (17)$$

where w_i 's are defined in Section III. With the above result, we then discuss the optimal power allocation for a dying channel under different conditions.

1) Optimal Power Allocation over i.i.d. Fading:

Theorem 1: When fading gains over blocks are i.i.d., the optimal power allocation profile is non-increasing.

Proof: The proof is provided in Appendix-A. ■

This is a general result regardless of the specific distributions of fading gains. That is, the optimal power vector lies in a convex cone $\mathcal{D}_+ = \{\mathbf{P}_K \in \mathbb{R}_+^K : P_1 \geq P_2 \geq \dots \geq P_K\}$, no matter what distribution the fading gain follows, as long as the i.i.d. assumption holds.

2) *Optimal Power Allocation over Identical Fading Gains:* Now we consider the case where the fading gains over all the blocks are the same, while they are still random. This represents the case where fading gains are highly correlated in time.

Theorem 2: When the fading gains α_i 's are the same, the optimal coding length is $K = 1$ with $P_1 = P$.

Proof: The proof is provided in Appendix-B. ■

This assertion implies that the optimal transmission scheme for a highly correlated fading channel is to simply transmit independent blocks instead of jointly-coded blocks.

B. Power Allocations for Some Special Cases

When the fading gain falls into some special distributions, we can further convert the corresponding optimization problem into convex ones and derive the optimal power vector efficiently.

1) *Optimal Power Allocation over i.i.d. Rayleigh Fading in High SNR Regime:* Given (8) and conditioned on the attack time T , the conditional outage probability can be written as:

$$\begin{aligned} p_{out|T} &= \Pr \left\{ \sum_{i=1}^L \log(1 + \alpha_i P_i) < KR \right\} \\ &\approx \prod_{i=1}^L \Pr \left\{ \log(1 + \alpha_i P_i) < \frac{K}{L} R \right\}. \end{aligned} \quad (18)$$

For Rayleigh fading, we have $\Pr(\alpha_i < 1/x) \approx 1/x$ when x is large. Thus, when SNR is high, we can simplify (18) as

$$p_{out|T} \approx \frac{(e^{KR/L} - 1)^L}{\prod_{i=1}^L P_i}. \quad (19)$$

The outage probability with Rayleigh fading in high SNR is approximated as below by substituting (19) into (17):

$$p_{out}(K) \approx w_0 + \frac{e^{KR} - 1}{P_1} w_1 + \frac{(e^{KR/2} - 1)^2}{P_1 P_2} w_2 + \dots + \frac{(e^{KR/K} - 1)^K}{\prod_{i=1}^K P_i} w_K^*. \quad (20)$$

Denoting $c_i = w_i(e^{KR/i} - 1)^i$, we further simplify (20) as

$$p_{out}(K) \approx w_0 + \frac{c_1}{P_1} + \frac{c_2}{P_1 P_2} + \cdots + \frac{c_K}{\prod_{i=1}^K P_i}.$$

Since the optimal power vector lies in a convex cone as shown in Theorem 1, the problem can be formulated as a convex optimization problem (refer to Appendix-C for the convexity proof):

$$\begin{aligned} \min_{\mathbf{P}_K \in D_+} \quad & w_0 + \frac{c_1}{P_1} + \frac{c_2}{P_1 P_2} + \cdots + \frac{c_K}{\prod_{i=1}^K P_i} \\ \text{s.t.} \quad & \sum_{i=1}^K P_i \leq KP, \end{aligned} \quad (21)$$

where $D_+ = \{\mathbf{P} \in \mathbb{R}_+^K : P_1 \geq P_2 \geq \cdots \geq P_K \geq 0\}$ is a convex cone. Thus, the optimal power vector can be efficiently solved with standard convex optimization algorithms such as the interior point method [12].

The simulation results are shown in Fig. 3, where we set the simulation parameters as: $R = 0.5$ nats/s/Hz, $1/\lambda = 5$ for the exponential random attack, and average power $P = 10$ dB. As we can see, the power vector derived by solving problem (21) achieves better performance in terms of the outage probability than the uniform power allocation case.

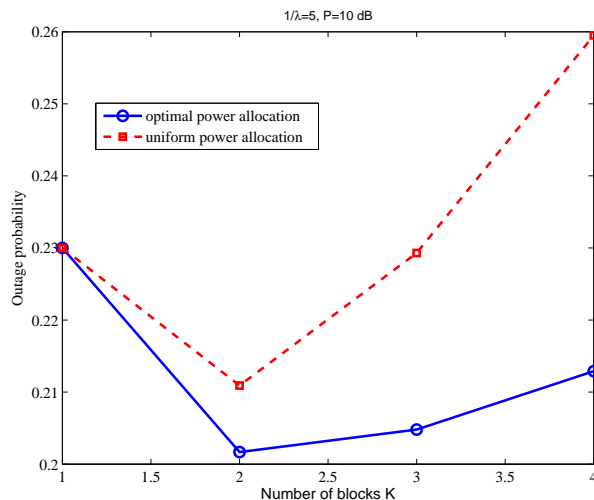


Fig. 3. Outage probability with non-uniform and uniform power allocation.

2) *Optimal Power Allocation over i.i.d. Log-normal Fading*: When the fading gain has a log-normal distribution, we can also approximate the problem as a convex one by minimizing the upper bound of the

objective function. Since we have

$$\sum_{i=1}^L \log(\alpha_i P_i) < \sum_{i=1}^L \log(1 + \alpha_i P_i),$$

the outage probability is upper-bounded as follows:

$$\begin{aligned} & \Pr\left\{\sum_{i=1}^L \log(1 + \alpha_i P_i) < KR\right\} \\ & < \Pr\left\{\sum_{i=1}^L \log(\alpha_i P_i) < KR\right\} \\ & = w_0 + \sum_{n=1}^K \Pr\left\{\sum_{i=1}^n \log \alpha_i < KR - \sum_{i=1}^n \log P_i\right\} w_n. \end{aligned}$$

Thus, the optimization problem of (16) is translated into the following problem, where we essentially minimize the upper bound:

$$\begin{aligned} \min_{\mathbf{P}_K} \quad & w_0 + \sum_{n=1}^K \Pr\left\{\sum_{i=1}^n \log \alpha_i < KR - \sum_{i=1}^n \log P_i\right\} w_n \\ \text{s.t.} \quad & \sum_{i=1}^K P_i \leq KP. \end{aligned} \tag{22}$$

Let the α_i 's be independent and log-normal random variables, i.e., $\log \alpha_i \sim \mathcal{N}(0, 1)$, $\forall i$. Since the sum of n standard normal random variables is a Gaussian random variable with zero mean and variance n , we have

$$\Pr\left\{\sum_{i=1}^n \log \alpha_i \leq x\right\} = \frac{1}{2} \left\{1 + \operatorname{erf}\left(\frac{x}{\sqrt{2n}}\right)\right\}, \tag{23}$$

where $\operatorname{erf}(x)$ is the error function. Substituting (23) into (22) yields the new objective function p_{out} :

$$p_{out} = w_0 + \sum_{n=1}^K \frac{1}{2} \left\{1 + \operatorname{erf}\left(\frac{KR - \sum_{i=1}^n \log P_i}{\sqrt{2n}}\right)\right\} w_n. \tag{24}$$

In general, (24) is not a convex function. However, under some special circumstances as described in Appendix-D, the problem in (22) with the objective replaced by (24) can be rewritten as a convex

problem, which is given as following:

$$\begin{aligned}
\min_{\mathbf{P}_K} \quad & w_0 + \sum_{n=1}^K \frac{1}{2} \left\{ 1 + \operatorname{erf} \left(\frac{KR - \sum_{i=1}^n \log P_i}{\sqrt{2n}} \right) \right\} w_n \\
\text{s.t.} \quad & \sum_{i=1}^K P_i \leq KP \\
& KR - \log P_1 \leq 0 \\
& KR - \log P_1 - \log P_2 \leq 0 \\
& \dots \dots \dots \\
& KR - \sum_{i=1}^K \log P_i \leq 0.
\end{aligned} \tag{25}$$

Therefore, efficient algorithms can be applied to solve the above problem.

Numerical results are provided as follows. Assume that the outage probability target is set as $\eta = 0.3$, the attack time is an exponential random variable with parameter $1/\lambda = 4$, and the fading gains are standard log-normal random variables. As we see from Fig. 4, the optimal power allocation leads to a significantly larger outage capacity over the uniform power allocation case. Moreover, as K increases, the outage capacity with the optimal power allocation may even increase to a maximum value while the outage capacity with uniform power allocation monotonically decreases. This suggests that, with the potential of a random attack, we can still span the codeword over more than one block to exploit diversity and achieve higher outage capacity if the power allocation and the codeword length K are smartly chosen.

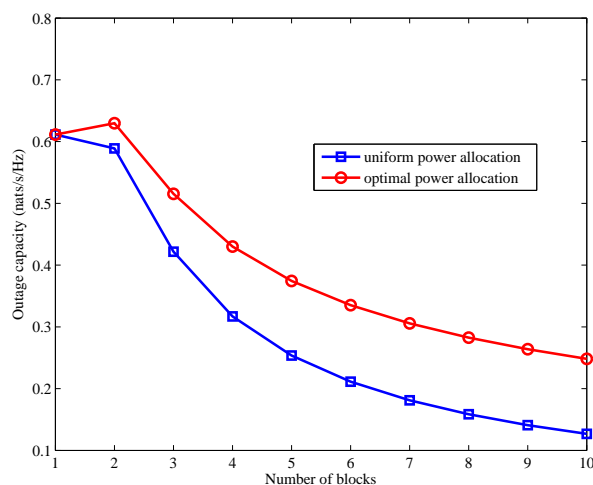


Fig. 4. Outage capacity v.s. number of blocks K , $1/\lambda = 4$, average power $P = 3$.

V. OUTAGE PROBABILITY OVER PARALLEL DYING CHANNELS

In the dying channel example of cognitive radio networks, secondary users have access to vacant frequency bands that are licensed to primary users. Some primary users may suddenly show up and take over some frequency bands, which results in connection losses if these frequency bands are being used by certain secondary users. Hence, each sub-channel (a frequency band) may have a different random delay constraint for information transmission due to the uncertainty of non-uniform primary user occupancy patterns. Specifically, the above system can be modeled as follows. Given a link with N parallel sub-channels as shown in Fig. 5, the codeword is spanned in time domain over K blocks and also across all the N sub-channels. In some sub-channels, random attacks terminate the transmission before it is completed such that less than K blocks are delivered. For other sub-channels, K blocks are assumed to be safely transmitted. What is the maximum rate for reliable communication over such a link? For the single channel case, it turns out that there is no way to achieve arbitrarily small outage with a finite transmit power. However, in this section we show that an arbitrarily small outage probability is achievable by exploiting the inherent multi-channel diversity.

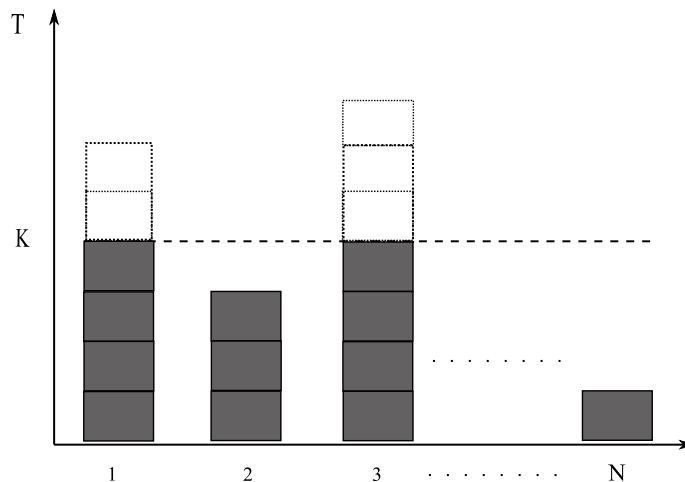


Fig. 5. Parallel dying channels.

In this section, we extend the results of the single dying channel to the parallel multi-channel case.

Definition 2: The outage probability of the parallel multi-channel case is given as

$$p_{out}(R, P, N) = \Pr \left\{ \sum_{i=1}^N \frac{1}{K} \sum_{k=1}^{L_i} \log(1 + \alpha_k^{(i)} P/N) < R \right\}, \quad (26)$$

where R is the total rate over N sub-channels, $\alpha_k^{(i)}$ is the fading gain of block k at sub-channel i , N is the number of sub-channels, $L_i = \min\{K, \lfloor T_i \rfloor\}$ is the random number of surviving blocks at sub-channel

i , K is the number of blocks over which a codeword is spanned in the time domain, and P is the total average power such that P/N is the average power for each sub-channel. Since the asymptotic behavior is concerned, uniform power allocation is assumed over N sub-channels. According to different attack models, in the next two sections we investigate the asymptotic behavior of the above outage probability in two cases: the independent random attack case and the m -dependent random attack case.

VI. INDEPENDENT RANDOM ATTACK CASE

Let the average power P be finite. Since $\log(1+x) \approx x$ if $|x| \ll 1$, when N is large, we rewrite (26) as

$$p_{out}(R, P, N) \approx \Pr \left\{ \frac{1}{N} \sum_{i=1}^N \frac{1}{K} \sum_{k=1}^{L_i} \alpha_k^{(i)} P < R \right\}. \quad (27)$$

We assume that the fading gains $\alpha_k^{(i)}$'s are i.i.d., and let the random variable Y_i be

$$Y_i = \frac{1}{K} \sum_{k=1}^{L_i} \alpha_k^{(i)}.$$

For the case of independent random attack, we assume that L_i 's are i.i.d., and hence Y_i 's are i.i.d..

The outage probability given by (27) can be recast as:

$$p_{out}(R, P, N) \approx \Pr \left\{ \frac{1}{N} \sum_{i=1}^N Y_i < R/P \right\}. \quad (28)$$

Since Y_i 's are i.i.d., according to the central limit theorem, as the number of sub-channels $N \rightarrow \infty$, we have

$$\frac{1}{N} \sum_{i=1}^N Y_i \rightarrow \mathcal{N}(\mu_Y, \sigma_Y^2/N). \quad (29)$$

According to Theorem 7.4 in [16] on the sum of a random number of random variables, we derive the following relations:

$$\mu_Y = \frac{1}{K} E(L) E(\alpha) \quad (30)$$

$$\sigma_Y^2 = \frac{1}{K^2} [E(L) Var(\alpha) + Var(L) E(\alpha)^2], \quad (31)$$

where α is a nominal random variable denoting the fading gain, L is a nominal integer random variable denoting the number of surviving blocks of each sub-channel, and $E(\cdot)$ and $Var(\cdot)$ denote the expectation

and variance, respectively. As such, the outage probability can be approximated as:

$$p_{out}(R, P, N) \approx \Phi\left(\frac{R/P - \mu_Y}{\sigma_Y/\sqrt{N}}\right). \quad (32)$$

As $N \rightarrow \infty$, $\frac{1}{N} \sum_{i=1}^N Y_i$ converges to μ_Y . The outage probability decreases to 0 over N if R/P is less than μ_Y , or converges to 1 if R/P is larger than μ_Y .¹ That is, even though all sub-channels are subject to fatal attacks, the outage probability can still be made arbitrarily small when N is large enough if the rate per unit cost is set in a conservative fashion, where μ_Y is a key threshold. This is remarkably different from the single dying channel case in which the outage probability is always finite since there are only a finite and random number of blocks to span a codeword.

VII. m -DEPENDENT RANDOM ATTACK CASE

In the previous section, we discussed the case where L_i 's are independent. However, in a practical system, such as cognitive radio networks, the primary users usually occupy a bunch of adjacent sub-channels instead of picking up sub-channels independently. Thus, the L_i 's across adjacent sub-channels are possibly correlated; and consequently the achievable rates across adjacent sub-channels are also correlated. On the other hand, if two sub-channels are far away from each other, it is reasonable to treat them as independent. Thus, we assume that Y_i 's are strictly stationary² and m -dependent³ with the same mean and variance.

A. Central limit theorem for m -dependent random variables

We first cite the central limit theorem for stationary and m -dependent summands from [18] (Theorem 9.1 therein).

Theorem 3 (Hoeffding and Robbins): Suppose $\{X_n, n \geq 1\}$ is a strictly stationary m -dependent sequence with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2 < \infty$. Then as $N \rightarrow \infty$, we have

$$\frac{1}{\sqrt{N}} \sum_{i=1}^N (X_i - \mu) \rightarrow \mathcal{N}(0, v_m), \quad (33)$$

where $v_m = \sigma^2 + 2 \sum_{i=1}^m Cov(X_t, X_{t+i})$ with $Cov(X_t, X_{t+i})$ the covariance of X_t and X_{t+i} .

Proof: The detailed proof can be found in [19]. ■

¹ R/P is interpreted as the rate per unit cost in [17]. It is interesting to see that the quantity of rate per unit cost plays an important role here, which is due to the fact that we operate over both a finite power and a finite coding length.

²Call a sequence $\{X_n, n \geq 1\}$ strictly stationary if, for every k , the joint distribution of $(X_{n+1}, \dots, X_{n+k})$ is independent of n .

³Call a sequence $\{X_n, n \geq 1\}$ m -dependent if for any integer t , the σ -fields $\sigma(X_j, j \leq t)$ and $\sigma(X_j, j \geq t + m + 1)$ are independent. Simply put, X_i and X_j are independent if $|i - j| > m$.

B. Asymptotic outage probability

As assumed, the random sequence $\{Y_1, Y_2, \dots, Y_N\}$ is stationary and m -dependent, and Y_i 's have the same mean and variance. Then the covariance is given as:

$$\text{Cov}(Y_i Y_{i+h}) = \begin{cases} 0 & |h| > m \\ \gamma(h) - \mu_Y^2 & |h| \leq m, \end{cases} \quad (34)$$

where μ_Y is the expectation of Y_i given in (30) and $\gamma(h) = E(Y_i Y_{i+h})$. Meanwhile,

$$v_m = \sigma_Y^2 + 2 \sum_{h=1}^m (\gamma(h) - \mu_Y^2). \quad (35)$$

Due to the fact that the fading gains $\alpha_p^{(i)}$ and $\alpha_q^{(i+h)}$ are independent if $p \neq q$ or $h \neq 0$, we could easily obtain $\gamma(h)$ for $|h| \leq m, h \neq 0$, as:

$$\begin{aligned} \gamma(h) &= \frac{1}{K^2} E \left[\sum_{p=1}^{L_i} \alpha_p^{(i)} \sum_{q=1}^{L_{i+h}} \alpha_q^{(i+h)} \right] \\ &= \frac{1}{K^2} E \left[E \left(\sum_{p=1}^{L_i} \alpha_p^{(i)} \sum_{q=1}^{L_{i+h}} \alpha_q^{(i+h)} \middle| L_i L_{i+h} \right) \right] \\ &= \frac{\mu_\alpha^2}{K^2} E(L_i L_{i+h}) \end{aligned} \quad (36)$$

Assume that L_i and L_j have the same correlation coefficient ρ if $|i - j| \leq m$ and $i \neq j$. The correlation matrix is given as

$$C = \begin{pmatrix} 1 & \rho & \cdots & 0 & 0 & 0 \\ \rho & 1 & \rho & \cdots & 0 & 0 \\ \vdots & \rho & 1 & \rho & \cdots & 0 \\ 0 & \cdots & \rho & \ddots & \rho & \cdots \\ 0 & 0 & \cdots & \rho & \ddots & \rho \\ 0 & 0 & 0 & \cdots & \rho & 1 \end{pmatrix}.$$

Note that the following main results can be also derived for other correlation matrices.

Then (36) is simplified as

$$\gamma(h) = \frac{\mu_\alpha^2}{K^2} (\rho \sigma_L^2 + \mu_L^2), \quad (37)$$

where ρ is a non-negative correlation coefficient, μ_L and σ_L are the mean and variance of the random variable L , respectively.

Substituting (30), (31), and (37) into (35), we have

$$v_m = \sigma_Y^2 + 2m \frac{\rho \mu_\alpha^2 \sigma_L^2}{K^2} \quad (38)$$

$$= \frac{\mu_L \sigma_\alpha^2}{K^2} + \frac{\mu_\alpha^2 \sigma_L^2}{K^2} (1 + 2m\rho). \quad (39)$$

According to Theorem 3, we have

$$\frac{1}{\sqrt{N}} \sum_{i=1}^N (Y_i - \mu_Y) \rightarrow \mathcal{N}(0, v_m).$$

By simple manipulation, we have

$$\frac{1}{N} \sum_{i=1}^N Y_i \rightarrow \mathcal{N}(\mu_Y, v_m/N), \quad (40)$$

where μ_Y is given in (30) and v_m is given in (39). Hence, the outage probability for the m -dependent random attack case can be approximated as follows when N is large,

$$p_{out}(R, P, N) \approx \Phi\left(\frac{R/P - \mu_Y}{\sqrt{v_m/N}}\right). \quad (41)$$

As we see from (38) that $v_m \geq \sigma_Y^2$, comparing (32) and (41), we conclude that the outage probability of the independent attack case is smaller than that of the m -dependent case given the same setting when the rate per unit cost R/P is less than μ_Y and the number of sub-channels N is large.

VIII. OUTAGE EXPONENT

As we learn from the previous sections, the outage probability over parallel multiple channels goes to zero as N increases if $R/P < \mu_Y$ for both of the two attack cases. In this section, we investigate how fast the outage probability decreases as N increases for both cases, which is measured by the outage exponent [20] defined as

$$\mathcal{E}(t) = \lim_{N \rightarrow \infty} \frac{-\log p_{out}(R, P, N)}{N}, \quad (42)$$

where $t = R/P$.

A. Independent Attack Case

According to the results in [20], we could derive the outage exponent for the independent attack case as

$$\mathcal{E}(t) = \sup_{s \leq 0} \{st - \Lambda(s)\}, \quad (43)$$

for $\forall t \leq t_0$, where $t_0 = \mu_Y$ and

$$\begin{aligned}\Lambda(s) &:= \log E[\exp(sY_i)] \\ &= \log M_Y(s),\end{aligned}\tag{44}$$

with $M_Y(s)$ the moment generating function of Y_i . According to Theorem 7.5 in [16], we have $M_Y(s) = h(f(s/K))$ where $h(z)$ and $f(s)$ are the probability generating function of the discrete random variable L_i and the moment generating function of the continuous random variable $\alpha_k^{(i)}$, respectively.⁴

Example: If Rayleigh fading is assumed, $\alpha_k^{(i)}$ is exponentially distributed; hence the corresponding moment generating function is $f(s) = (1 - s/\lambda_\alpha)^{-1}$, where λ_α is the parameter for the distribution of the $\alpha_k^{(i)}$. Assuming that the random attack time has an exponential distribution, L is an integer random variable with following distribution:

$$\begin{aligned}w_0 &= \Pr\{L = 0\} = \Pr\{0 \leq T < 1\}, & w_1 &= \Pr\{L = 1\} = \Pr\{1 \leq T < 2\}, \dots, \\ w_{K-1} &= \Pr\{L = K - 1\} = \Pr\{K - 1 \leq T < K\}, & w_K &= \Pr\{L = K\} = \Pr\{K \leq T\}.\end{aligned}$$

Thus, we have $h(z) = \sum_{i=0}^K w_i z^i$ and $M_Y(s) = \sum_{i=0}^K w_i (1 - s/(K\lambda_\alpha))^{-i}$. Then we can derive the outage exponent numerically by solving (43) for a given t .

B. m -dependent Attack Case

For the m -dependent attack case, the techniques used in deriving the outage exponent for the independent attack case does not apply any more since here Y_i 's are not independent. In this case, since the outage

⁴The moment generating function of the sum of a random number of random variables, i.e., $S_L = X_1 + X_2 + \dots + X_L$, is the compound function $h(f(s))$, where L is a random integer independent of X_i , $h(z)$ is the probability generating function of L , and $f(s)$ is the moment generating functions of X_i .

probability has an approximate normal distribution, we have

$$\begin{aligned}
p_{out}(R, P, N) &\approx \Phi\left(\frac{R/P - \mu_Y}{\sqrt{v(m)/N}}\right) \\
&= Q\left(\frac{\mu_Y - R/P}{\sqrt{v(m)/N}}\right) \\
&\leq \exp\left(\frac{-\left(\frac{\mu_Y - R/P}{\sqrt{v(m)/N}}\right)^2}{2}\right) \\
&= \exp\left(-N\frac{(\mu_Y - R/P)^2}{2v_m}\right)
\end{aligned} \tag{45}$$

Therefore, an approximate outage exponent can be quantified from the upper bound as

$$\mathcal{E}_{mdp}(R/P) \approx \frac{(\mu_Y - R/P)^2}{2v_m}, \tag{46}$$

where v_m is given in (39). The outage exponent obtained by (43) is derived by using the large deviation techniques. Thus, it is exact while the outage exponent given by (46) for the m -dependent attack case is approximate. However, when $R/P \ll \mu_Y$, this approximation is accurate since the exponential bound is tight for the Q -function when its argument is large.

Numerical results are provided here to validate our analysis for the parallel multi-channel case. We choose the random attack time T to be exponentially distributed with parameter $1/\lambda = 5$ and K is chosen to be 5. Rayleigh fading is assumed and the fading gain $\alpha_k^{(i)}$ is exponentially distributed with parameter 1 and the noise has unit power. First, we demonstrate the convergence of the outage probability for the independent attack case and the m -dependent attack case, where the value of μ_Y according to the above simulation setup is 0.571. For the independent attack case, as shown in Fig. 6, the solid and dashed curves are derived by (32) while the circles and crosses are obtained by simulations. We also observe similar convergence for the m -dependent attack case in Fig. 7. In both figures, the outage probability goes to 0 if $R/P < \mu_Y$, or goes to 1 if $R/P > \mu_Y$. We see that the accuracy of Gaussian approximations is acceptable with reasonably large N values.

Second, we compare the outage probability performance between the independent case and the m -dependent case. Here $P = 2$ and $R = 0.5$ nats/s. As shown in Fig. 8, the outage performance of the m -dependent case is worse than that of the independent case even when $m = 1$ and $\rho = 0.8$. This is due to the fact that when $R/P < \mu_Y$, the independent attack case is expected to have a smaller outage probability

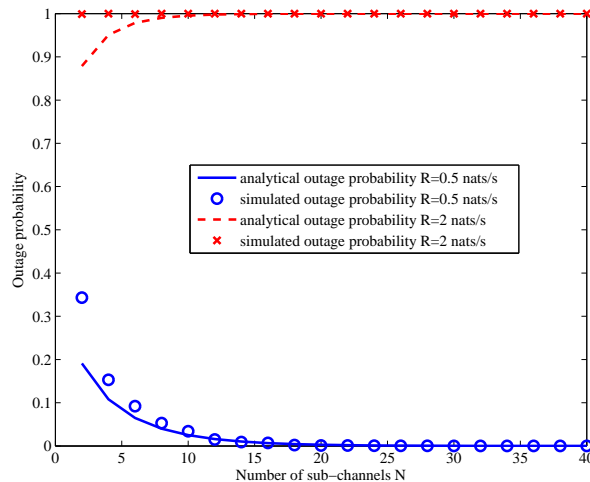


Fig. 6. Outage probability convergence behavior of the independent case: $\mu_Y=0.571$ and $P=2$.

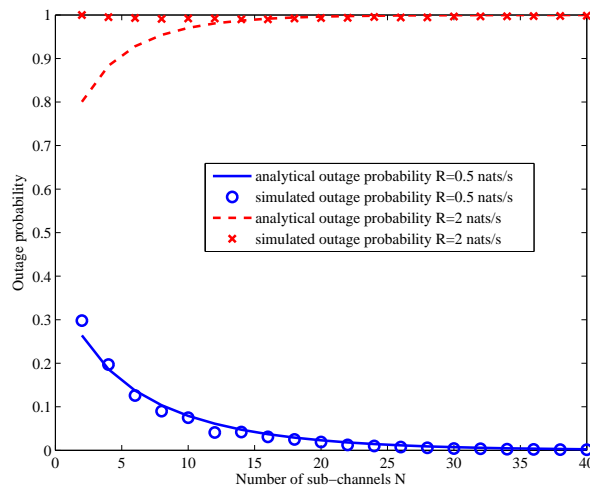


Fig. 7. Outage probability convergence behavior of the m -dependent case: $\mu_Y=0.571$, $m=1$, $\rho=0.8$, and $P=2$.

as we discussed at the end of Section VIII-B. However, the outage probability of the m -dependent case still decreases to 0 but at a slower rate as the number of sub-channels N increases, which is caused by the fact that the m -dependent attack case has a smaller outage exponent.

In Fig. 9, we compare the various outage exponent values between these two cases over the rate per unit cost R/P with the simulation setup as follows: $K = 5$, $m = 1$, and $\rho = 0.8$. First, we see that the outage exponent for the independent attack case is larger than that of the m -dependent attack case when the average attack time $1/\lambda$ is the same. Second, for both of the independent attack case and the m -dependent attack case, a larger average attack time $1/\lambda$ results in a larger outage exponent.

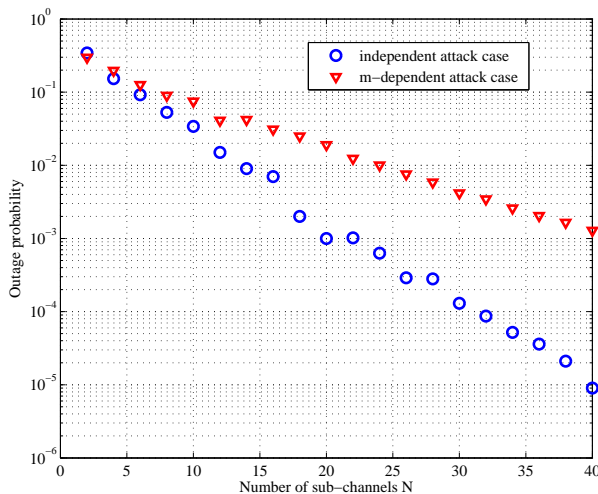


Fig. 8. Outage probabilities comparison. $P=2$, $R=0.5$ nats/s, $m=1$, and $\rho=0.8$.

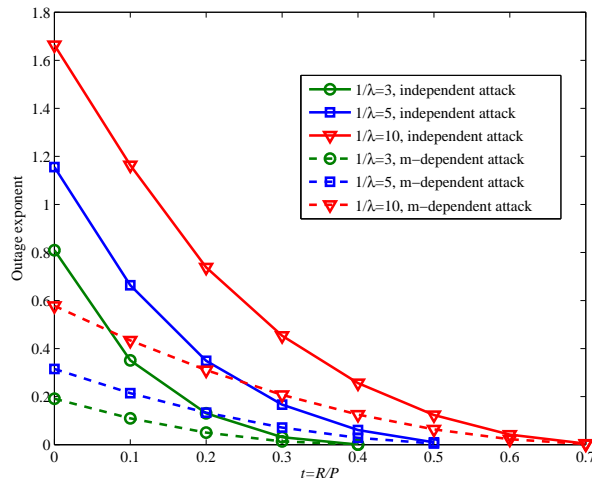


Fig. 9. Outage exponents for independent and m -dependent random attack cases: $m=1$, $\rho=0.8$, and $K=5$.

IX. CONCLUSION

In this paper, we considered a new type of channels called dying channels, where a random attack may happen during the transmission. We first investigated a single dying channel by modeling it as a K -block BF-AWGN channel with a random delay constraint. We obtained the optimal coding length K that minimizes the outage probability when uniform power allocation was assumed. Next, we investigated the general properties of the optimal power allocation for a given K . For some special cases, we cast the optimization problem into convex ones which can be efficiently solved. As an extension of the single dying channel result, we investigated the case of parallel dying channels and studied the asymptotic outage

behavior with two different attack models: the independent-attack case and the m -dependent-attack case. It has been shown that the outage probability diminishes to zero for both cases as the number of sub-channels increases if the target *rate per unit cost* is less than a given threshold. Moreover, the outage exponents for both cases were studied to reveal how fast the outage probability improves over the number of sub-channels.

APPENDIX

A. Proof of Theorem 1

Let us consider minimizing the outage probability given by (17). When $K = 1$, the proof is trivial.

When $K = 2$, the outage probability is

$$\begin{aligned} p_{out}(2) &= w_0 + \Pr\{\log(1 + \alpha_1 P_1) < 2R\}w_1 \\ &\quad + \Pr\{\log(1 + \alpha_1 P_1) + \log(1 + \alpha_2 P_2) < 2R\}w_2^*. \end{aligned}$$

As we see from the above equation, if $P_1 < P_2$, we have $\Pr\{\log(1 + \alpha_1 P_2) < 2R\} < \Pr\{\log(1 + \alpha_1 P_1) < 2R\}$. Hence, we can achieve a smaller $p_{out}(2)$ by swapping P_1 and P_2 , since the last term in $p_{out}(2)$ is not affected by such a swapping while the second term is decreased.

When $K \geq 3$, for any $j > i$, ($i, j \in \{1, \dots, K\}$), if $P_i < P_j$, by swapping P_i and P_j , all the terms containing both P_i and P_j , i.e., all the probability terms in the form of $\Pr\{\dots + \log(1 + \alpha_i P_i) + \dots + \log(1 + \alpha_j P_j) + \dots < KR\}$ will not be affected. However, the probability terms containing P_i but not P_j can be decreased by such a swapping. Thus, we could achieve a smaller outage probability in total.

Therefore, the optimal power allocation profile over i.i.d. fading is always non-increasing, i.e., $P_1 \geq P_2 \geq \dots \geq P_K \geq 0$.

B. Proof of Theorem 2

When the coding length $K = 1$, the outage probability is

$$p_{out}(1) = \Pr\{\log(1 + \alpha P) < R\} \Pr\{T > 1\} + w_0.$$

When we choose any other arbitrary values for K , i.e., $K = M$ and $M \neq 1$, according to (17), the outage probability is

$$\begin{aligned}
& p_{out}(M) \\
= & \Pr \left\{ \frac{1}{M} \sum_{i=1}^M \log(1 + \alpha P_i) < R \right\} \Pr\{T > M\} \\
& + w_0 + \sum_{i=1}^{M-1} \Pr \left\{ \frac{1}{M} \sum_{l=1}^i \log(1 + \alpha P_l) < R \right\} w_i.
\end{aligned} \tag{47}$$

Due to the concavity of the log function, we have $\frac{1}{M} \sum_{i=1}^M \log(1 + \alpha P_i) \leq \log(1 + \alpha P)$. Hence,

$$\Pr \left\{ \frac{1}{M} \sum_{l=1}^M \log(1 + \alpha P_l) < R \right\} \geq \Pr\{\log(1 + \alpha P) < R\}. \tag{48}$$

Moreover, it is obvious that summing over only a portion of the M blocks yields an even smaller value, i.e., $\frac{1}{M} \sum_{l=1}^i \log(1 + \alpha P_l) \leq \log(1 + \alpha P)$, with $1 \leq i \leq M - 1$. If $\exists P_j > 0$, for $i < j \leq M$, the strong inequality holds. Therefore, we have

$$\Pr \left\{ \frac{1}{M} \sum_{l=1}^i \log(1 + \alpha P_l) < R \right\} \geq \Pr\{\log(1 + \alpha P) < R\}. \tag{49}$$

Noting that $\sum_{i=1}^{M-1} w_i = \Pr\{1 < T \leq M\}$, and considering (48) and (49), the following inequality can be derived for (47):

$$\begin{aligned}
p_{out}(M) & \geq w_0 + \Pr\{\log(1 + \alpha P) < R\} \cdot \\
& (\Pr\{T > M\} + \Pr\{1 < T \leq M\}) \\
& = p_{out}(1).
\end{aligned} \tag{50}$$

From (50), we see that $p_{out}(1)$ has the smallest outage probability when fading gains are the same, which means that the optimal coding length is $K = 1$ with $P_1 = P$.

C. Convexity of the optimization problem in (21)

We first check the Hessian matrix of the objective function in terms of P_i .

$$\nabla^2 p_{out} = \nabla^2 \frac{c_1}{P_1} + \dots + \nabla^2 \frac{c_K}{\prod_{i=1}^K P_i}. \tag{51}$$

The j th term is:

$$\begin{aligned} & \nabla^2 \left(\frac{c_j}{\prod_{i=1}^j P_i} \right) \\ = & c_j \begin{pmatrix} \frac{2}{P_1^3 \prod_{i=2}^j P_i} & \frac{1}{P_1^2 P_2^2 \prod_{i=3}^j P_i} & \cdots & \frac{1}{P_1^2 P_j^2 \prod_{i=2}^{j-1} P_i} & \mathbf{0} \\ \frac{1}{P_1^2 P_2^2 \prod_{i=3}^j P_i} & \frac{2}{P_1 P_2^3 \prod_{i=3}^j P_i} & \cdots & \frac{1}{P_2^2 P_j^2 \prod_{i=1, i \neq 2}^{j-1} P_i} & \mathbf{0} \\ \cdots & \cdots & \ddots & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{aligned}$$

Let $\mathbf{z} \in \mathbb{R}^K$, then

$$\mathbf{z}^T \nabla^2 \left(\frac{c_j}{\prod_{i=1}^j P_i} \right) \mathbf{z} = \frac{1}{\prod_{i=1}^j P_i} \mathbf{z}^T \mathbf{P}^{(j)} (\mathbf{P}^{(j)})^T \mathbf{z} + \mathbf{z}^T \mathbf{M} \mathbf{z} \geq 0,$$

where $\mathbf{P}^{(j)} = (1/P_1, 1/P_2, \dots, 1/P_j, 0, \dots, 0)^T$, and $\mathbf{M} = \text{diag} \left(\frac{1}{P_1^2}, \frac{1}{P_2^2}, \dots, \frac{1}{P_j^2}, 0, \dots, 0 \right)$.

Therefore, (51) as the summation of all the K terms is positive semi-definite. Hence p_{out} is a convex function in terms of \mathbf{P}_K . In addition, \mathbf{P}_K lies in a convex cone as shown in Theorem. 1. Hence the problem is a convex problem.

D. Sufficient conditions for the convexity of the optimization problem in (25)

Let $h : \mathbb{R}^k \rightarrow \mathbb{R}$, $g : \mathbb{R}^n \rightarrow \mathbb{R}^k$, and $f = h \circ g : \mathbb{R}^n \rightarrow \mathbb{R}$ be defined as:

$$f(x) = h(g(x)), \quad \mathbf{dom} f = \{x \in \mathbf{dom} g \mid g(x) \in \mathbf{dom} h\},$$

where \mathbf{dom} is the domain of a function. The following two lemmas can be established.

Lemma 1: f is convex if h is convex and nondecreasing, and g is convex.

Proof: See Chapter 3 in [12]. ■

Lemma 2: The outage probability function given by (24) is convex, if

$$\begin{aligned} KR - \log P_1 & \leq 0, \\ KR - \log P_1 - \log P_2 & \leq 0, \\ & \dots\dots \\ KR - \sum_{i=1}^K \log P_i & \leq 0. \end{aligned}$$

Proof: As we know, the error function $\text{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2} dt$ is a convex function for $x \leq 0$ and it is non-decreasing.

Let

$$g_m(\mathbf{P}_K) = \frac{KR - \sum_{i=1}^m \log P_i}{\sqrt{2m}}.$$

Since $g_m(\mathbf{P}_K)$ is convex and $\text{erf}(x)$ is convex for $x \leq 0$ and nondecreasing, according to Lemma 1, if $g_m(\mathbf{P}_K) \leq 0$, $\text{erf}(g_m)$ is convex over \mathbf{P}_K . Hence, the objective function given by (24) is convex under the conditions given in the lemma.

Since the constraints in (25) are obviously convex and the objective function is convex under these constraints, the problem in (25) is convex. ■

REFERENCES

- [1] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: Information-theoretic and communications aspects," *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2619-2692, Oct. 1998.
- [2] A. Goldsmith and P. P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. Inf. Theory*, vol. 43, no. 6, pp. 1986-1992, Nov. 1997.
- [3] G. Caire and S. Shamai, "On the capacity of some channels with channel state information," *IEEE Trans. Inf. Theory*, vol. 45, no. 6, pp. 2007-2019, Sep. 1999.
- [4] G. Caire, G. Taricco, and E. Biglieri, "Optimal power control over fading channels," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1468-1489, Jul. 1999.
- [5] R. Berry, "Power and delay trade-offs in fading channels," *Ph.D. Dissertation, MIT, Cambridge, MA*, 2000.
- [6] P. Whiting and E. Yeh, "Broadcasting Over Uncertain Channels with Decoding Delay Constraints," *IEEE Trans. Inf. Theory*, vol. 52, no. 3, pp. 904-921, Mar. 2006.
- [7] L. H. Ozarow, S. Shamai, and A. D. Wyner, "Information theoretic considerations for cellular mobile radio," *IEEE Trans. Veh. Technol.*, vol. 43 no. 2, pp. 359-378, 1994.
- [8] S. Hanly and D. Tse, "Multi-access fading channels-Part II: Delay-limited capacities," *IEEE Trans. Inf. Theory*, vol. 44, no.7, pp. 2816-2831, Nov. 1998.
- [9] R. Negi and J. M. Cioffi, "Delay-constrained capacity with causal feedback," *IEEE Trans. Inf. Theory*, vol. 48, no. 9, pp. 2478-2494, Sep. 2002.
- [10] J. Wolfowitz, *Coding Theorems of Information Theory*, 3rd ed. Berlin/Heidelberg: Springer-Verlag, 1978.
- [11] T. Cover and J. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.
- [12] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.
- [13] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2005.
- [14] M. Zeng, R. Zhang, and S. Cui, "On the outage capacity of a dying channel," in *Proc. of Global Communications Conference (GLOBECOM)*, pp.1-5, New Orleans, Dec. 2008.
- [15] M. Zeng, R. Zhang, and S. Cui, "Optimal transmission for dying channels," to appear at ICC, Dresden, Germany, Jun. 2009.

- [16] S. Miller and D. Childers, *Probability and Random Processes: With Applications to Signal Processing and Communications*, Elsevier Academic Press, 2004.
- [17] S. Verdú, "On channel capacity per unit cost," *IEEE Trans. Inf. Theory*, vol. 36, no. 5, pp. 1019-1030, Sep. 1990.
- [18] A. DasGupta, *Asymptotic Theory of Statistics and Probability*, Springer, 2008.
- [19] W. Hoeffding and H. Robbins, "The central limit theorem for dependent random variables", *Duke Math. J.*, vol 15, no. 3, pp. 773-780, 1948.
- [20] W. Zhang and U. Mitra, "On outage behavior of wideband slow-fading channels", arXiv:0710.0291v1, <http://arxiv.org/abs/cs/0509001>.
- [21] M. Zeng, R. Zhang, and S. Cui, "Asymptotic behavior of the outage probability of a dying channel," submitted to Milcom'09.