Optimizing Adaptive Modulation in Wireless Networks via Utility Maximization

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***Abstract*— We investigate adaptive modulation using the network utility maximization framework. We derive new crosslayer optimal power and rate adaptation policies for several practical modulation schemes. The behavior of these crosslayer policies is found to differ from policies based on physical-layer optimization only. The multiple flow single link case is analyzed and optimal power and rate policies found. The multiple interfering link case is investigated and a numerical method presented to find optimal policies for this case.**

I. INTRODUCTION

Adaptive Modulation, AM, is a physical layer technique to maximize the performance of a single wireless link over flat fading channels, subject to constraints on average transmission power. AM yields policies that adapt to changes in the channel condition. Spectral efficiency, the average transmission rate for a fixed bandwidth, is the metric generally used to measure performance. We term this technique SE/AM for Spectrally Efficient Adaptive Modulation. SE/AM is optimized only for a single link and does not take into account the demands of upper layer network protocols, creating a possible mismatch between the optimum behavior expected by the network and supplied by the link. To address these limitations, we extend AM using the Network Utility Maximization, NUM, which we term NUM/AM. NUM/AM is a crosslayer technique to maximize network performance over a set of possibly interfering links. Performance is measured by utility functions, which model the metrics associated with network protocols or applications. NUM/AM balances the demands of the network with what the links can supply. In particular, NUM/AM policies optimally adjust individual link parameters to maximize overall network performance.

SE/AM has generated considerable research interest and commercial activity [1], [2], [3], [4], [5], [6]. The fundamental concept is the real time adjustment of transmitter parameters, such as rate, power, BER, coding rate, etc., under flat fading while meeting an average link budget constraint. SE/AM rate and power policies are greedy, taking advantage of good channel conditions and budgeting little or no transmitter power to poor channel conditions.

NUM has been extensively studied in the context of wireline networks and is a rapidly expanding area of research in wireless networks [7], [8]. In NUM the goal is to maximize network performance as measured at an upper layer protocol or network application level. The network is modeled as a collection of links, generally of deterministic capacity, that can carry one or more flows. In this paper we consider capacities that are functions of the channel state and vary over time. Conceptually, NUM and AM are symbiotic, with NUM modeling the upper layers of the protocol stack and AM the lower, as illustrated in figure 1.

In this paper we study NUM/AM at several levels of increasing complexity. A single link is initially considered and a new, optimal, closed form policy found for a general class of utility functions. The adaptive rate and power policies are found to be very different than those of SE/AM for several practical modulation schemes. At the next level of complexity, multiple flows over a single link is investigated and yields results similar to the single flow case. Finally, the case of multiple interfering links is investigated, and a method to compute the jointly optimal NUM/AM rate and power policies for each link is described.

The remainder of this paper is organized as follows: Section II describes the system model and a general class of utility functions. Section III formally presents the NUM/AM problem and describes an analytical method for finding the solution. Section IV uses the general framework of section III to find optimal single link policies for the specific modulations of MQAM and MPSK. Section V considers multiple interfering links and describes a numerical method for finding the optimal NUM/AM strategies. Conclusions and future work are presented in Section VI.

II. SYSTEM MODEL

A. Wireless Model

In this section we present our system model and notation, following that of [6]. The link model is illustrated in Figure 1. The channel scales the transmitted signal by the channel state variable G and adds white Guassian noise with power N. The channel gains vary over time but are assumed to be stationary and ergodic with distribution $p(G)$. Let $\bar{S}$ be the average transmit power and $\bar{G}$ the average channel gain. We set $\bar{G}=1$ by appropriately scaling $\bar{S}$. The SNR before adaptation is $\gamma = \frac{\bar{S}G}{N}$. There is a feedback channel from receiver to transmitter for sending channel estimates. These estimates are assumed to be instantaneous and error free.

The system can adapt to changing channel conditions by estimating G and adapting its parameters such as transmit power S, transmit rate R, BER, etc. We consider the case of continuous link rate adaptation subject to instantaneous BER constraints, where the link rate and transmitter power adjust simultaneously to changes in the channel. The instantaneous link rate is given by $R(S(\gamma))$, and the instantaneous system power policy is given by $S(\gamma)$. The instantaneous SNR after adaptation is $\frac{\gamma S(\gamma)}{\bar{S}}$.

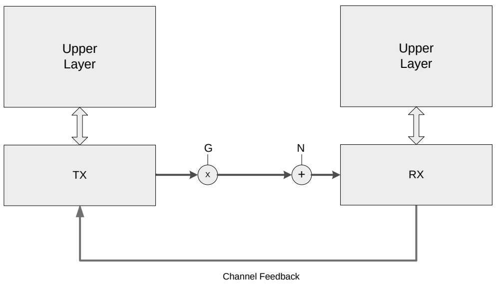

Fig. 1. System Model

B. Utility Functions

Utility functions are a key component of NUM, since they are used as the core metric of network performance [9], [10]. Utility functions can model network protocols, applications, or user preferences. TCP in particular [11] has been modeled in this way. Each flow in a network is associated with a utility function $U(r)$, where r is the flow rate. Each $U(r)$ is generally assumed to be continuously differentiable, nondecreasing, and strictly concave. Strictly concave utility functions exhibit diminishing returns with rate, that is, as rate increases the incremental utility grows by smaller amounts.

In this paper we consider the following general class of utility functions often used in the literature:

$$U(r) = \begin{cases} \frac{r^{1-\alpha}}{1-\alpha} & \alpha > 0 \quad \alpha \neq 1 \\ \ln(r) & \alpha = 1 \end{cases} \tag{1}$$

The parameter α corresponds to different properties of the utility function. For $\alpha = 1$ the utility function has the property of proportional fairness, and for $\alpha = 2$ the utility function has the property of minimum potential delay fairness. The case of $\alpha = 0$ corresponds to maximizing the rate.

C. Lambert W Function

We make use of the Lambert W function [12] to find-closed form adaptive modulation policies. The Lambert W function, $\mathscr{W}(x)$, satisfies the equation

$$x = \mathscr{W}(x)\exp(\mathscr{W}(x)) \tag{2}$$

for $x \geq -\frac{1}{e}$. It is positive for $x > 0$ and is strictly concave.

III. Network Utility Maximization and Adaptive Modulation

The objective of NUM/AM is to find adaptive rate and power policies that maximize the time average utility of the network, under constraints on the average power transmitted. Mathematically this objective is described as

$$\begin{array}{ll} \text{maximize} & \lim \frac{1}{T} \int_T U(r)dt \\ \text{s.t.} & \\ & \lim \frac{1}{T} \int_T S dt = \bar{S}. \end{array} \tag{3}$$

Under the assumptions of channel stationarity and ergodicity the problem can be formulated as

$$\begin{array}{ll} \underset{S(\gamma)\geq 0}{\text{maximize}} & \mathbf{E}[U(r)] \\ \text{s.t.} & \\ & \mathbf{E}[S(\gamma)] = \bar{S} \\ & r = R(S(\gamma)) \end{array} \tag{4}$$

where $\mathbf{E}$ is the expectation operator and the optimization is over functions $S(\gamma)$. The functions $R(S(\gamma))$ and $S(\gamma)$ are policies in the sense that given only the current channel state, γ, they return values that maximize average performance and meet the average power constraint. Knowledge of the systems' prior transmission powers and rates is not required.

When $R(S(\gamma))$ is concave, (4) is a convex optimization problem and a globally optimal policy exists. The following Euler-Lagrange equation gives necessary conditions for the optimal policy:

$$\nabla_{S(\gamma)}(U(R(S(\gamma)) + \lambda(S(\gamma) - \bar{S}) = 0, \tag{5}$$

where λ is determined by the power constraint and the underlying channel state probability distribution. When solvable, the result is an analytical expression for the policies $S(\gamma)$ and $R(S(\gamma))$. More generally, (4) can be solved numerically, expressing the policies in look-up table form. Methods of finding λ on-line, without prior knowledge of $p(\gamma)$, remain an open problem.

It is worth noting by Jensen's inequality that

$$E[U(R(\gamma))] \leq U(E[R(\gamma)]) \tag{6}$$

and that the utility of an average rate overestimates the average utility of the network. In this sense fading not only reduces channel capacity but also reduces utility, for any concave utility function.

IV. Single Link Case

In this section we determine NUM/AM policies to maximize the utility of a single link. Two cases are investigated. The first case considers practical modulation schemes constrained to meet instantaneous BER rates. The second case generalizes the first to multiple flows sharing a single link.

A. BER Constrained Network Utility Maximization

In this section we consider NUM/AM under the instantaneous BER constraints $\overline{\text{BER}}$. Instantaneous BER thresholds are appropriate for many data networks, where packet error is a factor. In this section optimal adaptive rate and transmitter power policies are found that maximize average utility. Both MQAM and MPSK are modeled parametrically.

We use the BER approximations of [13], [6] which are tight (to within 1.5 dB) for BER $\leq 10^{-3}$. The BER approximation applies to different modulation schemes through different parameterizations. In particular, MQAM is modeled for $c=1$ and MPSK for $c=0$ or $c=-1$ as

$$BER(S(\gamma),\gamma,R)=\exp[\frac{-\gamma\frac{S(\gamma)}{\bar{S}}}{f(r)}] \qquad f(r)=2^{r(\gamma)}-c. \tag{7}$$

Under the instantaneous BER constraint, $BER(S(\gamma),\gamma,R)=\overline{BER}$, (7) implicitly defines the link rate function:

$$R(S(\gamma))=\log(c+K\gamma\frac{S(\gamma)}{\bar{S}}) \qquad K=\frac{-1}{\ln(\overline{BER})}. \tag{8}$$

The Lagrangian of (4) is

$$\int U(r(\gamma))p(\gamma)d\gamma+\lambda\left(\int S(\gamma)p(\gamma)d\gamma-\bar{S}\right). \tag{9}$$

The Lagrange multiplier, λ, captures the trade-off between power and utility. Increasing the average power constraint by $\Delta\bar{S}$ will result in $\Delta U=-\lambda\Delta\bar{S}$ improvement in utility. It can be shown that $\lambda\leq 0$. Applying (5) yields

$$\frac{\delta U}{\delta R}\frac{\delta R}{\delta S(\gamma)}=-\lambda \tag{10}$$

The result is valid for any continuous distribution of SNR.

For the class of utility functions considered here, we can solve (10) for $S(\gamma)$ by expressing it in two ways and applying the Lambert W function. Rewriting (10) as

$$\frac{1}{R^{\alpha}}\frac{\frac{\gamma}{\bar{S}}}{(1+\gamma\frac{S(\gamma)}{\bar{S}})}=-\lambda \tag{11}$$

gives

$$\frac{S(\gamma)}{\bar{S}}=\frac{-1}{R^{\alpha}\bar{S}\lambda}-\frac{c}{K\gamma} \tag{12}$$

and

$$\frac{R}{\alpha}\exp\frac{R}{\alpha}=\theta \quad \theta=\frac{[-\frac{K\gamma}{\bar{S}\lambda}]^{\frac{1}{\alpha}}}{\alpha} \tag{13}$$

Equation (13) is in the form of equation (2), yielding

$$R(\gamma)=\alpha\mathscr{W}(\theta). \tag{14}$$

and

$$\frac{S(\gamma)}{\bar{S}}=\frac{-1}{[\alpha\mathscr{W}(\phi)]^{\alpha}\bar{S}\lambda}-\frac{c}{K\gamma}. \tag{15}$$

The behavior of the system strongly depends on the value of c. For MQAM, $c=1$, system transmitter power is positive for any channel condition. This can be seen by noting that (15) is positive iff $\mathscr{W}(x)<x$ for all positive x, which is true for the Lambert W function. The average power constraint can always be met for $|\lambda|$ sufficiently small. As channel conditions improve, utility and rate improve, but power decreases. This is in contrast to SE/AM where the performance metric is spectral efficiency, as analyzed in [6], where the system ceases to transmit below a channel threshold, and above this threshold increases both rate and power with improving channel conditions. The adaptation of [6] to maximize spectral efficiency is often referred to as "water-filling" in time, and our analysis shows that utility maximization for utility functions of the form in (1) leads to a very different optimal adaptation than water-filling.

For the case of MPSK, $c=0$, the system does not transmit when channel conditions fall below a threshold γ_1. Above this threshold the utility and rate increase, while power decreases with improving channel conditions. Here

$$\frac{S(\gamma)}{\bar{S}}=\frac{-1}{[\alpha\mathscr{W}(\phi)]^{\alpha}\bar{S}\lambda}, \tag{16}$$

so transmitter power grows rapidly for small and decreasing values of γ. The system stops transmitting below γ_1 to meet the average power constraint. This is also in contrast to [6], where the transmitter either transmits at a constant power or simply does not transmit.

When $c=-1$ the power relationship

$$\frac{S(\gamma)}{\bar{S}}=\frac{-1}{[\alpha\mathscr{W}(\phi)]^{\alpha}\bar{S}\lambda}+\frac{1}{K\gamma} \tag{17}$$

is positive for all γ and decreases with improving channel conditions. As in the $c=0$ MPSK case the transmitter power grows quickly with decreasing γ. To meet the average power constraint, the transmitter ceases to transmit below a threshold value. Intuitively, the power threshold marks a "balance point" between the benefit of increasing marginal utility at poorer channel conditions and the cost of dramatically reducing power at other channel conditions.

B. Multiple Flows Over a Single Link

In this section we consider multiple flows over a single link. The goal is to find the best rate adaptation policy and to optimally allocate the links' capacity to the different flows. As channel conditions change, the allocations may also change. Each flow $i=1\ldots N$ has rate $r_i=\beta_i(\gamma)R(S(\gamma))$, where β_i is the percentage of the links capacity allocated to flow i, and has utility function $U(r_i)$. The performance of the link is given by the sum of these utility functions. The simplest approach is to separately find the best link rate, from the previous section, and then to allocate it optimally to the different flows. The optimal allocation occurs when

$$\dot{U}_i(\beta_i(\gamma)R(S(\gamma)))=\dot{U}_j(\beta_j(\gamma)R(S(\gamma))) \tag{18}$$

$$\textstyle\sum_i\beta_i(\gamma)=1. \tag{19}$$

That is, the marginal utilities are identical. As γ changes the allocations will generally also change.

An alternative is to allow approximate flow buffering. In this approach, $\sum_i\beta_i(\gamma)$ need not equal one for every channel state, but only on average. This problem can be written as

$$\begin{array}{lll}\underset{S(\gamma)\beta\gamma}{\text{maximize}} & \mathbf{E}[\sum_i U_i(r_i)] & \\ \text{s.t.} & & \\ & \mathbf{E}[S(\gamma)]=\bar{S} & S(\gamma)\geq 0 \\ & \mathbf{E}[\sum_i\beta_i]=1 & \beta_i\geq 0 \\ & r_i=\beta_i(\gamma)R(S(\gamma)) & \end{array} \tag{20}$$

where the optimization is over the set of possible power policies $S(\gamma)$ and link capacity allocations. Since the link rate function is given here, the optimal rate function is $R(S(\gamma))$. Applying the Euler-Lagrange approach to the Lagrangian

$$L(S(\gamma),\lambda_1,\lambda_2) = \mathbf{E}[\sum_i U_i(r_i)+\lambda_1(S(\gamma)-\overline{S}) +\lambda_2(\sum_i \beta_i - 1)] \quad (21)$$

yields the following optimality conditions

$$\dot{U}_i = \dot{U}_j \quad (22)$$

$$\dot{R}\sum_i \dot{U}_i \beta_i(\gamma) = -\lambda_1 \quad (23)$$

$$\dot{U}_i R(S(\gamma)) = -\lambda_2 \quad (24)$$

Equation (22) states that the marginal utility of each of the flows must be equal for every channel state γ. As channel conditions change the relative allocations may also change. When $U_i = U_j$ the system divides the link capacity equally for every channel state.

Our analysis can be considered in economic terms by interpreting the Lagrange multipliers as prices. Specifically, λ_1 is the price, in utility units, of deviating from the average power constraint for any channel state. Similarly, λ_2 is the price of deviating from a non-zero buffer state. Further, λ_1 relates the change in average utility when the power constraint is adjusted $\Delta\mathbf{E}[\sum U] = -\lambda_1 \Delta\overline{S}$, and λ_2 can be similarly interpreted.

The optimal rate and power policies are similar to those of the single flow case, but adjusted by the Lagrange multiplier λ_2 and $\sum_i \beta_i(\gamma)$. The optimal policies are given by

$$\frac{S(\gamma)}{S} = \frac{-1}{\alpha[\mathscr{W}(\theta)]^{\alpha} S\lambda} - \frac{c}{\gamma} \quad (25)$$

where

$$\theta = \frac{\left[-\frac{\gamma\lambda_2(\sum_i \beta_i(\gamma))}{S\lambda_1}\right]^{\frac{1}{\alpha}}}{\alpha} \quad (26)$$

which are identical to the single link case when $(\lambda_2 \sum_i \beta_i(\gamma)) = 1$.

C. Comparing SE/AM and NUM/AM

In this section we compare SE/AM and NUM/AM with a numerical example. We consider MQAM modulation with $\overline{S} = 20$ dB, $\mathbf{E}[G] = 0$ dB with flat Rayleigh fading, $N = 0$ dB and α taking on the four values 0.0, 0.5, 0.9, 2.0. In the single link case, SE/AM corresponds to (1) with $\alpha = 0$. This is not correct for the multiple link case.

Figure 2 graphs $S(\gamma)$, the optimal power policy as a function of channel state G. For SE/AM, transmit power improves with improving channel state. This is not necessarily the case for NUM/AM. When $\alpha = 0.5$ and the channel is poor, transmitter power increases, and then decreases. For $\alpha = 2$, the transmitter power decreases with improving channel conditions for all values of G shown. In general, the SE/AM and NUM/AM power curves cross at a point determined by $\overline{S}$, $p(G)$ and α. Below this point NUM/AM expends more power and above this point SE/AM does.

Figure 3 shows the optimal rate policy for SE/AM and NUM/AM. The behavior is similar, with NUM/AM improving link rate performance at poor channel conditions at the expense of performance at good channel conditions. Increasing α moves the crossover point, and enhances this effect. This is indicated by the arrow shown in the figure. In this sense, NUM/AM policies seek to better "equalize" the links' rate over a wider range of channel states. In economic terms utility curves are conservative; they forgo a portion of the upside for greater certainty on the downside.

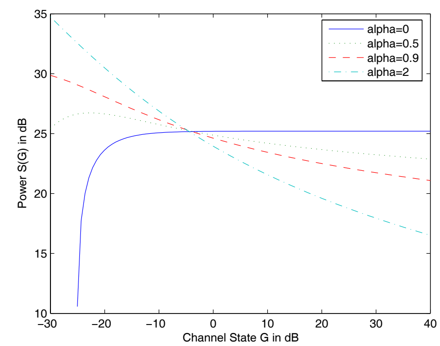

Fig. 2. Power Performance

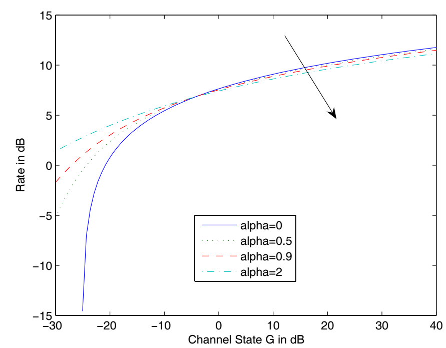

Fig. 3. Rate Performance

V. Multiple Interfering Links

In this section we consider multiple interfering links. To simplify the exposition, each link carries exactly one flow. The goal is to find a set of policies that maximize the performance of the network over randomly changing channel conditions. The performance of the network is taken as the sum of the performance of each individual link, $\sum U_l(r_l)$ $l = 1,\ldots L$. The links can interfere with each other, effecting the throughput of a given link through its SIR and link rate functions.

The channel is modeled as a random channel state (gain) matrix G, where G_{ij} is the power gain from the transmitter on link j to the receiver on link i and $G \sim p(G)$. We assume the channel state is estimated without error and is known at the set of transmitters. The vector of transmitter powers is given by $S(G)$. Because of interference, the channel state G is explicitly considered, rather than the SNR γ used in the previous section. The link rate function is assumed to be of the form

$$R_l(S(G)) = \log\left(1 + \frac{G_{ii}S_l}{\sum_{j\neq l} G_{lj}S_j + N}\right), \qquad (27)$$

so a change in the transmitter power of one link is immediately expressed as changes in the other links SIR's and rates.

Mathematically the goal is to find the best rate $R(S(G))$ and power $S(G)$ policies that maximize system performance. This can be expressed as

$$\begin{array}{ll} \underset{S(\gamma)}{\text{maximize}} & \mathbf{E}[\sum_i U_i(r_i] \\ \text{subject to} & \\ & \mathbf{E}[\mathbf{1}^T S(\gamma)] = \bar{S} \quad S(\gamma) \geq 0 \\ & r_l = R_l(S(\gamma)) \end{array} \qquad (28)$$

where the power constraint limits the average power used by the network to $\bar{S}$. This constraint allows the network an additional degree of freedom. It can allocate power between links, not just over channel states as in the single link case. It is worth noting that when $\alpha = 0$ for all utility functions, (28) maximizes the sum of the rates on all of links in the network.

Unfortunately (28) is not a convex problem and global policies may not exist. It can be made convex by assuming $SIR_l >> 1$, moderate SIR's, and transforming the variables $S_l = \exp(x_l)$, $G_{ij} = \exp(g_{ij})$, $N = \exp(n)$, where x_l g_{ij} and n are proportional to transmitter power, channel gain, and noise in dB. The link rate model can now be expressed as

$$R_i(G, S(G)) = -\ln(\mathrm{e}^{(-x_i - g_{ii})}(\textstyle\sum_{j\neq i} \mathrm{e}^{(x_j+g_{ij})} + \mathrm{e}^n)). \qquad (29)$$

Applying Euler-Lagrange yields the necessary optimality conditions

$$\dot{U}_l - \sum_{k\neq l} \dot{U}_k \left[\frac{\mathrm{e}^{g_{jk}+x_k}}{\sum \mathrm{e}^{g_{jk}+x_k} + \mathrm{e}^n}\right] = -\lambda \qquad (30)$$

which, unfortunately, are difficult to solve analytically.

Equation (28) can, however, be solved numerically. Figure 4 shows the rate behavior of a six link interfering network. Average total network power is constrained to 20 dB. The y-axis are the link rates, and the x-axis is the channel state ordered by increasing rate. Twenty representative channel states were selected. As in the single link case, rates improve with improving channel state. Figure 5 shows the transmitter powers. The network allocates more power to poorer channel conditions deemphasizing rate performance at good channel states, to improve performance at poorer ones. Further, the network allocates relatively more power to links with smaller link rates, seeking to balance the marginal utility of each of the flows.

VI. Conclusion

NUM/AM extends SE/AM from isolated links to general networks, optimizing metrics that model the behavior of higher level protocols. In the single link case, the optimal policies for NUM/AM are very different from the otimal "water filling in time" policies of SE/AM, with NUM/AM tending to make rates more equal over changing channel conditions. A similar behavior is exhibited for the multi link.

Future research directions include developing on-line algorithms for the multi-link case and modeling buffering in the system. A second area is investigating adaptive coding or elastic symbol periods in the NUM/AN framework.

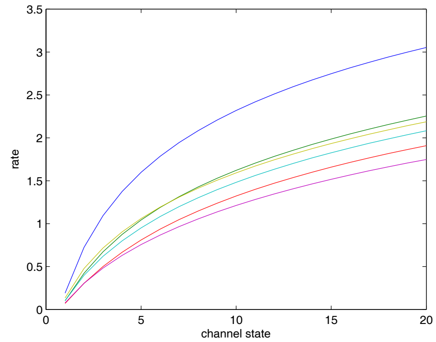

Fig. 4. Multi-Link Rate Performance

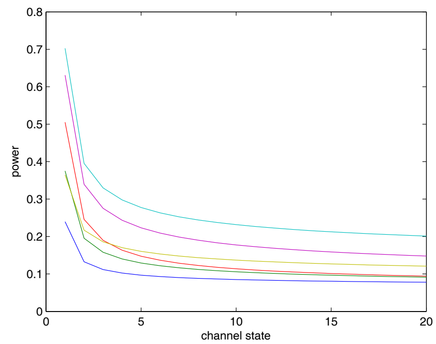

Fig. 5. Multi-Link Power Performance

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