PAPER

Optimal Beamforming in Two-Way Relay Networks with Cognitive Radio Capabilities

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SUMMARY In this paper, we present a cognitive relay network with two primary transceivers that communicate via several distributed relay terminals. Spectrum sensing is deployed at the relays to sense the absence/presence of the primary transceivers based on energy detection. The primary network utilizes a two-step two-way amplify-and-forward (AF) scheme by using the cognitive radio (CR) terminals as its relay nodes when the primary network is not in operation, in contrast, the CRs communicate with their own base station (BS). In the first relaying step, the primary transceivers send their signal to the CRs/relays. Distributed beamforming is then performed in the second relaying step. Our aim is to set the beamforming weights so as to minimize the total power dissipated in the relay network while satisfying a target signal-to-noise ratio (SNR) at the primary transceivers and at the cognitive BS. This is achieved by solving an optimization problem that we formulate as a nonconvex quadratically constrained quadratic program (QCQP). This problem is solved efficiently by semidefinite relaxation (SDR) and Lagrangian duality. Simulation results are provided to demonstrate the superiority of our proposed technique, compared to classical beamforming techniques, in terms of power reduction.

key words: *cognitive radio, two-way relaying, cooperative networks, cooperative spectrum sensing, beamforming, convex optimization*

1. Introduction

The lack of spectrum resources is considered to be the main challenge faced by emerging high-speed wireless access technologies. Cognitive radio (CR) technology [1] is a promising method to utilize the limited and scarce spectrum in an efficient way. It allows unlicensed (secondary) users to use the licensed (primary) users frequency bands when the licensed users are not operating. In this way, spectrum utilization can be significantly improved by allowing secondary users to access *spectrum holes* unoccupied by primary users. To prevent harmful interference at the primary transceiver, the CR network has to sense the licensed spectrum constantly to detect the absence/presence of the primary transmitter. The hidden terminal problem is a great challenge when implementing a CR system. This happens when the CR node is shadowed or is in severe multipath fading [2]. Cooperation among different cognitive users is an efficient solution to overcome the hidden terminal limitation due to the increase of the sensing/detection reliability. The basic idea behind cooperative communications is presented in many references such as [3]–[5].

Recently, the research community has paid an increasing amount of attention to the problem of two-way (bidirectional) relay channels (see e.g., [6]), where two senders exchange information via one or several relays. In 1961, Shannon investigated two-way relay systems and derived the inner and outer bound on the capacity region of a binary multiplying channel (BMC) [7]. By allowing concurrent transmission of two information flows, two-way networks can reduce the spectral loss caused by the half-duplex mode of relays [8], [9]. The key advantage of two-way networks is that each user can cancel the self-interference from its received signal to help the decoding of the information from the other user. Recently, different distributed beamforming schemes have been proposed for relaying schemes where the beamforming is done so as to satisfy some system requirements. For instance, different beamforming problems are presented in [10], [11] for one-way and in [12] for two-way relay networks that minimize the transmit power or maximize the received signal-to-noise ratio (SNR). In [13], the authors proposed the main idea of enabling CR capabilities for a two-way relay network without providing an analytical analysis of the investigated problem.

In this paper, we start by considering a conventional two-way relay network composed of a pair of transceivers which are not in each other's line-of-sight. To increase the link reliability, some relays are deployed to forward the data between the two transceivers. Thus, the primary network considered here includes two transceivers and some relays in its basic structure. We propose here to perform spectrum sensing at the relays in order to provide a CR capability for the considered two-way network. In other words, in this paper, the relays deployed in a conventional two-way relay network plays also the role of CR terminals by communicating with a cognitive BS, only when the spectrum sensing decision is that the primary transceivers are absent. Our aim in this paper is to derive an optimal beamforming parameters (weighting) so as to minimize the required power at the relay network while satisfying some constraints at the primary transceivers and at the cognitive BS, in terms of average received SNR. Note that the SNR constraint at the cognitive BS is chosen so as to provide a target spectrum sensing performance.

We show that the investigated beamforming optimization problem can be formulated as a nonconvex quadratically constrained quadratic program (QCQP) [14]. Then, mathematical techniques such as semidefinite relaxation

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(SDR) and Lagrangian duality are used to change the problem into a convex form and obtain an efficient solution for the convex problem. The so-obtained simplified problem is solved with available semidefinite programming (SDP) tools such as CVX [15].

The rest of this paper is organized as follows. In Sect. 2, the cognitive two-way relay network system model is presented. In Sect. 3, a modified spectrum sensing method based on the received SNR at the cognitive BS is proposed. The transmit power minimization of cognitive relay nodes is provided in Sect. 4 and the feasibility conditions of the problem are investigated. In Sect. 5, different mathematical tools for solving the optimization problem are presented. Section 6 contains the numerical results and finally, Sect. 7 concludes the paper.

2. System Model

We consider a CR network with n_r CR terminals and one cognitive BS (also denoted as fusion center). As shown in Fig. 1, the primary network is composed of two transceivers with no direct link. Therefore, we assume that the CR terminals play the role of relays for the primary transceivers. In this scenario, the CR network uses the licensed spectrum for communicating with a cognitive BS when there is no data transmission between the two transceivers. The CR network relay-assists the primary transceivers to work as a two-step two-way amplify-and-forward scheme. In the first step of relaying, both transceivers transmit their data to the CR users, simultaneously. In the second step, each CR/relay multiplies its received signal by a complex coefficient and broadcasts it toward the two transceivers as well as the cognitive BS.

2.1 Signal Transmission

During the first step, the received complex signal vector $\mathbf{x}$ of size $n_r \times 1$ at the CR terminals can be written as:

$$\mathbf{x} = \sqrt{P_1}\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{f}_2 s_2 + \boldsymbol{\nu}, \tag{1}$$

where P_1 and P_2 are the transmit power of transceivers 1 and 2, respectively, s_1 and s_2 are the information symbols transmitted by transceivers 1 and 2, respectively, $\boldsymbol{\nu}$ is the $n_r \times 1$ complex noise vector at the CR terminals, and

$$\mathbf{f}_k \triangleq \left[f_{1k}\ f_{2k}\ \ldots\ f_{n_r k}\right]^T, \tag{2}$$

is the vector of the channel coefficients between the k-th ($k = 1, 2$) transceiver and the CR terminals, and $(.)^T$ is the transpose operator. We assume that the channel coefficients in uplink and downlink transmissions (for the two relaying steps) are identical. Moreover, we assume that channel state information (CSI) is known at the two transceivers.

In the second step of relaying, the i-th CR terminal ($i = 1, 2, \ldots, n_r$) multiplies its received signal by a complex weight w_i^* and broadcasts it toward the network. The $n_r \times 1$ complex vector $\mathbf{t}$ denoting the transmitted signal CR terminals is written as:

$$\mathbf{t} = \mathbf{W}\mathbf{x} \tag{3}$$

where the weighting matrix $\mathbf{W}$ is:

$$\mathbf{W} = \mathrm{diag}\left\{\left[w_1^*\ w_2^*\ \ldots\ w_{n_r}^*\right]\right\} \tag{4}$$

with diag$\{\mathbf{a}\}$ denoting a diagonal matrix with diagonal elements equal to vector $\mathbf{a}$. Therefore, each relay amplifies the received signal by gain $|w_i|$ and adds a phase equal to $\angle w_i$ to the phase of the received signals as a beamforming weight factor.

2.2 Received Signals at the Primary Transceivers

In the second step of relaying, the received signals at the two transceivers, y_1 and y_2, are respectively equal to:

$$\begin{aligned} y_1 &= \mathbf{f}_1^T\mathbf{W}\mathbf{x} + n_1 \\ &= \mathbf{f}_1^T\mathbf{W}\left(\sqrt{P_1}\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{f}_2 s_2 + \boldsymbol{\nu}\right) + n_1, \end{aligned} \tag{5}$$

and

$$\begin{aligned} y_2 &= \mathbf{f}_2^T\mathbf{W}\mathbf{x} + n_2 \\ &= \mathbf{f}_2^T\mathbf{W}\left(\sqrt{P_1}\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{f}_2 s_2 + \boldsymbol{\nu}\right) + n_2, \end{aligned} \tag{6}$$

where n_k is the received noise at the k-th transceiver during the second step, for $k = 1, 2$. Noting that $\mathbf{a}^T\mathrm{diag}(\mathbf{b}) = \mathbf{b}^T\mathrm{diag}(\mathbf{a})$, we can rewrite (5) and (6) respectively as:

$$\begin{aligned} y_1 = &\sqrt{P_1}\mathbf{w}^H\mathbf{F}_1\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{w}^H\mathbf{F}_1\mathbf{f}_2 s_2 \\ &+ \mathbf{w}^H\mathbf{F}_1\boldsymbol{\nu} + n_1, \end{aligned} \tag{7}$$

and

$$\begin{aligned} y_2 = &\sqrt{P_1}\mathbf{w}^H\mathbf{F}_2\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{w}^H\mathbf{F}_2\mathbf{f}_2 s_2 \\ &+ \mathbf{w}^H\mathbf{F}_2\boldsymbol{\nu} + n_2 \end{aligned} \tag{8}$$

where $\mathbf{F}_k \triangleq \mathrm{diag}(\mathbf{f}_k)$ for $k = 1, 2$, and $\mathbf{w} = \mathrm{diag}\{\mathbf{W}^H\}$, and $(.)^H$ denotes the Hermitian transpose. Also, diag$\{\mathbf{A}\}$ is a vector formed by the diagonal elements of the square matrix $\mathbf{A}$.

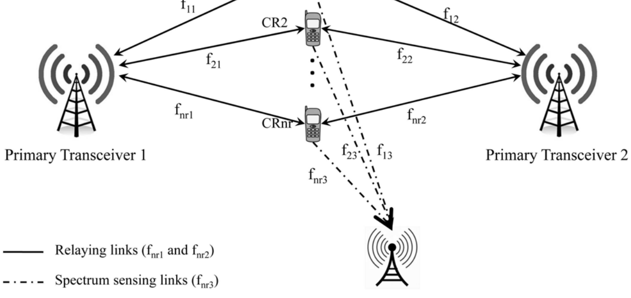

Fig. 1 Architecture of the proposed two-way relay network with cognitive capabilities.

Note that the beamforming weight vector $\mathbf{w}$ is obtained from an optimization problem solved at the cognitive BS and then transmitted among CR/relays via a control channel. Since s_1 is known at transceiver 1, $\sqrt{P_1}\mathbf{w}^H\mathbf{F}_1\mathbf{f}_1 s_1$ is also known at the primary transceiver 1 and thus the first term in (7) is known. Hence, this term can be subtracted from y_1 and the residual signal can be processed at transceiver 1 to extract the information s_2. Similarly, the second term in (8) can be subtracted from y_2 and the residual signal can be processed at transceiver 2 to extract the information s_1. Therefore, we define the residual signals $\tilde{y}_1 = y_1 - \sqrt{P_1}\mathbf{w}^H\mathbf{F}_1\mathbf{f}_1 s_1$ and $\tilde{y}_2 = y_2 - \sqrt{P_2}\mathbf{w}^H\mathbf{F}_2\mathbf{f}_2 s_2$, as the observation signals used at their corresponding transceivers to extract the symbol of the other transceiver.

2.3 Received Signals at the Cognitive Base Station

In our model, we assume that the cognitive radio network does not impose any interference on the primary network. Thus, the primary network operates as a two-way relay network and the cognitive BS senses the channel for eventual opportunistic transmission. The signal y_3 received at the cognitive BS during the second step of relaying is written as:

$$\begin{aligned} y_3 &= \mathbf{f}_3^T\mathbf{W}\mathbf{x} + n_3 \\ &= \mathbf{f}_3^T\mathbf{W}\left(\sqrt{P_1}\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{f}_2 s_2 + \boldsymbol{\nu}\right) + n_3, \end{aligned} \tag{9}$$

where

$$\mathbf{f}_3 = [f_{13}\ f_{23}\ \ldots\ f_{n_r 3}]^T \tag{10}$$

is the vector of channel coefficients between the CR users and the cognitive BS, and n_3 is the noise present at the cognitive BS. Similar to (7) and (8), we can rewrite y_3 as:

$$\begin{aligned} y_3 = \ &\sqrt{P_1}\mathbf{w}^H\mathbf{F}_3\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{w}^H\mathbf{F}_3\mathbf{f}_2 s_2 \\ &+ \mathbf{w}^H\mathbf{F}_3\boldsymbol{\nu} + n_3, \end{aligned} \tag{11}$$

where $\mathbf{F}_3 \overset{\Delta}{=} \mathrm{diag}\,(\mathbf{f}_3)$.

Different samples of y_3 are gathered at the cognitive BS to make a decision about the absence or the presence of the primary transmission during the second step of relaying, as explained in the next section.

3. Spectrum Sensing at Cognitive BS

Here, we explain the energy detection scheme adopted for spectrum sensing [2], [16] at the cognitive BS. As illustrated in Fig. 2, spectrum sensing can be viewed as a binary hypothesis testing problem. At the i-th time instant, based on the received signal at the cognitive BS denoted by y_3^i, from (11) we have [16]:

$$y_3^i = \begin{cases} \mathbf{w}^H\mathbf{F}_3^i\boldsymbol{\nu}^i + n_3^i, & \mathcal{H}_0, \\ \sqrt{P_1}\mathbf{w}^H\mathbf{F}_3^i\mathbf{f}_1^i s_1^i + \sqrt{P_2}\mathbf{w}^H\mathbf{F}_3^i\mathbf{f}_2^i s_2^i & \\ \qquad + \mathbf{w}^H\mathbf{F}_3^i\boldsymbol{\nu}^i + n_3^i, & \mathcal{H}_1, \end{cases} \tag{12}$$

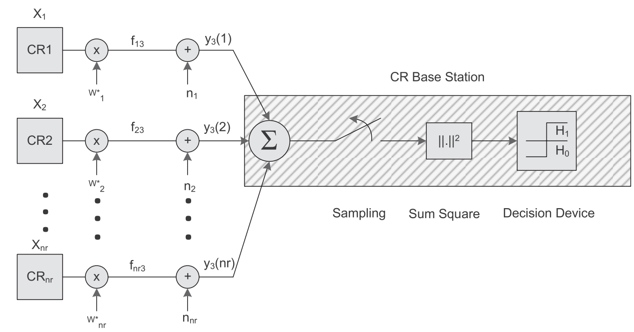

Fig. 2 Energy detection metric evaluation by data fusion at the cognitive BS.

where hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ denote that primary signal is absent or present, respectively.

We assume that n_3^i and $\boldsymbol{\nu}^i$ are zero-mean complex additive white Gaussian noise (AWGN) with equal variance, i.e., $\mathcal{CN}(0,\sigma^2)$ and $\mathcal{CN}([0]_{n_r\times 1},\sigma^2\boldsymbol{I}_{n_r})$. Furthermore, the signal samples s_1^i, s_2^i are assumed to be zero-mean complex Gaussian random processes with unit variances, i.e., $E[|s_1^i|^2] = E[|s_2^i|^2] = 1$. Without loss of generality, s_1^i, s_2^i, $\boldsymbol{\nu}^i$ and n_3^i are assumed to be independent of each other. We also assume that the channel gains $\mathbf{f}_k$ for $k = 1, 2, 3$ are constant during the detection interval. By these assumptions, the received signal at the cognitive BS, i.e., y_3^i, is a Gaussian random variable (RV) distributed as:

$$y_3^i \sim \begin{cases} \mathcal{CN}(0,\sigma_1^2) & \mathcal{H}_0, \\ \mathcal{CN}(0,\sigma_2^2) & \mathcal{H}_1, \end{cases} \tag{13}$$

where σ_1^2 and σ_2^2 are the variances of each sample in the two hypotheses and are respectively equal to:

$$\begin{aligned} \sigma_1^2 &= E[\left|\mathbf{w}^H\mathbf{F}_3\boldsymbol{\nu}^i + n_3^i\right|^2] \\ &= E[(\mathbf{w}^H\mathbf{F}_3\boldsymbol{\nu}^i + n_3^i)(\boldsymbol{\nu}^{i\,H}\mathbf{F}_3^H\mathbf{w} + n_3^i)] \\ &= \sigma^2(\mathbf{w}^H\mathbf{F}_3\mathbf{F}_3^H\mathbf{w} + 1) \end{aligned} \tag{14}$$

and

$$\begin{aligned} \sigma_2^2 = &\ E\,[\,|\ \sqrt{P_1}\mathbf{w}^H\mathbf{F}_3\mathbf{f}_1 s_1^i + \sqrt{P_2}\mathbf{w}^H\mathbf{F}_3\mathbf{f}_2 s_2^i \\ &\qquad + \mathbf{w}^H\mathbf{F}_3\boldsymbol{\nu}^i + n_3^i\big|^2\ ] \\ = &\ P_1\mathbf{w}^H\mathbf{F}_3\mathbf{f}_1\mathbf{f}_1^H\mathbf{F}_3^H\mathbf{w} + P_2\mathbf{w}^H\mathbf{F}_3\mathbf{f}_2\mathbf{f}_2^H\mathbf{F}_3^H\mathbf{w} \\ &\qquad + \sigma^2(\mathbf{w}^H\mathbf{F}_3\mathbf{F}_3^H\mathbf{w} + 1), \end{aligned} \tag{15}$$

where $E[.]$ denotes expectation.

The measured energy summation at the cognitive BS over a detection interval composed of N samples y_3^i ($i = 1, 2, ..., N$) is thus given by:

$$\begin{aligned} \mathbf{Z} &= \sum_{i=1}^{N} |y_3^i|^2 \\ &= \begin{cases} \sum_{i=1}^{N}\left|\mathbf{w}^H\mathbf{F}_3\boldsymbol{\nu}^i + n_2^i\right|^2, & \mathcal{H}_0, \\ \sum_{i=1}^{N}\left|\sqrt{P_1}\mathbf{w}^H\mathbf{F}_3\mathbf{f}_1 s_1^i + \sqrt{P_2}\mathbf{w}^H\mathbf{F}_3\mathbf{f}_2 s_2^i\right. & \\ \qquad \left. + \mathbf{w}^H\mathbf{F}_3\boldsymbol{\nu}^i + n_3^i\right|^2, & \mathcal{H}_1. \end{cases} \end{aligned} \tag{16}$$

The test statistic $\mathbf{Z}$ is the sum of the squares of N Normal random variables. Then, $\mathbf{Z}$ has a central chi-square $\mathcal{X}^2$ distribution with N degrees of freedom for hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$.

According to the central limit theorem (CLT), if the number of samples N is large enough the test statistic follows normal distributions [2]. Then, we can write the test statistic $\mathbf{Z}$ as [17]:

$$\mathbf{Z} \sim \begin{cases} \mathcal{N}(N\sigma_1^2, 2N\sigma_1^4), & \mathcal{H}_0, \\ \mathcal{N}(N\sigma_2^2, 2N\sigma_2^4), & \mathcal{H}_1. \end{cases} \tag{17}$$

Let λ be the decision threshold used at the cognitive BS to decide between the two above hypothesis. Therefore, the false-alarm probability, P_F, and detection probability, P_D, usually defined in CR systems can be obtained from (17) as:

$$P_F = Pr(\mathbf{Z} > \lambda|\mathcal{H}_0) = Q\left(\frac{\lambda - N\sigma_1^2}{\sqrt{2N\sigma_1^4}}\right), \tag{18}$$

and

$$P_D = Pr(\mathbf{Z} > \lambda|\mathcal{H}_1) = Q\left(\frac{\lambda - N\sigma_2^2}{\sqrt{2N\sigma_2^4}}\right), \tag{19}$$

where $Q(.)$ is the standard Gaussian complementary cumulative distribution function (CDF) which is defined as $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-\frac{u^2}{2})\,du$. By setting the threshold λ at the cognitive BS so as to have a desired probability of false-alarm equal to $\bar{P}_F$, we obtain the threshold as:

$$\lambda = \sqrt{2N\sigma_1^4}Q^{-1}(\bar{P}_F) + N\sigma_1^2, \tag{20}$$

where $Q^{-1}(.)$ is the inverse standard Gaussian complementary CDF. Substituting λ from (20) in (19) we can rewrite the probability of detection as follows:

$$\begin{aligned} P_D &= Q\left(\frac{\sqrt{2N\sigma_1^4}Q^{-1}(\bar{P}_F) + N\sigma_1^2 - N\sigma_2^2}{\sqrt{2N\sigma_2^4}}\right) \\ &= Q\left(\frac{\sqrt{2N}Q^{-1}(\bar{P}_F) + N - N\left(\frac{\sigma_2}{\sigma_1}\right)^2}{\left(\frac{\sigma_2}{\sigma_1}\right)^2\sqrt{2N}}\right). \end{aligned} \tag{21}$$

Let us define SNR_{cr} as the ratio of the two transceivers' received energy to the energy of the additive noise at the cognitive BS. This parameter can be formulated as follows:

$$SNR_{cr} = \frac{P_1\mathbf{w}^H\mathbf{F}_3\mathbf{f}_1\mathbf{f}_1^H\mathbf{F}_3^H\mathbf{w} + P_2\mathbf{w}^H\mathbf{F}_3\mathbf{f}_2\mathbf{f}_2^H\mathbf{F}_3^H\mathbf{w}}{\sigma^2\mathbf{w}^H\mathbf{F}_3\mathbf{F}_3^H\mathbf{w} + \sigma^2}. \tag{22}$$

From (14) and (15), we can rewrite $\left(\frac{\sigma_2}{\sigma_1}\right)^2$ involved in (21) as:

$$\begin{aligned} \left(\frac{\sigma_2}{\sigma_1}\right)^2 &= 1 + \frac{P_1\mathbf{w}^H\mathbf{F}_3\mathbf{f}_1\mathbf{f}_1^H\mathbf{F}_3^H\mathbf{w} + P_2\mathbf{w}^H\mathbf{F}_3\mathbf{f}_2\mathbf{f}_2^H\mathbf{F}_3^H\mathbf{w}}{\sigma^2\mathbf{w}^H\mathbf{F}_3\mathbf{F}_3^H\mathbf{w} + \sigma^2}, \\ &= 1 + SNR_{cr} \end{aligned} \tag{23}$$

and consequently, the probability of detection in our proposed two-way cognitive relay model can be rewritten as:

$$P_D = Q\left(\frac{\sqrt{2N}Q^{-1}(\bar{P}_F) - SNR_{cr}N}{(1 + SNR_{cr})\sqrt{2N}}\right). \tag{24}$$

We observe from (24) that the increase of SNR_{cr} increases the Q-function. Thus, the probability of detection is increased when SNR_{cr} increases.

We propose here to use SNR_{cr} as a detection criterion in the fusion center. More precisely, instead of using the energy summation as in (16), we propose to use SNR_{cr} as discriminator between the two hypothesis. In this way, the binary hypothesis involved in spectrum sensing at the cognitive BS can be written as:

$$\begin{cases} SNR_{cr} \le \gamma_{cr}, & \mathcal{H}_0, \\ SNR_{cr} > \gamma_{cr}, & \mathcal{H}_1, \end{cases} \tag{25}$$

where γ_{cr} is a predefined SNR threshold ensuring a target detection probability. In next section, we set this decision threshold as a constraint of the optimization problem.

4. Minimization of the Total Power Dissipated in the Relay Network

Here, we aim at finding the beamforming coefficients so as to minimize the power dissipated in the relay network. Minimizing the power of the relay network is of great importance since the relays are in fact cognitive users with limited power. We formulate the relay power minimization problem as a nonconvex QCQP. In next section, a solution is proposed by semidefinite relaxation and Lagrangian duality of the optimization problem to obtain an efficient solution.

4.1 Optimization Problem Formulation

An optimization problem is solved to find the beamforming vector $\mathbf{w}$ that minimizes the total relaying power while the received SNR at the two transceivers and at the cognitive BS are kept above a certain threshold. This constrained minimization problem can be formulated as:

$$\begin{aligned} &\min_{\mathbf{w}} P_r \\ &\text{subject to: } SNR_1 \ge \gamma_1, \\ &\qquad SNR_2 \ge \gamma_2, \\ &\qquad SNR_{cr} \ge \gamma_{cr}, \end{aligned} \tag{26}$$

where the relay transmit power P_r is given by:

$$\begin{aligned} P_r &= E\left\{\mathbf{t}^H\mathbf{t}\right\} \\ &= \mathbf{w}^H\left(P_1\mathbf{F}_1\mathbf{F}_1^H + P_2\mathbf{F}_2\mathbf{F}_2^H + \sigma^2\boldsymbol{I}\right)\mathbf{w} \\ &= \mathbf{w}^H\left(P_1\mathbf{D}_1 + P_2\mathbf{D}_2 + \sigma^2\boldsymbol{I}\right)\mathbf{w} \\ &= \mathbf{w}^H\mathbf{D}\mathbf{w}, \end{aligned} \tag{27}$$

where $\mathbf{D}_1 \triangleq \mathbf{F}_1\mathbf{F}_1^H$, $\mathbf{D}_2 \triangleq \mathbf{F}_2\mathbf{F}_2^H$ and $\mathbf{D} = \left(P_1\mathbf{D}_1 + P_2\mathbf{D}_2 + \sigma^2\boldsymbol{I}\right)$ are diagonal matrices with real and positive elements.

The received SNRs in the second transmission step at transceivers 1 and 2 can be expressed respectively as:

$$SNR_1 = \frac{P_2\mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}}{\sigma^2 + \sigma^2\mathbf{w}^H\mathbf{D}_1\mathbf{w}}, \tag{28}$$

and

$$SNR_2 = \frac{P_1\mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}}{\sigma^2 + \sigma^2\mathbf{w}^H\mathbf{D}_2\mathbf{w}}\ . \tag{29}$$

where $\mathbf{h} \triangleq \mathbf{F}_1\mathbf{f}_2 = \mathbf{F}_2\mathbf{f}_1$. Similarly, we can rewrite (22) as follows:

$$SNR_{cr} = \frac{P_1\mathbf{w}^H\mathbf{h}'\mathbf{h}'^H\mathbf{w} + P_2\mathbf{w}^H\mathbf{h}''\mathbf{h}''^H\mathbf{w}}{\sigma^2 + \sigma^2\mathbf{w}^H\mathbf{D}_3\mathbf{w}}. \tag{30}$$

where $\mathbf{h}' \triangleq \mathbf{F}_3\mathbf{f}_1$, $\mathbf{h}'' \triangleq \mathbf{F}_3\mathbf{f}_2$ and $\mathbf{D}_3 \triangleq \mathbf{F}_3\mathbf{F}_3^H$. Then, the constrained minimization problem in (26) can be rewritten in an equivalent form as:

$$\begin{aligned} &\min_{\mathbf{w}}\ \mathbf{w}^H\left(P_1\mathbf{D}_1 + P_2\mathbf{D}_2 + \sigma^2\boldsymbol{I}\right)\mathbf{w} \\ &\quad\text{subject to:} \\ &\quad c_1 : \frac{P_2\mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}}{\sigma^2 + \sigma^2\mathbf{w}^H\mathbf{D}_1\mathbf{w}} \geq \gamma_1, \\ &\quad c_2 : \frac{P_1\mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}}{\sigma^2 + \sigma^2\mathbf{w}^H\mathbf{D}_2\mathbf{w}} \geq \gamma_2, \ \text{and} \\ &\quad c_3 : \frac{P_1\mathbf{w}^H\mathbf{h}'\mathbf{h}'^H\mathbf{w} + P_2\mathbf{w}^H\mathbf{h}''\mathbf{h}''^H\mathbf{w}}{\sigma^2 + \sigma^2\mathbf{w}^H\mathbf{D}_3\mathbf{w}} \geq \gamma_{cr}. \end{aligned} \tag{31}$$

Furthermore, without loss of generality, we assume that the weighting vector $\mathbf{w}$ is normalized among relay/CR terminals i.e., $\mathbf{w}^H\mathbf{w} = ||\mathbf{w}||^2 = 1$.

Remark 1: The constraints in the optimization problem (31) are not satisfied for every transmit power P_1 and P_2. Moreover, it is not guaranteed that for every power of transceivers 1 and 2, the constraint c_3 at the cognitive BS is satisfied. Hence, we introduce the following lemma that defines a feasibility region for powers P_1 and P_2 where the optimization problem (31) has a solution.

Lemma (derivation of lower bounds for the primary transceivers' power): The optimization problem of (31) is feasible if the transmit power of the two transceivers, i.e., P_1 and P_2 are selected such that the following inequalities are satisfied (see also Fig. 3):
$P_2\lambda_{max}\{\mathbf{H}\} - \gamma_1\lambda_{max}\{\mathbf{D}_1\} - \gamma_1 \geq 0$,
$P_1\lambda_{max}\{\mathbf{H}\} - \gamma_2\lambda_{max}\{\mathbf{D}_2\} - \gamma_2 \geq 0$ and
$P_1\lambda_{max}\{\mathbf{H}'\} + P_2\lambda_{max}\{\mathbf{H}''\} - \gamma_{cr}\lambda_{max}\{\mathbf{D}_3\} - \gamma_{cr} \geq 0$ where $\lambda_{max}\{\mathbf{A}\}$ denotes the maximum eigenvalue of a given matrix $\mathbf{A}$.

Proof. The proof is provided in the Appendix. □

For notational convenience in the sequel, let us write

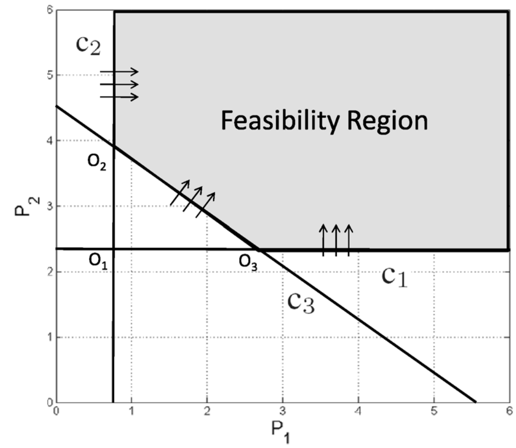

Fig. 3 Solution area for P_1 and P_2 according to Lemma.

the beamforming vector $\mathbf{w}$ as:

$$\mathbf{w} = \mathbf{D}^{-1/2}\tilde{\mathbf{w}}. \tag{32}$$

Now, the optimization problem (31) for finding $\tilde{\mathbf{w}}$ can be written in an equivalent form as:

$$\begin{aligned} &\min_{\tilde{\mathbf{w}}}\ ||\tilde{\mathbf{w}}||^2 \\ &\quad\text{subject to:} \\ &\quad \tilde{\mathbf{w}}^H\mathbf{C}_1\tilde{\mathbf{w}} \geq \gamma_1, \\ &\quad \tilde{\mathbf{w}}^H\mathbf{C}_2\tilde{\mathbf{w}} \geq \gamma_2, \ \text{and} \\ &\quad \tilde{\mathbf{w}}^H\mathbf{C}_3\tilde{\mathbf{w}} \geq \gamma_{cr}. \end{aligned} \tag{33}$$

where $\mathbf{C}_1$, $\mathbf{C}_2$ and $\mathbf{C}_3$ are defined as follows:

$$\mathbf{C}_1 = P_2\mathbf{D}^{-1/2\,H}\mathbf{h}\mathbf{h}^H\mathbf{D}^{-1/2} - \gamma_1\mathbf{D}^{-1/2\,H}\mathbf{D}_1\mathbf{D}^{-1/2}, \tag{34}$$

$$\mathbf{C}_2 = P_1\mathbf{D}^{-1/2\,H}\mathbf{h}\mathbf{h}^H\mathbf{D}^{-1/2} - \gamma_2\mathbf{D}^{-1/2\,H}\mathbf{D}_2\mathbf{D}^{-1/2} \text{and} \tag{35}$$

$$\begin{aligned}\mathbf{C}_3 = {} & P_1\mathbf{D}^{-1/2\,H}\mathbf{h}'\mathbf{h}'^H\mathbf{D}^{-1/2} + P_2\mathbf{D}^{-1/2\,H}\mathbf{h}''\mathbf{h}''^H\mathbf{D}^{-1/2} \\ & - \gamma_{cr}\mathbf{D}^{-1/2\,H}\mathbf{D}_3\mathbf{D}^{-1/2}. \end{aligned} \tag{36}$$

Remark 2: The objective function of (33) is a convex function in quadratic form but constraints may not be convex in general. Given the realization of wireless channels and depending on the SNR threshold γ_1 and γ_2 with a given transmit power P_1 and P_2, it is not guaranteed that $\mathbf{C}_1$, $\mathbf{C}_2$ and $\mathbf{C}_3$ are positive semidefinite. Then, the constraints are generally nonconvex.

Remark 3: The objective function and constraints of the optimization problem (33) are both quadratic functions. Such problems are referred to as quadratically constrained quadratic programs (QCQP). Since the constraints may be nonconvex in general, this nonconvex QCQP is NP-hard. We thus perform relaxation techniques to efficiently solve this optimization problem by convex programming.

5. Convex Relaxation of Nonconvex QCQP

There are two widely-used relaxation techniques for solving

SDP problems: SDR and Lagrangian duality. SDR is a powerful and computationally efficient approximation technique for the class of very difficult optimization problems [14], [18]. In particular, it can be applied to many nonconvex QCQPs. Apart from the rank relaxation interpretation of SDR, there is another interpretation that is based on Lagrangian duality. The basic idea in Lagrangian duality is to take the constraints in (26) into account by augmenting the objective function with a weighted sum of the constraint functions. Notice that both of these techniques provide lower bounds on the optimal value of the nonconvex QCQP. For the sake of completeness and to clarify the specificity of each technique, in what follows, we provide an optimal solution for the nonconvex relay power minimization problem of Sect. 4 obtained by direct semidefinite relaxation and Lagrangian duality.

5.1 Semidefinite Relaxation

Noting that $\mathbf{w}^H\mathbf{A}\mathbf{w} = \mathrm{Tr}\left[\mathbf{A}(\mathbf{w}\mathbf{w}^H)\right]$, where Tr[.] denotes matrix trace, the optimization problem (33) writes:

$$\begin{aligned}&\min_{\tilde{\mathbf{W}}}\ \mathrm{Tr}\left[\tilde{\mathbf{W}}\right]\\&\text{subject to:}\\&\quad \mathrm{Tr}\left[(\tilde{P}_2\tilde{\mathbf{H}} - \gamma_1\tilde{\mathbf{D}}_1)\tilde{\mathbf{W}}\right] \geq \gamma_1,\\&\quad \mathrm{Tr}\left[(\tilde{P}_1\tilde{\mathbf{H}} - \gamma_2\tilde{\mathbf{D}}_2)\tilde{\mathbf{W}}\right] \geq \gamma_2,\\&\quad \mathrm{Tr}\left[(\tilde{P}_1\tilde{\mathbf{H}}' + \tilde{P}_2\tilde{\mathbf{H}}'' - \gamma_{cr}\tilde{\mathbf{D}}_3)\tilde{\mathbf{W}}\right] \geq \gamma_3,\\&\quad \tilde{\mathbf{W}} = \tilde{\mathbf{w}}\tilde{\mathbf{w}}^H,\ \tilde{\mathbf{W}} \geq 0 \ \text{and}\ \mathrm{rank}(\tilde{\mathbf{W}}) = 1.\end{aligned} \tag{37}$$

where
$\tilde{\mathbf{H}} = \mathbf{D}^{-1/2\,H}\mathbf{h}\mathbf{h}^H\mathbf{D}^{-1/2}$,
$\tilde{\mathbf{H}}' = \mathbf{D}^{-1/2\,H}\mathbf{h}'\mathbf{h}'^H\mathbf{D}^{-1/2}$,
$\tilde{\mathbf{H}}'' = \mathbf{D}^{-1/2\,H}\mathbf{h}''\mathbf{h}''^H\mathbf{D}^{-1/2}$,
$\tilde{\mathbf{D}}_1 = \mathbf{D}^{-1/2\,H}\mathbf{D}_1\mathbf{D}^{-1/2}$,
$\tilde{\mathbf{D}}_2 = \mathbf{D}^{-1/2\,H}\mathbf{D}_2\mathbf{D}^{-1/2}$ and
$\tilde{\mathbf{D}}_3 = \mathbf{D}^{-1/2\,H}\mathbf{D}_3\mathbf{D}^{-1/2}$.
P_1 and P_2 are the relay transmit power satisfying the inequalities in Lemma.

The problem is transformed into a convex form by removing the rank-one nonconvex constraint. It can be solved efficiently by convex optimization programs, such as CVX [15]. Then, it is noted that the optimization problem in (33) cannot always guarantee that the optimal solution has a rank equal to one. If matrix $\mathbf{W}$ has a rank greater than one, the rank reduction program in [19] can be used efficiently.

5.2 Relaxation by Lagrangian Duality

Lagrangian duality provides a computable lower bound on the optimal value of the nonconvex QCQP. The Lagrangian of the optimization problem (33) is [18]:

$$\begin{aligned}\mathcal{L}(\tilde{\mathbf{w}}, \lambda_1, \lambda_2, \lambda_3) &= \|\tilde{\mathbf{w}}\|^2\\&- \lambda_1\left[\tilde{\mathbf{w}}^H\left(\tilde{P}_2\tilde{\mathbf{H}} - \gamma_1\tilde{\mathbf{D}}_1\right)\tilde{\mathbf{w}} - \gamma_1\right]\\&- \lambda_2\left[\tilde{\mathbf{w}}^H\left(\tilde{P}_1\tilde{\mathbf{H}} - \gamma_2\tilde{\mathbf{D}}_2\right)\tilde{\mathbf{w}} - \gamma_2\right]\\&- \lambda_3\left[\tilde{\mathbf{w}}^H\left(\tilde{P}_1\tilde{\mathbf{H}}' + \tilde{P}_2\tilde{\mathbf{H}}'' - \gamma_{cr}\tilde{\mathbf{D}}_3\right)\tilde{\mathbf{w}} - \gamma_{cr}\right].\end{aligned} \tag{38}$$

Thus, the Lagrangian dual function is defined as:

$$g(\lambda_1, \lambda_2, \lambda_3) = \inf_{\tilde{\mathbf{w}}} \mathcal{L}(\tilde{\mathbf{w}}, \lambda_1, \lambda_2, \lambda_3). \tag{39}$$

The problem is feasible when the matrix

$$\begin{aligned}\mathbf{A} =& \lambda_1\left(\tilde{P}_2\tilde{\mathbf{H}} - \gamma_1\tilde{\mathbf{D}}_1\right) + \lambda_2\left(\tilde{P}_1\tilde{\mathbf{H}} - \gamma_2\tilde{\mathbf{D}}_2\right)\\&+ \lambda_3\left(\tilde{P}_1\tilde{\mathbf{H}}' + \tilde{P}_2\tilde{\mathbf{H}}'' - \gamma_{cr}\tilde{\mathbf{D}}_3\right)\end{aligned} \tag{40}$$

is positive semidefinite. Therefore, the dual problem is formed as:

$$\begin{aligned}&\max_{\lambda_1,\lambda_2,\lambda_3}\ (\lambda_1\gamma_1 + \lambda_2\gamma_2 + \lambda_3\gamma_{cr})\\&\text{subject to:}\\&\quad \boldsymbol{I} - \lambda_1\left(\tilde{P}_2\tilde{\mathbf{H}} - \gamma_1\tilde{\mathbf{D}}_1\right)\\&\quad - \lambda_2\left(\tilde{P}_1\tilde{\mathbf{H}} - \gamma_2\tilde{\mathbf{D}}_2\right)\\&\quad - \lambda_2\left(\tilde{P}_1\tilde{\mathbf{H}}' + \tilde{P}_2\tilde{\mathbf{H}}'' - \gamma_{cr}\tilde{\mathbf{D}}_3\right) \geq 0\end{aligned} \tag{41}$$

The above dual problem is a semidefinite program which is generally quicker to solve than (37).

6. Simulation Results and Discussion

In this section, we provide some numerical results to analyse the sensing performance achieved by the CR network. We also compare the performance obtained with our proposed mixed network with a conventional relay network without any cognitive capabilities. A uniform normalized weight for our distributed beamforming method also presents to provide a fair comparison. Each element in channel vectors $\mathbf{f}_1$, $\mathbf{f}_2$ and $\mathbf{f}_3$ (defined in Sect. 2) is assumed to be distributed as a complex zero-mean Gaussian random variable with variance equal, respectively, to $\sigma^2_{f_1}$, $\sigma^2_{f_2}$ and $\sigma^2_{f_3}$. We assume that the noise power for the three channels (denoted by σ^2) is also equal to one. We have used a number of 200 random channel realizations for averaging the solution of our optimization problem. In our optimization problem, we set the primary transceivers' power P_1 and P_2 so as to have a point inside the feasibility region of Fig. 3. Here, we have used the SDR technique for solving our nonconvex QCQP problem by employing the CVX toolbox of Matlab.

Figure 4 plots the average relay transmit power versus the SNR threshold defined at the cognitive BS. For comparison, we have provided the performance archived by using a conventional beamforming (i.e., without considering the third constraint in (26) and uniform beamforming. A first observation is that since the conventional method does not have to satisfy the third constraint in (26), a lower power is consumed in the relay network compared to the power consumed in the considered network. However, one should be aware that the conventional two-way relay network does not

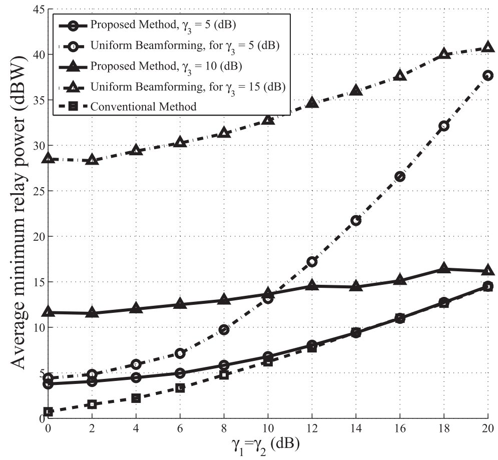

Fig. 4 Average minimum relay power versus the equal required SNR at transceivers in dB for the conventional, the cognitive two-way relay networks and uniform weight of beamforming for different spectrum sensing requirements at the cognitive BS, $\sigma^2_{f_1} = \sigma^2_{f_2} = \sigma^2_{f_3} = 5$ dB and $N_r = 10$.

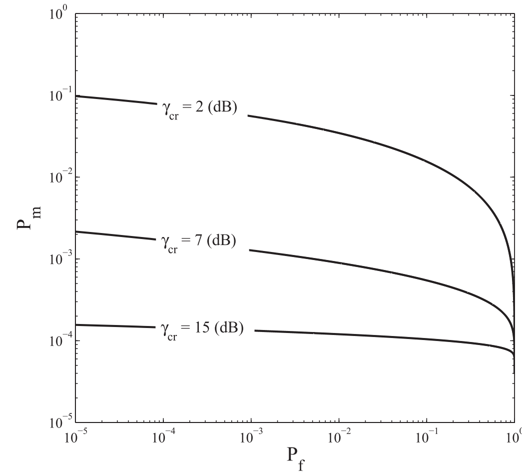

Fig. 6 Probability of miss-detection versus the probability of false-alarm (ROC) for different values of SNR threshold at cognitive BS, $\gamma_1 = \gamma_2 = 10$ dB, $\sigma^2_{f_1} = \sigma^2_{f_2} = \sigma^2_{f_3} = 0$ dB and $N = 20$.

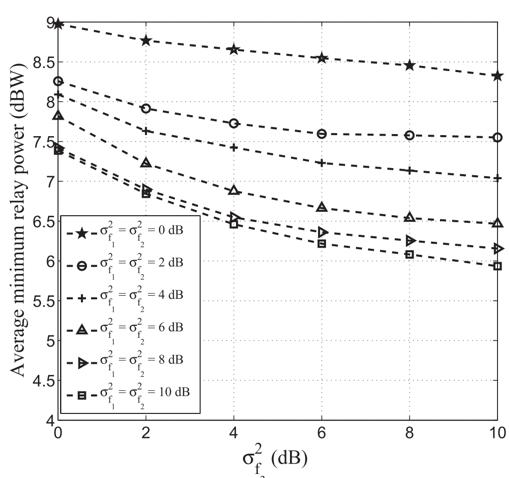

Fig. 5 Average minimum relay power versus $\sigma^2_{f_3}$ for different values of $\sigma^2_{f_1} = \sigma^2_{f_2}$, $\gamma_1 = \gamma_2 = \gamma_{cr} = 10$ dB.

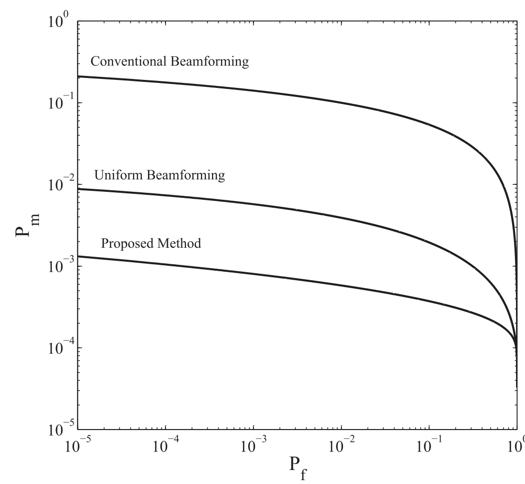

Fig. 7 Probability of miss-detection versus the probability of false-alarm for proposed distributed beamforming, uniform weighting and conventional beamforming, $n_r = 10$, $\gamma_1 = \gamma_2 = \gamma_{cr} = 10$ dB and $N = 20$.

have any CR capability. In other words, this scheme satisfies less constraints than the proposed method and for this reason it consumes less power. The optimality of the proposed method is better understood by comparing the curves between the proposed method and the uniform beamforming method, showing that under similar conditions, the consumed power at the relay network is reduced significantly when the proposed technique is employed rather than uniform beamforming.

Similar curves are shown in Fig. 5, where the average minimum relay power is plotted versus the channels' quality. As can be seen from this figure, when the network enjoys favorable channel conditions (characterized by larger variances, i.e., larger channel gains on average), the minimum relay transmit power is decreased. This can be explained by the fact that for good channel conditions, all constraints can be satisfied by smaller relay powers.

Spectrum sensing performance evaluation is usually done via the so-called receiver operating characteristic (ROC) curves that plot the miss-detection probability versus the false-alarm probability. In Fig. 6, we show different ROC curves for different values of the γ_{cr} threshold. As shown, the probability of miss-detection is decreased (leading to a more accurate spectrum sensing) when a higher SNR threshold is considered at the cognitive BS.

Figure 7 compares the ROC performance obtained by using our proposed method in comparison with uniform and conventional beamforming methods. We observe that for a target SNR requirement at the cognitive BS, using our proposed method leads to an improved spectrum sensing, characterized by a lower miss-detection probability for a given false-alarm probability. Note that this is due to the consideration of the target SNR requirement at the cognitive BS, which leads to a more accurate spectrum sensing.

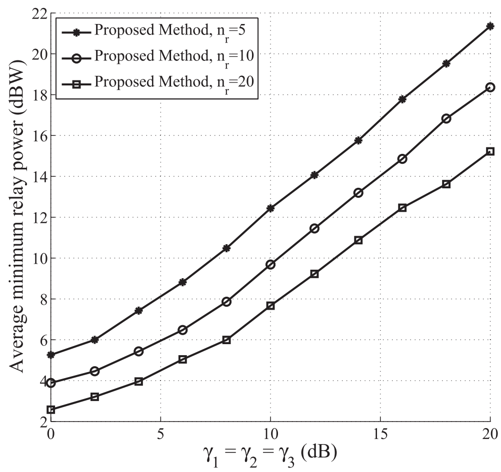

Fig. 8 Average relay transmit power versus equal SNR thresholds at the primary transceivers and at the CR base station for different number of CR/relays.

The average relay power consumption for different number of CR/relay nodes is presented in Fig. 8. As can be seen, when the SNR thresholds increase, the required power at the relays increases too. Moreover, it is observed that when the number of CR/relay nodes increases, the average minimum relay power decreases. This behavior can be intuitively explained by the fact that when the number of relays increases, the probability of having relays enjoying favorable channel conditions would increase (due to the space diversity) leading to a decrease of the power dissipated at relays (see also Fig. 5).

7. Conclusion

In this work, we investigated the problem of distributed beamforming in a two-step two-way relay network with cognitive radio capabilities. In our considered network, the relays also play the role of a cognitive user that communicates with a cognitive BS when the primary network is not in operation. We derived the optimal beamforming weight vector that minimizes the transmitted power from the relays under constraints that guarantee target SNR requirements at the primary transceivers and at the cognitive BS. Furthermore, we derived the feasibility region for the primary network transmit power indicating the domain where the optimization problem has a solution. Moreover, by appropriate modifications, we transformed the optimization problem into the well-known nonconvex QCQP, which can be efficiently solved by relaxation and Lagrangian duality techniques. Simulation results indicated that the probability of detection at the cognitive BS is improved by using the proposed beamforming scheme compared to a conventional relay network and a uniform AF relaying.

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Appendix: Proof of Lemma

In what follows, we first show that the constrained optimization problem (31) is not feasible for every value of the primary transmission powers P_1 and P_2 and then we derive a lower bound for the primary transmission powers. For notational simplification, let us define $\mathbf{H} = \mathbf{h}\mathbf{h}^H$, $\mathbf{H}' = \mathbf{h}'\mathbf{h}'^H$, $\mathbf{H}'' = \mathbf{h}''\mathbf{h}''^H$ and assume a unit noise variance, i.e., $\sigma^2 = 1$.

Since $\mathbf{w}^H\mathbf{w} = 1$, the constraints c_1, c_2 and c_3 of the optimization problem (31) can be written as:

$$\mathbf{w}^H(P_2\mathbf{H} - \gamma_1\mathbf{D}_1)\mathbf{w} - \mathbf{w}^H\mathbf{w}\gamma_1 \geq 0, \quad \text{(A· 1)}$$
$$\mathbf{w}^H(P_1\mathbf{H} - \gamma_2\mathbf{D}_2)\mathbf{w} - \mathbf{w}^H\mathbf{w}\gamma_2 \geq 0 \text{ and} \quad \text{(A· 2)}$$
$$\mathbf{w}^H(P_1\mathbf{H}' + P_2\mathbf{H}'' - \gamma_{cr}\mathbf{D}_3)\mathbf{w} - \mathbf{w}^H\mathbf{w}\gamma_{cr} \geq 0. \quad \text{(A· 3)}$$

The above constraints are not satisfied if the following matrices are negative semidefinite:

$$P_2\mathbf{H} - \gamma_1\mathbf{D}_1 - \gamma_1\boldsymbol{I} \leq 0, \quad \text{(A· 4)}$$
$$P_1\mathbf{H} - \gamma_2\mathbf{D}_2 - \gamma_2\boldsymbol{I} \leq 0 \text{ and} \quad \text{(A· 5)}$$
$$P_1\mathbf{H}' + P_2\mathbf{H}'' - \gamma_{cr}\mathbf{D}_3 - \gamma_{cr}\boldsymbol{I} \leq 0. \quad \text{(A· 6)}$$

It is well known that a test for positive definiteness of a matrix is that the eigenvalues are all positive real numbers. Let $\lambda_{max}\{\mathbf{A}\}$ denote the maximum eigenvalue of a given matrix $\mathbf{A}$. If the maximum eigenvalues of matrices in the left side of equations (A· 4), (A· 5) and (A· 6) are negative, the optimization problem will be infeasible. Then, the infeasibility condition can be written as:

$$\lambda_{max}\{P_2\mathbf{H} - \gamma_1\mathbf{D}_1 - \gamma_1\boldsymbol{I}\} \leq 0, \quad \text{(A· 7)}$$
$$\lambda_{max}\{P_1\mathbf{H} - \gamma_2\mathbf{D}_2 - \gamma_2\boldsymbol{I}\} \leq 0 \text{ and} \quad \text{(A· 8)}$$
$$\lambda_{max}\{P_1\mathbf{H}' + P_2\mathbf{H}'' - \gamma_3\mathbf{D}_3 - \gamma_{cr}\boldsymbol{I}\} \leq 0. \quad \text{(A· 9)}$$

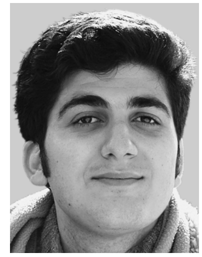
As a special case of Weyl's inequality (see Theorem 4.3.1 in [20]), we can write the maximum eigenvalue for the sum of multiple matrices in an inequality form. For example, if $\mathbf{A}$ and $\mathbf{B}$ are Hermitian matrices, this theorem says that $\lambda_{max}\{\mathbf{A} + \mathbf{B}\} \leq \lambda_{max}\{\mathbf{A}\} + \lambda_{max}\{\mathbf{B}\}$. Then, we can rewrite the above infeasibility conditions as:

$$\lambda_{max}\{P_2\mathbf{H}\} - \lambda_{max}\{\gamma_1\mathbf{D}_1\} - \gamma_1 \leq 0, \quad \text{(A· 10)}$$
$$\lambda_{max}\{P_1\mathbf{H}\} - \lambda_{max}\{\gamma_2\mathbf{D}_2\} - \gamma_2 \leq 0 \text{ and} \quad \text{(A· 11)}$$
$$\lambda_{max}\{P_1\mathbf{H}'\} + \lambda_{max}\{P_2\mathbf{H}''\} - \lambda_{max}\{\gamma_{cr}\mathbf{D}_3\} - \gamma_{cr} \leq 0. \quad \text{(A· 12)}$$

We can now derive a lower bound for the transmit power of the two primary transceivers, that makes the constraints feasible:

$$P_2\lambda_{max}\{\mathbf{H}\} - \gamma_1\lambda_{max}\{\mathbf{D}_1\} - \gamma_1 \geq 0, \quad \text{(A· 13)}$$
$$P_1\lambda_{max}\{\mathbf{H}\} - \gamma_2\lambda_{max}\{\mathbf{D}_2\} - \gamma_2 \geq 0 \text{ and} \quad \text{(A· 14)}$$
$$P_1\lambda_{max}\{\mathbf{H}'\} + P_2\lambda_{max}\{\mathbf{H}''\} - \gamma_{cr}\lambda_{max}\{\mathbf{D}_3\} - \gamma_{cr} \geq 0. \quad \text{(A· 15)}$$

As shown in Fig. 3, the above three equations are in fact the equations of three lines in the $P_1 - P_2$ plane that define a feasibility region for powers P_1 and P_2 where the constraints involved in the optimization problem (31) are feasible.



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