LETTER

On Ergodic Capacity of Spectrum-Sharing Systems in Fading Channels*

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SUMMARY In spectrum-sharing systems where the secondary user (SU) opportunistically accesses the primary user (PU)'s licensed channel, the SU should satisfy both the transmit power constraint of the SU transmitter and the received power constraint at the PU receiver. This letter studies the ergodic capacity of spectrum-sharing systems in fading channels. The ergodic capacity expression along with the optimal power allocation scheme is derived considering both the average transmit and received power constraints. The capacity function in terms of the two power constraints is found to be divided into transmit power limited region, received power limited region and dual limited region. Numerical results in Rayleigh fading channels are presented to verify our analysis.

key words: *ergodic capacity, opportunistic spectrum access, received power constraint, spectrum-sharing*

1. Introduction

Spectrum scarcity is becoming a serious problem due to the rapid development of wireless communications resulting in high demands for spectrum resources. Recent spectrum measurements, however, show that the licensed spectrum is actually severely underutilized by the primary users (PUs). In order to improve the spectrum utilization, the secondary users (SUs) are introduced to opportunistically access the licensed spectrum of the PU. For the spectrum-sharing concept to be acceptable, it is important to guarantee the performance of PU not compromised by the participation of the SU. To this end, the regulator requires the SU's transmission to satisfy the tolerable interference power level at the PU receiver, which is also described by the concept of interference temperature [1]. Therefore, in addition to the transmit power constraint that is often related to the hardware capabilities of the transmitter, e.g., the capability of the power amplifier or the energy limit of the battery, the SU transmitter should also satisfy the received power constraint at the PU receiver.

This letter studies the ergodic capacity of spectrum-sharing systems in fading channels under both the average transmit and received power constraints. It is found that the capacity function in terms of the two power constraints can be divided into three regions. The result further reveals that relaxing the received power constraint does not give more capacity in the transmit power limited region while increasing the value of the average transmit power constraint does not help to improve the capacity in the received power limited region.

Related works can be seen in [2], where the author studies the channel capacity of the SU system in different AWGN channels. Also, the capacity of spectrum-sharing systems in fading channels is investigated under the average or peak received power constraint in [3] and further under joint peak and average received power constraints in [4]. The transmit power constraint, however, is ignored in the derivations of both [3] and [4].

2. Problem Formulation

We consider a basic spectrum-sharing system as shown in Fig. 1. The SU transmitter is permitted to access the licensed channel of the PU if the transmission satisfies the tolerable interference power level of the PU receiver. Let g_0 and g_1 denote the instantaneous channel power gains from the SU transmitter to the PU and SU receivers respectively. g_0 and g_1 are assumed to be independent of each other. We also assume that perfect knowledge of g_0 and g_1 are known by both the SU transmitter and receiver. The SU transmitter may obtain g_0 by periodically sensing the pilot of the PU receiver provided that time division duplex (TDD) is employed by the PU transmission. g_0 may also be acquired by feeding back directly from the primary user or through a band manager [5] which mediates between the two parties [4]. We note that in fading environments, there are cases where g_0 may be difficult to perfectly estimate. Our results, however, provide capacity upper bounds in such cases.

If the average interference power level is the concerned performance of the PU, which implies that the PU's quality of service (QoS) is mainly determined by the average SNR, the transmission of the SU should satisfy the average re-

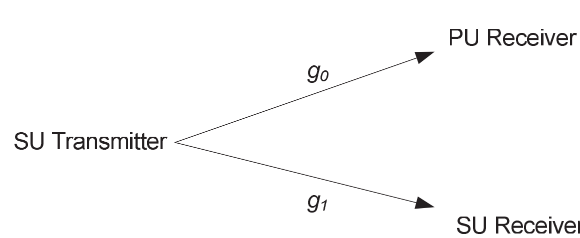

Fig. 1 Spectrum-sharing system model.

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ceived power constraint at the PU receiver. In addition, as in the conventional wireless systems, the SU is also subject to the average transmit power constraint, which is related to the hardware capabilities of the transmitter, e.g., the capability of power amplifier of a cell phone. In fading channels, it is straightforward to show that the channel capacity can be achieved by optimally distributing the transmit power across time such that both the average transmit and received power constraints are satisfied. Without loss of generality, the power of the white Gaussian noise at the SU receiver is assumed to be 1. Then, the optimal power allocation scheme can be obtained by solving the following optimization problem:

$$\begin{aligned} C = & \max_{P(g_0,g_1)\geq 0} \mathrm{E}_{g_0,g_1}\{\log(1+g_1 P(g_0,g_1))\} \\ \text{s.t.} \quad & \mathrm{E}_{g_0,g_1}\{P(g_0,g_1)\} \leq G, \\ & \mathrm{E}_{g_0,g_1}\{g_0 P(g_0,g_1)\} \leq Q, \end{aligned} \tag{1}$$

where $P(g_0, g_1)$ is the transmit power under certain fading states g_0 and g_1, G is the average transmit power constraint, Q is the average received power constraint, and $\mathrm{E}_{g_0,g_1}\{\cdot\}$ denotes the expectation over the joint probability density function (PDF) of g_0 and g_1.

3. Ergodic Capacity Analysis

It is complicated to solve (1) directly. Instead, we derive the solution by three steps, each of which is related with solving a sub-problem decomposed from (1). Interestingly, each solution of the sub-problem leads to a specialized power allocation scheme as well as a corresponding capacity function region of the original problem.

3.1 Transmit Power Limited Region

We begin with considering the sub-problem of (1) with only average transmit power constraint, which is given by

$$\begin{aligned} C = & \max_{P(g_0,g_1)\geq 0} \mathrm{E}_{g_0,g_1}\{\log(1+g_1 P(g_0,g_1))\}. \\ \text{s.t.} \quad & \mathrm{E}_{g_0,g_1}\{P(g_0,g_1)\} \leq G \end{aligned} \tag{2}$$

It is obvious to see that solving (2) is actually the same as the derivation of the capacity of the point to point fading channel with side information at both the transmitter and receiver [6]. The optimal power allocation, which is independent with g_0, is the water-filling over the fading states g_1 across time. Let $P_t^*(g_1)$ be the solution of (2). Then we have

$$P_t^*(g_1) = \left(\frac{1}{\lambda_1} - \frac{1}{g_1}\right)^+, \tag{3}$$

where λ_1 is determined by $\mathrm{E}_{g_1}\left\{\left(\frac{1}{\lambda_1} - \frac{1}{g_1}\right)^+\right\} = G$ and $(\cdot)^+$ denotes $\max\{\cdot, 0\}$. If $\mathrm{E}_{g_0,g_1}\{g_0 P_t^*(g_1)\} < Q$, then $P_t^*(g_1)$ is also the solution of (1). Since g_0 and g_1 are independent with each other, it follows that

$$\mathrm{E}_{g_0,g_1}\{g_0 P_t^*(g_1)\} = \mathrm{E}\{g_0\}\mathrm{E}_{g_1}\{P_t^*(g_1)\} = \mathrm{E}\{g_0\}G.$$

Therefore, we obtain that if $G < \frac{Q}{\mathrm{E}\{g_0\}}$, the capacity can be expressed as

$$C = \mathrm{E}_{g_1}\{\log(1+g_1 P_t^*(g_1))\}. \tag{4}$$

It is clear to see that (4) is independent with Q. Therefore, as long as $G < \frac{Q}{\mathrm{E}\{g_0\}}$ holds, the capacity of the spectrum-sharing system only depends on the average transmit power constraint, i.e., the capacity in this region is transmit power limited.

3.2 Received Power Limited Region

If $G < \frac{Q}{\mathrm{E}\{g_0\}}$ does not hold, we turn to consider the sub-problem with only average received power constraint, which is given by

$$\begin{aligned} C = & \max_{P(g_0,g_1)\geq 0} \mathrm{E}_{g_0,g_1}\{\log(1+g_1 P(g_0,g_1))\}. \\ \text{s.t.} \quad & \mathrm{E}_{g_0,g_1}\{g_0 P(g_0,g_1)\} \leq Q \end{aligned} \tag{5}$$

The solution, which is derived in [3], can be expressed as

$$P_r^*(g_0, g_1) = \left(\frac{1}{\lambda_0 g_0} - \frac{1}{g_1}\right)^+, \tag{6}$$

where λ_0 is determined by $\mathrm{E}_{g_0,g_1}\left\{\left(\frac{1}{\lambda_0} - \frac{g_0}{g_1}\right)^+\right\} = Q$. If $\mathrm{E}_{g_0,g_1}\{P_r^*(g_0,g_1)\} < G$, then $P_r^*(g_0, g_1)$ is also the solution of (1). Notice that $\mathrm{E}_{g_0,g_1}\{P_r^*(g_0,g_1)\}$ is a monotonically increasing function of Q, we define $\gamma(Q) = \mathrm{E}_{g_0,g_1}\{P_r^*(g_0,g_1)\}$. Therefore, if $G > \gamma(Q)$, the capacity can be expressed as

$$C = \mathrm{E}_{g_0,g_1}\{\log(1+g_1 P_r^*(g_0,g_1))\}, \tag{7}$$

which is the same as the result only considering the average received power constraint in [3]. Therefore, as long as $G > \gamma(Q)$ holds, the capacity of the spectrum-sharing system only depends on the average received power constraint, i.e., the capacity in this region is received power limited.

3.3 Dual Limited Region

In order to derive the capacity in the area outside the above two regions, we propose two lemmas first.

Lemma 1: $\frac{Q}{\mathrm{E}\{g_0\}} \leq \gamma(Q)$ always holds regardless of the value of Q and the PDFs of g_0 and g_1.

proof: If $\exists Q' \geq 0$ such that $\frac{Q'}{\mathrm{E}\{g_0\}} > \gamma(Q')$, there exists G' such that $\gamma(Q') < G' < \frac{Q'}{\mathrm{E}\{g_0\}}$. Thus, both $P_t^*(g_1)$ and $P_r^*(g_0, g_1)$ are the solutions of (1) given that $Q = Q'$ and $G = G'$. Since the objective function of (1) is strictly concave, there is only one solution for (1). Then, we have $P_t^*(g_1) = P_r^*(g_0, g_1)$. As $\mathrm{E}_{g_0,g_1}\{g_0 P_r^*(g_0,g_1)\} = Q'$ and $\mathrm{E}_{g_0,g_1}\{g_0 P_r^*(g_0,g_1)\} = \mathrm{E}_{g_0,g_1}\{g_0 P_t^*(g_1)\} = \mathrm{E}\{g_0\}G'$, it follows that $G' = \frac{Q'}{\mathrm{E}\{g_0\}}$, which contradicts the premise $\gamma(Q') < G' < \frac{Q'}{\mathrm{E}\{g_0\}}$. Therefore, we come to the conclusion. ∎

Lemma 2: Define a convex optimization problem (**OP**$_0$) with differentiable objective function and inequality constraints satisfying Slater's conditions:

$$\begin{aligned} \min_{x} \quad & h_0(x) \\ \text{s.t.} \quad & h_i(x) \le 0, i = 1, \cdots, n. \end{aligned} \tag{8}$$

and let x_0 be any optimal point of $\mathbf{OP}_0$. Assume x_k is any optimal point of a sub-problem ($\mathbf{OP}_{-k}$) of $\mathbf{OP}_0$ with $n-1$ inequality constraints:

$$\begin{aligned} \min_{x} \quad & h_0(x) \\ \text{s.t.} \quad & h_i(x) \le 0, i = 1, \cdots, k-1, k+1, \cdots, n. \end{aligned} \tag{9}$$

Then if $h_k(x_k) \ge 0$, we have $h_k(x_0) = 0$.

proof: We use reduction to absurdity to prove the lemma. Assume $h_k(x_0) \ne 0$, then we have $h_k(x_0) < 0$. Since x_0 is the optimal point of the convex optimization problem $\mathbf{OP}_0$, x_0 satisfies all the KKT conditions of $\mathbf{OP}_0$, which are given by

$$h_i(x_0) \le 0,\ \alpha_i \ge 0,\ \alpha_i h_i(x_0) = 0, \quad i = 1, \cdots, n \tag{10a}$$

$$\nabla h_0(x_0) + \sum_{i=1}^{n} \alpha_i \nabla h_i(x_0) = 0, \tag{10b}$$

where α_i is the Lagrange multiplier. Since $h_k(x_0) < 0$, according to $\alpha_k h_k(x_0) = 0$ in (10a), we have $\alpha_k = 0$. Then (10b) can be expressed as

$$\nabla h_0(x_0) + \sum_{i=1, i\ne k}^{n} \alpha_i \nabla h_i(x_0) = 0. \tag{11}$$

Based on (10a) and (11), it is clear that x_0 also satisfies the KKT conditions of $\mathbf{OP}_{-k}$. Therefore, x_0 is also an optimal point of $\mathbf{OP}_{-k}$. Consequently, we know that there is an optimal point of $\mathbf{OP}_{-k}$ satisfying $h_k(x_k) < 0$, which contradicts the premise that $h_k(x_k) \ge 0$. Finally, we come to the conclusion that $h_k(x_0) = 0$. ∎

Based on Lemma 1, the area outside the transmit power limited region and received power limited region can be described by the inequality $\frac{Q}{\mathrm{E}\{g_0\}} \le G \le \gamma(Q)$. According to the derivations in Sect. 3.1 and Sect. 3.2, $\frac{Q}{\mathrm{E}\{g_0\}} \le G \le \gamma(Q)$ implies that $\mathrm{E}_{g_0,g_1}\{g_0 P_t^*(g_1)\} \ge Q$ and $\mathrm{E}_{g_0,g_1}\{P_r^*(g_0,g_1)\} \ge G$. Based on Lemma 2, we can derive that the solution of (1) must satisfy the two power constraints with equalities when $\frac{Q}{\mathrm{E}\{g_0\}} \le G \le \gamma(Q)$. Thus, solving (1) in this case is equivalent to solving the following optimization problem:

$$\begin{aligned} C = \quad & \max_{P(g_0,g_1)\ge 0} \mathrm{E}_{g_0,g_1}\{\log(1 + g_1 P(g_0,g_1))\} \\ \text{s.t.} \quad & \mathrm{E}_{g_0,g_1}\{P(g_0,g_1)\} = G, \\ & \mathrm{E}_{g_0,g_1}\{g_0 P(g_0,g_1)\} = Q. \end{aligned} \tag{12}$$

To find the solution, we form the Lagrangian:

$$\begin{aligned} & L(P(g_0,g_1), \lambda_0, \lambda_1) = \mathrm{E}_{g_0,g_1}\{\log(1 + g_1 P(g_0,g_1))\} \\ & -\lambda_1 \left(\mathrm{E}_{g_0,g_1}\{P(g_0,g_1)\} - G\right) \\ & -\lambda_0 \left(\mathrm{E}_{g_0,g_1}\{g_0 P(g_0,g_1)\} - Q\right). \end{aligned} \tag{13}$$

The first order KKT condition is given by

$$\begin{aligned} & \frac{\partial L(P(g_0,g_1),\lambda_0,\lambda_1)}{\partial P(g_0,g_1)} \\ & = \mathrm{E}_{g_0,g_1}\left\{\frac{g_1}{g_1 P(g_1,g_1)+1} - \lambda_1 - \lambda_0 g_0\right\} \\ & = 0. \end{aligned} \tag{14}$$

The optimal power allocation can be obtained by solving (14) with the constraint $P(g_0,g_1) \ge 0$, which is given by

$$P_d^*(g_0,g_1) = \left(\frac{1}{\lambda_0 g_0 + \lambda_1} - \frac{1}{g_1}\right)^+, \tag{15}$$

where λ_0 and λ_1 are determined by the following equations:

$$\begin{cases} \mathrm{E}_{g_0,g_1}\left\{\left(\frac{1}{\lambda_0 g_0+\lambda_1} - \frac{1}{g_1}\right)^+\right\} & = G, \\ \mathrm{E}_{g_0,g_1}\left\{\left(\frac{g_0}{\lambda_0 g_0+\lambda_1} - \frac{g_0}{g_1}\right)^+\right\} & = Q. \end{cases}$$

Thus, the capacity can be expressed as

$$C = \mathrm{E}_{g_0,g_1}\left\{\log(1 + g_1 P_d^*(g_0,g_1))\right\}. \tag{16}$$

Therefore, as long as $\frac{Q}{\mathrm{E}\{g_0\}} \le G \le \gamma(Q)$ holds, the capacity of the spectrum-sharing system depends on both the average transmit and received power constraints, i.e., the capacity in this region is dual limited.

Combining the above results, we finally obtain the expression of the capacity given by (4), (7) and (16), along with the optimal power allocation determined by (3), (6) and (15) respectively.

4. Numerical Results

In this section, we present numerical results to verify our analysis. Assume both the channels are Rayleigh fading, i.e., g_0 and g_1 are both subject to exponential distribution. Without loss of generality, we further assume $\mathrm{E}\{g_0\} = \mathrm{E}\{g_1\} = 1$.

Figure 2 shows how the division of the capacity function regions varies the values of Q and G vary. The three regions are separated by the curves $G = \gamma(Q)$ and $G = \frac{Q}{\mathrm{E}\{g_0\}}$. Figure 3 illustrates the ergodic capacity versus the average transmit power constraint under different values of Q. Note that $Q = \infty$ implies that no received power constraint is placed on the SU transmitter. Take the curve $Q = 0$ dB for example. If G is less than 0 dB, the curve is overlapped with

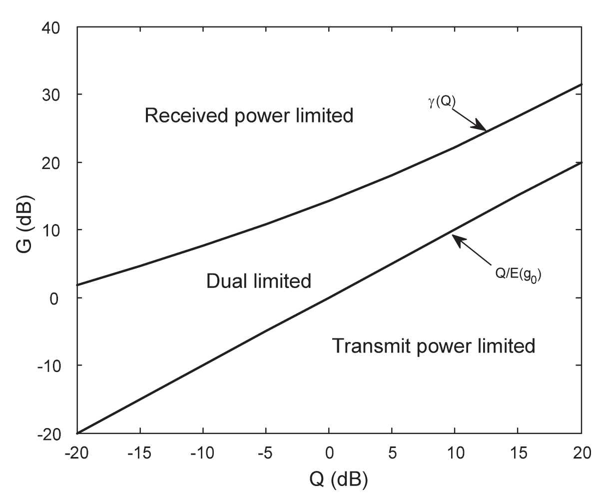

Fig. 2 Capacity function regions division.

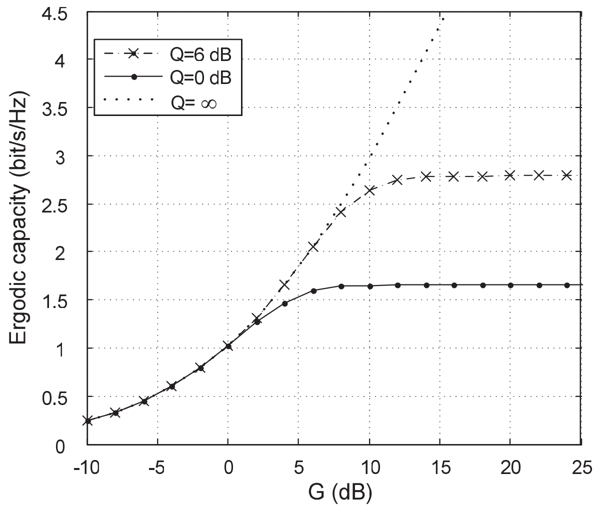

Fig. 3 Ergodic capacity versus average transmit power constraint under different average received power constraints.

the curves $Q = \infty$ and $Q = 6$ dB, which implies that increasing the value of average received power constraint does not help to increase the capacity. This is because $\frac{Q}{\mathrm{E}\{g_0\}} = 0$ dB and the capacity lies in the transmit power limited region. The curve of $Q = 0$ dB is no longer overlapped with the other two curves when G is greater than 0 dB. This can be explained by the fact that the capacity lies in the dual limited region and is impacted by both the average transmit and received power constraints. Since it can be calculated that $\gamma(Q) = 14.2$ dB given $Q = 0$ dB, the capacity enters the received power limited region when G is greater than 14.2 dB. As shown in Fig. 3, as the value of G increases, the capacity remains the same, which implies that increasing the value of average transmit power constraint does not help to improve the capacity in this region.

5. Conclusions

In this letter, we investigate the ergodic capacity of spectrum-sharing systems in fading channels. Different from [3], the capacity is studied under both the average transmit and received power constraints. The capacity function in terms of the two power constraints is found to be divided into three regions. We can also see that relaxing the average received power constraint does not give more capacity in the transmit power limited region while increasing the value of average transmit power constraint does not help to improve the capacity in the received power limited region.

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