LETTER

Multiuser Detection for MC-CDMA System Using an RQP Approach**

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SUMMARY A recursive quadratic programming (RQP) approach is proposed for multiuser detection in multicarrier code-division multiple-access (MC-CDMA) systems. In this approach, the combinatorial problem associated with the optimal maximum likelihood (ML) detection is relaxed to a quadratic programming (QP) problem first and then a recursive approach is developed to improve the detection performance. Computer simulations are presented which demonstrate that the detector developed based on the proposed approach offers close-to-optimal symbol-error rate (SER) performance which outperforms several existing suboptimal detectors.

key words: *quadratic programming, recursive approach, multiuser detection, M-ary modulation, MC-CDMA*

1. Introduction

In multicarrier code-division multiple-access (MC-CDMA) systems, the information symbols of each user are spread over a specific spreading code. After the spreading, information-bearing chips are modulated onto and transmitted over a group of subcarriers. The bandwidth of each subcarrier is assumed to be sufficiently narrow so that the propagation channel for each subcarrier can be considered as frequency-flat fading. In general, the probability that all subcarriers undergo deep fades at the same time is low, and hence frequency diversity is achieved. This is among one of the most important properties that make MC-CDMA scheme a promising candidate for wideband wireless communication systems.

In a typical MC-CDMA system experiencing frequency selective fading, the attenuation magnitude and phase shift for different subcarriers are independent. If a number of users simultaneously transmit information-bearing symbols by sharing the subcarriers, multiuser interference (MUI) is inevitably in presence at the received signal. To alleviate the MUI for recovering the transmitted symbols, various detection or interference cancellation approaches have been proposed for MC-CDMA systems [1]–[3].

In this paper, a new multiuser detector based on a *recursive quadratic programming* (RQP) approach is proposed, which is motivated by an early work in [4]. In the proposed detector, the combinatorial problem associated with the optimal maximum-likelihood (ML) detection is first relaxed into a quadratic programming (QP) problem, then a recursive approach is developed to improve the detection performance. Computer simulations are presented to demonstrate that the proposed detector outperforms several existing suboptimal detectors in terms of symbol-error rate (SER) performance. The computational complexity of the proposed detector is also investigated and compared with other detectors.

2. Signal Model

We consider a K-user synchronous MC-CDMA system of L subcarriers. The block diagram of the system is illustrated in Fig. 1. The signal received at the front-end of the receiver is denoted as [5]

$$r(t) = \sum_{k=1}^{K} \sqrt{a_k} b_k \sum_{l=1}^{L} c_{kl} h_{kl} e^{j\omega_l t} + n(t), \quad t \in [0,\ T_s) \quad (1)$$

where b_k denotes the information-bearing symbol which belongs to an alphabet set $\mathcal{A}$ of cardinality M, $\sqrt{a_k}$ is the amplitude of the received signal of user k, T_s is the symbol interval, $\omega_l = 2\pi l/T_s$, and c_{kl} is the lth component of the kth user's spreading code $\boldsymbol{c}_k = [c_{k1}\ c_{k2}\ \cdots\ c_{kL}]^T$. The propagation channel for each subcarrier is assumed frequency-flat fading for all users and thus can be described by using a complex coefficient h_{kl}, $k = 1, \ldots, K$ and $l = 1, \ldots, L$. $n(t)$ denotes an additive white Gaussian noise (AWGN) pro-

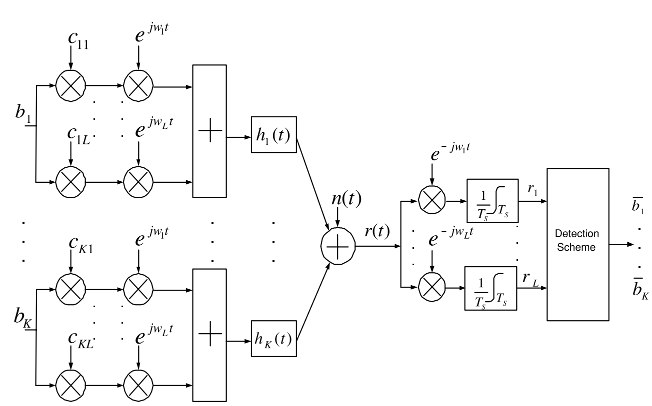

Fig. 1 System diagram of synchronous MC-CDMA system.

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cess with independent real and imaginary components, each of which has zero mean and variance σ^2. The spreading codes are normalized as $\|\boldsymbol{c}_k\|^2 = 1$ for $k = 1, \ldots, K$.

The symbol b_k for $k = 1, \ldots, K$ belongs to the alphabet set $\mathcal{A}$. They usually can be represented as complex constellation points in signal space diagram, i.e., $b_k \in \mathcal{A} = \{I_m + jQ_m\}$ with I_m and Q_m ($m = 1, \ldots, M$) representing the values of In-phase (I) and Quadrature(Q) components of b_k, respectively.

At receiver side, $r(t)$ in (1) is first correlated by corresponding subcarriers and then sampled at the end of each symbol interval. The outputs of the correlators can be denoted as

$$\begin{aligned} r_n &= \frac{1}{T_s}\int_0^{T_s} r(t)e^{-j2\pi nt/T_s}\, dt \\ &= \sum_{k=1}^{K} \sqrt{a_k} b_k c_{kn} h_{kn} + \eta_n \quad \text{for} \quad n = 1, \ldots, L \end{aligned} \tag{2}$$

where η_n is a complex Gaussian random variable with zero mean and variance $2\sigma^2/T_s$. In matrix form, (2) can be expressed as

$$\boldsymbol{r} = \boldsymbol{CAb} + \boldsymbol{\eta} \tag{3}$$

where $\boldsymbol{r} = [r_1\ r_2\ \cdots\ r_L]^T$, $\boldsymbol{A} = \mathrm{diag}\{\sqrt{a_1}, \sqrt{a_2}, \ldots, \sqrt{a_K}\}$, $\boldsymbol{b} = [b_1\ b_2\ \cdots\ b_K]^T$, $\boldsymbol{C}$ is an $L \times K$ matrix given as

$$\boldsymbol{C} = \begin{bmatrix} c_{11}h_{11} & c_{21}h_{21} & \cdots & c_{K1}h_{K1} \\ c_{12}h_{12} & c_{22}h_{22} & \cdots & c_{K2}h_{K2} \\ \vdots & \vdots & \ddots & \vdots \\ c_{1L}h_{1L} & c_{2L}h_{2L} & \cdots & c_{KL}h_{KL} \end{bmatrix} \tag{4}$$

and $\boldsymbol{\eta} = [\eta_1\ \eta_2\ \cdots\ \eta_L]^T$ is a vector of zero-mean complex Gaussian variables with covariance matrix $2\sigma^2/T_s\boldsymbol{I}$.

The objective of multiuser detection is to recover the information-bearing symbols $\boldsymbol{b}$ from $r(t)$ in (1). It is shown that the ML detection can be carried out by solving the following optimization problem [1]

$$\text{minimize} \quad \boldsymbol{x}^H\boldsymbol{Hx} + \mathrm{Re}\{\boldsymbol{x}^H\boldsymbol{p}\} \tag{5a}$$

$$\begin{aligned} &\text{subject to}: x_i \in \mathcal{A} = \{I_m + jQ_m\} \\ &\quad \text{for} \ \ i = 1, \ldots, K; m = 1, \ldots, M \end{aligned} \tag{5b}$$

where $\boldsymbol{H} = \boldsymbol{AC}^H\boldsymbol{CA}$, $\boldsymbol{p} = -2\boldsymbol{Ay}$, $\boldsymbol{y} = \boldsymbol{C}^H\boldsymbol{r}$, and x_i denotes the ith component of $\boldsymbol{x}$.

Note that the feasible region of the problem in (5) is composed of all possible constellation points in the alphabet set $\mathcal{A}$. Because of the constraints in (5b), the optimization problem in (5) is a combinatorial problem in nature. In principle, its solution can be obtained only by exhaustive evaluation of the objective function over M^K possible values of $\boldsymbol{x}$.

3. Quadratic Relaxation for the Combinatorial Problem

The problem in (5) is converted into the following one that only involves real-valued variables, i.e,

$$\text{minimize} \quad \tilde{\boldsymbol{x}}^T\tilde{\boldsymbol{H}}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{x}}^T\tilde{\boldsymbol{p}} \tag{6a}$$

$$\text{subject to}: \tilde{x}_i + j\tilde{x}_{i+K} \in \mathcal{A} \ \ \text{for} \ \ i = 1, \ldots, K \tag{6b}$$

where

$$\tilde{\boldsymbol{x}} = [\ \mathrm{Re}\{\boldsymbol{x}^T\}\ \mathrm{Im}\{\boldsymbol{x}^T\}\ ]^T$$

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \mathrm{Re}\{\boldsymbol{H}\} & -\mathrm{Im}\{\boldsymbol{H}\} \\ \mathrm{Im}\{\boldsymbol{H}\} & \mathrm{Re}\{\boldsymbol{H}\} \end{bmatrix}$$

$$\tilde{\boldsymbol{p}} = [\ \mathrm{Re}\{\boldsymbol{p}^T\}\ \mathrm{Im}\{\boldsymbol{p}^T\}\ ]^T$$

and $\tilde{x}_i$ denotes the ith component of $\tilde{\boldsymbol{x}}$.

If the constraint set in (6b) is relaxed into (7b) that just covers the region containing all constellation points, the problem in (6) can be relaxed to

$$\text{minimize} \quad \tilde{\boldsymbol{x}}^T\tilde{\boldsymbol{H}}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{x}}^T\tilde{\boldsymbol{p}} \tag{7a}$$

$$\begin{aligned} &\text{subject to}: \mathcal{S}_R^{\min} \le \tilde{x}_i \le \mathcal{S}_R^{\max} \ \ i = 1, \ldots, K \\ &\quad \mathcal{S}_I^{\min} \le \tilde{x}_i \le \mathcal{S}_I^{\max} \ \ i = K+1, \ldots, 2K \end{aligned} \tag{7b}$$

where $\mathcal{S}_R$ and $\mathcal{S}_I$ are the sets of values that can be assumed by I_m and Q_m, respectively. $\mathcal{S}_R^{\max}$ and $\mathcal{S}_R^{\min}$ denote the maximal and minimal values for $\mathcal{S}_R$, and $\mathcal{S}_I^{\max}$ and $\mathcal{S}_I^{\min}$ denote the maximal and minimal values for $\mathcal{S}_I$. After the relaxation, the problem in (7) becomes a QP problem and hence is a QP relaxation of the combinatorial problem in (6). In what follows, the detector developed based on the solution of the problem in (7) is termed as a QP detector.

Once the solution to the problem in (7), denoted as $\tilde{\boldsymbol{x}}^*$, is obtained, the associated complex constellation diagram can be obtained as $x_i^* = \tilde{x}_i^* + j\tilde{x}_{i+K}^*$ for user i. Then the decisions for the transmitted information symbol can be made based on shortest Euclidean distance criterion as follows

$$\begin{aligned} \bar{b}_i &= dec\{x_i^*\} = \{I_n + jQ_n : |I_n + jQ_n - x_i^*| \le \\ &\quad |I_m + jQ_m - x_i^*|, \forall m \ne n\} \ \ \text{for} \ \ i = 1, \ldots, K \end{aligned} \tag{8}$$

Obviously, the QP detector achieves suboptimal detection performance only, since the problem in (7) of the proposed QP detector is a relaxation of the problem in (6) for the optimal ML detector.

4. Multiuser Detection Based on Recursive Quadratic Programming Approach

In this section, we consider to improve the performance of the QP detector.

Denote Ω_j the set of indices of the users whose information symbols have been detected before the jth iteration and $\bar{b}_i$ ($i \in \Omega_j$) the determined information symbol for the ith user. The QP problem in (7) can be modified into a recursive form as

$$\text{minimize} \quad \tilde{\boldsymbol{x}}^T\tilde{\boldsymbol{H}}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{x}}^T\tilde{\boldsymbol{p}} \tag{9a}$$

$$\begin{aligned} &\text{subject to}: \ \mathcal{S}_R^{\min} \le \tilde{x}_i \le \mathcal{S}_R^{\max} \ \ i \notin \tilde{\Omega}_j,\ 1 \le i \le K \\ &\mathcal{S}_I^{\min} \le \tilde{x}_i \le \mathcal{S}_I^{\max} \ \ i \notin \tilde{\Omega}_j,\ K+1 \le i \le 2K \end{aligned} \tag{9b}$$

$$\tilde{x}_i = \mathrm{Re}\{\bar{b}_i\},\ \tilde{x}_{i+K} = \mathrm{Im}\{\bar{b}_i\} \ \ \text{for} \ \ i \in \Omega_j \tag{9c}$$

where $\tilde{\Omega}_j$ denotes the set of indices of the real and imaginary components of the information symbols which have been detected before the jth iteration. Due to the fact that $\tilde{x}_i$ and $\tilde{x}_{i+K}$ for $i \in \Omega_j$ are known, the variables in (9) are $\{\tilde{x}_i,\ \tilde{x}_{i+K}$ for $i \notin \Omega_j\}$. Obviously, the problem in (9) is a size-reduced problem of (7). Once the solution of (9) is obtained, Ω_j is expanded to Ω_{j+1} by including the indices of the users whose information symbols are detected in the jth iteration. By following this rule, new problems can be formulated sequentially until all information symbols are detected.

As can be shown, the proposed recursive approach tries to improve the detection performance of the QP detector proposed in Sect. 3 with the help of determined information symbols obtained in previous iterations. This motivation is similar to that of many other decision-aided multiuser detectors. Substituting (9c) into (9a), the problem in (9) is shown to be equivalent to

$$\text{minimize} \quad \tilde{\boldsymbol{x}}_j^T \tilde{\boldsymbol{H}}_j \tilde{\boldsymbol{x}}_j + \tilde{\boldsymbol{x}}_j^T (\tilde{\boldsymbol{p}}_j + 2\hat{\boldsymbol{H}}_j \hat{\boldsymbol{b}}_j) \tag{10a}$$

$$\text{subject to}: \ \mathcal{S}_R^{\min} \le \tilde{x}_i \le \mathcal{S}_R^{\max} \ \ i \notin \tilde{\Omega}_j,\ 1 \le i \le K$$

$$\mathcal{S}_I^{\min} \le \tilde{x}_i \le \mathcal{S}_I^{\max} \ \ i \notin \tilde{\Omega}_j,\ K+1 \le i \le 2K \tag{10b}$$

where $\tilde{\boldsymbol{x}}_j = \{\tilde{x}_i, i \notin \tilde{\Omega}_j\}$ denotes the variable vector obtained by removing the components of $\tilde{\boldsymbol{x}}$ whose indices are in $\tilde{\Omega}_j$, $\hat{\boldsymbol{b}}_j = [\ \mathrm{Re}\{\bar{\boldsymbol{b}}_j^T\}\ \ \mathrm{Im}\{\bar{\boldsymbol{b}}_j^T\}\ ]^T$ with $\bar{\boldsymbol{b}}_j = \left\{\bar{b}_i, i \in \Omega_j\right\}$ denoting the vector of M-ary decisions that have been determined before the jth iteration, $\tilde{\boldsymbol{p}}_j$ is obtained by removing the components of $\tilde{\boldsymbol{p}}$ whose indices are in $\tilde{\Omega}_j$, $\tilde{\boldsymbol{H}}_j$ and $\hat{\boldsymbol{H}}_j$ denote submatrices of $\tilde{\boldsymbol{H}}$ where $\tilde{\boldsymbol{H}}_j$ is obtained by removing the rows and columns of $\tilde{\boldsymbol{H}}$ whose indices are in $\tilde{\Omega}_j$, and $\hat{\boldsymbol{H}}_j$ is obtained by removing the rows of $\tilde{\boldsymbol{H}}$ whose indices are in $\tilde{\Omega}_j$ and columns of $\tilde{\boldsymbol{H}}$ whose indices are not in $\tilde{\Omega}_j$.

Denoting the solution of the problem in (10) as $\tilde{\boldsymbol{x}}_j^* = \{\tilde{x}_i^*, i \notin \tilde{\Omega}_j\}$, $\{x_i^*, i \notin \Omega_j\}$ and $\{\bar{b}_i, i \notin \Omega_j\}$ can be obtained according to (8). The information symbols can then be determined by using a thresholding process described as follows. Denote d_i the shortest Euclidean distance between x_i^* and all constellation points in $\mathcal{A}$, i.e.,

$$d_i = |x_i^* - \bar{b}_i| \quad \text{for} \quad i \notin \Omega_j \tag{11}$$

For each $i \notin \Omega_j$, if d_i is no greater than a prescribed threshold, the information symbol of the ith user is determined as $\bar{b}_i$; otherwise, the information symbol is left to be detected in subsequent iterations.

The RQP approach is summarized in Table 1. In Step 2, $\min(\boldsymbol{d}_j)$ denotes the component of $\boldsymbol{d}_j = \{d_i, i \notin \Omega_j\}$ of least magnitude, and the threshold ξ_j assumes a value greater than $\min(\boldsymbol{d}_j)$. Therefore, in each iteration, at least one information symbol will be determined. Consequently, all information symbols can be detected in at most K iterations. In general, the magnitude of threshold affects the trade-off between detection performance and computational complexity. In the simulations to be presented in the next section, the threshold for the RQP detector is specified in the form of

$$\xi_j = \alpha \cdot \min(\boldsymbol{d}_j) \quad \text{for} \quad j = 1,\ 2, \dots \tag{12}$$

Table 1 Recursive QP detection.

Parameters:
n_j: number of detected information symbols before the jth iteration.
ξ_j: threshold used in the jth iteration.
Initialization:
$j = 0$, $n_0 = 0$, $\Omega_0 = null$, $\tilde{\Omega}_0 = null$, $\tilde{\boldsymbol{H}}_0 = \tilde{\boldsymbol{H}}$, $\tilde{\boldsymbol{p}}_0 = \tilde{\boldsymbol{p}}$, $\hat{\boldsymbol{b}}_0 = \boldsymbol{0}$, and $\hat{\boldsymbol{H}}_0 = \boldsymbol{0}$.
Step 1: Solve the QP problem in (10) and denote the solution as $\tilde{\boldsymbol{x}}_j^* = \{\tilde{x}_i^*, i \notin \tilde{\Omega}_j\}$.
Step 2: Let $x_i^* = \tilde{x}_i^* + j\tilde{x}_{i+K}^*$. Calculate $\bar{b}_i = dec\{x_i^*\}$ and $\boldsymbol{d}_j = \{d_i, i \notin \Omega_j\}$. Let $\xi_j \in [\min(\boldsymbol{d}_j), +\infty)$. For $i \notin \Omega_j$, if $d_i \le \xi_j$, make a decision for the corresponding information symbol as $\bar{b}_i$.
Step 3: Denote Ω_{j+1} the index set which includes all indices of Ω_j and those of the users whose information symbols are detected in Step 2. Update n_{j+1}. If $n_{j+1} = K$, stop and output $\bar{\boldsymbol{b}}_{j+1} = \{\bar{b}_i, i \in \Omega_{j+1}\}$.
Step 4: Update $\tilde{\Omega}_{j+1}$, $\tilde{\boldsymbol{H}}_{j+1}$, $\tilde{\boldsymbol{p}}_{j+1}$, $\hat{\boldsymbol{H}}_{j+1}$, and $\hat{\boldsymbol{b}}_{j+1}$ according to the definitions for (10). Set $j = j + 1$ and repeat from Step 1.

where α is a scalar greater than or equal to one.

As shown in Table 1, decisions are made only for the information symbols that can be detected with higher accuracy in each iteration. Then the determined information symbols are fixed in the following iterations and a QP problem of reduced size for the other information symbols is formulated. This process continues until the detection of all information symbols are completed.

5. Simulation Results

Computer simulations were conducted to evaluate the SER performance of the detectors developed based on the proposed RQP approach. An uplink MC-CDMA system with two-ray frequency-selective multipath Rayleigh fading channel is considered. The attenuation magnitudes for Rayleigh fading are generated by following the probability density function (PDF) below

$$p(r) = re^{-r^2/2}, \quad r \ge 0 \tag{13}$$

which change once over each symbol period. Comparisons with the ML, the QP, the Equal Gain Combining (EGC), the Maximal Ratio Combining (MRC) [2], and the Global MMSE (GMMSE) detectors [3] were also carried out. The modulation scheme used in the simulations is 16-Quadrature Amplitude Modulation (QAM) with rectangular constellation diagram of equal protection [6]. All users in the simulated systems are assumed to have equal average transmission signal energy. The spreading codes of all users are randomly generated and the same spreading code is shared by both I and Q components for each user. In simulations, a "trial-and-fail" approach was adopted for choosing the value of α in (12), i.e., many different values of α are tried until the requirements of performance and complexity are well balanced. It is found that an α value slightly greater than one demonstrate a good balance of satisfactory detection

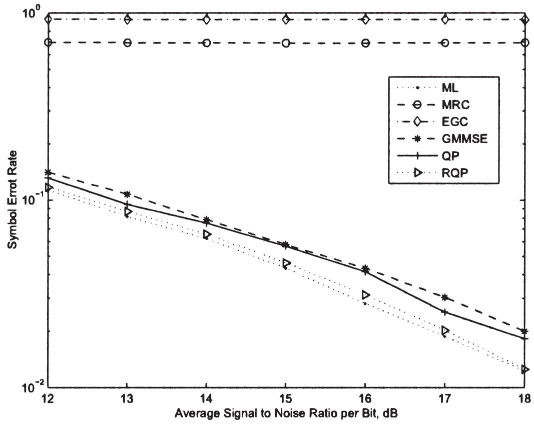

Fig. 2 SERs obtained in the first example.

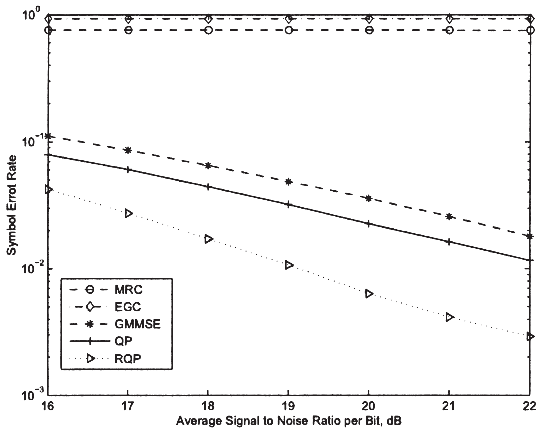

Fig. 3 SERs obtained in the second example.

performance and reasonably low computational complexity. Therefore, in the simulations, α is chosen and fixed to be 1.01 as an example to convey valid results of the proposed approach.

In the first simulation we considered a four-user system. The spreading codes used are of length fifteen. The SER obtained by using the RQP detector is plotted in Fig. 2. For comparison purposes, the SERs obtained by using the ML, the QP, the EGC, the MRC, and the GMMSE detectors are also plotted in the same figure. It can be observed in Fig. 2 that the SER of the RQP detector is close to that of the ML detector and superior relative to those obtained by using the QP and other suboptimal detectors.

In the second simulation a ten-user system using spreading codes of length fifteen was considered. Due to the larger number of users in the system, the received signal is subject to more significant MUI than that in the first

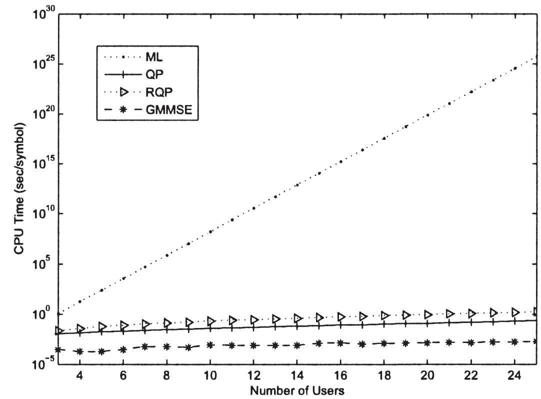

Fig. 4 Comparison of computational complexity.

example. The SER of the RQP detector is plotted in Fig. 3. Note that the extremely high computational complexity for the ML detector in this example prevents us demonstrating the SER curve of the ML detector. As similar to what has been observed in the first example, the RQP detector outperforms other suboptimal detectors.

The computational complexity of the proposed RQP detector was evaluated and compared with those of the ML, the QP, and the GMMSE detectors. The measurement of interest is average CPU time involved for detecting one symbol. The results obtained are shown in Fig. 4. It is seen that the computational complexity of the RQP detector is significantly lower than that of the ML detector but higher than those of the QP and the GMMSE detectors.

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