Minimum SINR Maximization for Multiuser MIMO Downlink with Per BS Power Constraints

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***Abstract*— The joint cooperative processing of transmitted signal from several multiple-input multiple-output (MIMO) base station (BS) antenna heads is considered for users located within a soft handover (SHO) region. The mathematical framework for the SHO based MIMO system is derived and the joint design of linear transmit and receive beamformers in a MIMO multiuser transmission subject to per BS power constraints is considered. Solution for the maximization of the minimum weighted SINR per data stream criterion is proposed. The proposed algorithm is shown to provide very efficient solutions despite of the fact that the global optimum cannot be guaranteed due to the non-convexity of the problem. Moreover, a less complex but still efficient allocation method based on zero forcing transmission is provided for the same optimization criterion.**

I. Introduction

There has been increasing interest to consider network infrastructure based *cooperative processing between base stations (BSs)* with a cellular system [1]–[7] or fixed relay stations. Recently, [2]–[5], [8]–[11] studied the downlink sum rate and spectral efficiency optimization for cooperative multiple-input-multiple-output (MIMO) systems with perfect data cooperation between base stations. Although BS cooperation naturally increases the system complexity, it has potentially significant capacity and coverage benefits making it worth more detailed consideration.

The sum capacity and the capacity region of MIMO downlink (DL) with per antenna or per BS power constraints were recently discovered in [12] and in [11], [13], respectively. Furthermore, the minimum-power beamformer design for multiple-input single-output (MISO) DL under per antenna or per BS power constraints was investigated in [11], where the original DL problem was transformed into a dual uplink (UL) minimax optimization problem with an uncertain noise covariance. Convex optimization methods [14], such as second-order cone programming [15], semidefinite programming [16] and geometric programming [17], are very powerful tools which allow for efficient numerical solution for many signal processing and communications problems, e.g., [11], [18]–[22]. In particular, they were used to solve a wide range of optimal transmit and receive beamformer design problems [23]–[26].

The purpose of this paper is to analyze the *BS cooperation with linear processing* in a more practical scenario. We assume a time division duplex (TDD) system with adaptive MIMO transmission, where the transmission parameters in reciprocal uplink (UL) and downlink (DL) can be adapted according to the channel conditions. In order to attain the channel state information (CSI) between all users and BS antennas in the cellular network the channels should be jointly estimated at each BS, which may be difficult to implement in practice. We consider the case where the joint cooperative processing of transmitted signal from several MIMO BS antenna heads is restricted to an area where the users have comparable signal strengths from adjacent BS antenna heads. We assume that cooperative signal processing can be performed in a centralized manner so that the MIMO antenna heads are distributed over a larger geographical area (e.g., hundreds of meters), as illustrated for example in [7, Fig. 4]. The distributed antenna heads are then connected to the central processing unit via radio over fiber technology or wireless microwave links, for example. Similarly to the soft handover (SHO) feature in (W)CDMA systems [27], SHO region is defined for users with similar received power levels from adjacent distributed BS antenna heads. Since the signal processing of the BS antenna heads is concentrated at one central controller, joint beamforming from all the antennas belonging to the ”active set” can be performed to the user(s) within the SHO region.

In [28], we studied single-user DL space-frequency bit and power allocation for rate maximization criterion with different BS power constraints in an adaptive MIMO-OFDM system. In [29], the system level gains and trade-offs from cooperative SHO processing were investigated. The impact of the size of the SHO region, overhead from increased physical (time, frequency) resource utilization, different non-reciprocal inter-cell interference distributions due to SHO was evaluated by system level simulations. Large link and system level gains were shown to be available from cooperative processing.

In this paper, we limit our attention to linear multiuser precoding schemes with per BS power constraints. We propose a general method for joint design of the linear transmit and the receive beamformers for *maximizing the minimum weighted SINR per data stream subject to per BS power constraints*. Due to non-convexity of the problem, globally optimal solutions cannot be guaranteed, but simulation results demonstrate that the solutions are efficient in practically relevant scenarios. We extend the precoder design via conic optimization for fixed receivers introduced recently in [26] to include per BS (and/or per antenna) power constraints. We propose an iterative solu-

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tion where the transmit and receive beamformers are separately optimized, and where each step can be efficiently solved by using standard convex optimization tools [30]. In addition, we focus on generalized zero forcing (ZF) transmission due to its simplicity [31], [32]. It enables to decouple the data streams, and allows, for example, efficient implementation of the bit and power loading algorithms in practical systems.

II. System Model

The cellular MIMO system consists of N_B base stations, each BS has N_T transmit antennas and user k is equipped with N_{R_k} receive antennas. A user is served by M_k BSs which define the SHO active set $\mathcal{S}_k$ for the user k. The signal vector $\mathbf{y}_k \in \mathbb{C}^{N_{R_k}}$ received by the user k can be expressed as

$$\begin{aligned}\mathbf{y}_k &= \sum_{b\in\mathcal{S}_k} a_{b,k}\mathbf{H}_{b,k}\left(\mathbf{x}_{b,k}+\sum_{i\neq k}\mathbf{x}_{b,i}\right)\\ &+ \sum_{b\notin\mathcal{S}_k} a_{b,k}\mathbf{H}_{b,k}\mathbf{x'}_b+\mathbf{n}_k\\ &= \widetilde{\mathbf{H}}_k\widetilde{\mathbf{x}}_k+\mathbf{i}_k^{\text{intra}}+\mathbf{i}_k^{\text{inter}}+\mathbf{n}_k \end{aligned} \tag{1}$$

where $\mathbf{x}_{b,k} \in \mathbb{C}^{N_T}$ is the transmitted signal from the b'th base station to user k, $\mathbf{x'}_b \in \mathbb{C}^{N_T}$ denotes the total transmitted signal vector from BS transmitter (TX) b, $\mathbf{n}_k \sim \mathcal{CN}(0, N_0\mathbf{I}_{N_{R_k}})$ represents the additive noise sample vector, and $a_{b,k}\mathbf{H}_{b,k} \in \mathbb{C}^{N_{R_k}\times N_T}$ is the channel matrix from BS b to user k with large scale fading coefficient $a_{b,k}$. The elements of $\mathbf{H}_{b,k}$ are normalized to have unitary variance.

The signal $\widetilde{\mathbf{x}}_k = \left[\mathbf{x}_{\mathcal{S}_k(1),k}{}^{\text{T}}, \ldots, \mathbf{x}_{\mathcal{S}_k(M_k),k}{}^{\text{T}}\right]^{\text{T}} \in \mathbb{C}^{M_kN_T}$ transmitted for user k is distributed over M_k base stations being in SHO active set $\mathcal{S}_k$. The global channel matrix $\widetilde{\mathbf{H}}_k \in \mathbb{C}^{N_{R_k}\times M_kN_T}$ for user k from all M_k BSs is

$$\widetilde{\mathbf{H}}_k = \left[a_{\mathcal{S}_k(1),k}\mathbf{H}_{\mathcal{S}_k(1),k}, \ldots, a_{\mathcal{S}_k(M_k),k}\mathbf{H}_{\mathcal{S}_k(M_k),k}\right]. \tag{2}$$

The vectors $\mathbf{i}_k^{\text{intra}} = \sum_{b\in\mathcal{S}_k} a_{b,k}\mathbf{H}_{b,k}\sum_{i\neq k}\mathbf{x}_{b,i}$ and $\mathbf{i}_k^{\text{inter}} = \sum_{b\notin\mathcal{S}_k} a_{b,k}\mathbf{H}_{b,k}\mathbf{x'}_b$ include the received intra- and inter-cell interference, respectively.

The transmitted vector for user k is generated as $\widetilde{\mathbf{x}}_k = \mathbf{M}_k\mathbf{d}_k$, where $\mathbf{M}_k \in \mathbb{C}^{N_TM_k\times m_k}$ is the pre-coding matrix, $\mathbf{d}_k = [d_{1,k}, \ldots, d_{m_k,k}]^{\text{T}}$ is the vector of normalized complex transmitted data symbols, and $m_k \leq \min(N_TM_k, N_{R_k})$ denotes the number of active data streams. $\mathbf{M}_{k,c}$ can be further split into $\mathbf{M}_{k,c} = \mathbf{V}_{k,c}\mathbf{P}_{k,c}^{1/2}$, where $\mathbf{V}_{k,c} = [\mathbf{v}_{k,1,c}, \ldots, \mathbf{v}_{k,m_{k,c},c}]$ contains the normalized TX beamformers and $\mathbf{P}_{k,c} = \text{diag}\left(p_{k,1,c}, \ldots, p_{k,m_{k,c},c}\right)$ controls the powers allocated to each of $m_{k,c}$ streams.

The receiver (RX) is assumed to be equipped with a linear minimum mean square error (LMMSE) filter and the decision variables are generated as $\widehat{\mathbf{d}}_k = \mathbf{W}_k^{\text{H}}\mathbf{y}_k$. The weight matrix $\mathbf{W}_k \in \mathbb{C}^{N_{R_k}\times m_k}$ of the LMMSE filter is found by minimizing $\mathbf{W}_k = \arg\min_{\mathbf{W}_k} \text{E}\left[\left\|\mathbf{d}_k - \mathbf{W}_k^{\text{H}}\mathbf{y}_k\right\|^2\right]$ and is given as

$$\mathbf{W}_k^{\text{H}} = \mathbf{M}_k^{\text{H}}\widetilde{\mathbf{H}}_k^{\text{H}}\left(\widetilde{\mathbf{H}}_k\mathbf{M}_k\mathbf{M}_k^{\text{H}}\widetilde{\mathbf{H}}_k^{\text{H}}+\mathbf{Z}_k+\mathbf{R}_k\right)^{-1} \tag{3}$$

where $\mathbf{Z}_k$ and $\mathbf{R}_k$ are the intra- and inter-cell interference covariance matrices, respectively. The matrix $\mathbf{Z}_k = \sum_{i\neq k}\widetilde{\mathbf{H}}_k\mathbf{M}_i\mathbf{M}_i^{\text{H}}\widetilde{\mathbf{H}}_k^{\text{H}}$ consists of transmissions to the users i that have an identical SHO active set composition with user k, $\mathcal{S}_i = \mathcal{S}_k$. In this paper, we assume $\mathbf{R}_k = N_0\mathbf{I}$. Ideally, the whitening of colored inter-cell interference can be also contained in $\widetilde{\mathbf{H}}_k$ [28]. The practical interference scenarios are considered in the system level study [29].

III. Maximization of Minimum Weighted SINR per data stream with Per BS Power Constraints

It is possible to serve several users having identical SHO active sets $\mathcal{S}_k$ in the same time-frequency transmission slot using some space division multiple access (SDMA) methods to separate them in space domain [2], [31], [33]. SDMA can be used to improve the utilization of the physical resources (space, time, frequency) by exploiting the available spatial degrees of freedom in downlink multi-user MIMO channel, with an expense of somewhat increased complexity. In the following, we restrict our focus to a single set of users $\mathcal{A}$, where all users $k \in \mathcal{A}$ have identical active set composition, $\mathcal{S}_k = \mathcal{S}_i,\ \forall\ k,i \in \mathcal{A}$. We denote by $M = |\mathcal{S}_k|$ the SHO active set size, which is common to all $k \in \mathcal{A}$. Moreover, we focus on linear transmission schemes, where the transmitters send S independent streams, $S \leq \min(MN_T,\ \sum_{k\in\mathcal{A}} N_{R_k})$. Note that it is possible to associate more than one stream to one user, i.e., the cardinality of the set of scheduled users, $\mathcal{A} = \{k_s | s = 1, \ldots, S\}$, is less than or equal to S.

Different power constraints can be considered for cooperative BS processing [2], [10]–[13]. We consider two general power constraints: a sum power constraint for all M BSs in the SHO active set $\mathcal{S}_k$ and an individual power constraint for each BS. The total power transmitted by the BS n is

$$\begin{aligned}\text{Tr}\left(\text{E}\left[\mathbf{x'}_n\mathbf{x'}_n^{\text{H}}\right]\right) &= \text{Tr}\left(\sum_{k\in\mathcal{A}}\mathbf{M}_k^{[n]}\mathbf{M}_k^{[n]\text{H}}\right)\\ &= \sum_{k\in\mathcal{A}}\sum_{i=1}^{m_k}\left\|\mathbf{v}_{k,i}^{[n]}\right\|_2^2 p_{k,i}\end{aligned} \tag{4}$$

where $\mathbf{M}_k^{[n]} \in \mathbb{C}^{N_T\times m_k}$ is the pre-coder matrix of user k that corresponds to n'th base station belonging to $\mathcal{S}_k$, i.e., $\mathbf{M}_k^{[n]} = [\mathbf{M}_k]_{[(n-1)N_T+1:nN_T,\ :\ ]}$, $n = 1, \ldots, M$. Similarly, $\mathbf{v}_{k,i}^{[n]} \in \mathbb{C}^{N_T}$ is the transmit vector for the i'th stream of user k from BS n, i.e., $\mathbf{v}_{k,i}^{[n]} = [\mathbf{v}_{k,i}]_{(n-1)N_T+1:nN_T}$, $n = 1, \ldots, M$,

A. Joint Design of Linear Tx and Rx Beamformers

In this section, we consider the problem of joint design of the linear transmit and receive beamformers for finding a maximum weighted SINR value achievable for each stream subject to the per BS power constraints P_n, $n = 1, \ldots, M$. Per data stream processing is considered, where for each data stream s, $s = 1, \ldots, S$ the central base station's scheduler unit associates an intended user k_s, with the channel matrix $\widetilde{\mathbf{H}}_{k_s} \in \mathbb{C}^{MN_T\times N_{R_{k_s}}}$.

Let $\mathbf{m}_s \in \mathbb{C}^{MN_T}$ and $\mathbf{w}_s \in \mathbb{C}^{N_{R_{k_s}}}$ be arbitrary transmit and receive beamformers for the stream s. The SINR of the data stream s can be expressed as

$$\gamma_s = \frac{\left|\mathbf{w}_s^{\mathrm{H}}\widetilde{\mathbf{H}}_{k_s}\mathbf{m}_s\right|^2}{1+\sum_{i=1,i\neq s}^{S}\left|\mathbf{w}_s^{\mathrm{H}}\widetilde{\mathbf{H}}_{k_s}\mathbf{m}_i\right|^2}. \tag{5}$$

Similarly to (4), the total power transmitted by n'th BS is given by $\sum_{s=1}^{S}\left\|\mathbf{m}_s^{[n]}\right\|_2^2$, where $\mathbf{m}_s^{[n]} \in \mathbb{C}^{N_T}$ is the transmit vector for the s'th data steam associated with n'th BS, i.e., $\mathbf{m}_s = \left[\mathbf{m}_s^{[1]^{\mathrm{T}}}, \ldots, \mathbf{m}_s^{[M]^{\mathrm{T}}}\right]^{\mathrm{T}}$.

Suppose now that the system has to keep SINR per data stream γ_s in some fixed ratios, i.e., $\gamma_s/\beta_s = \gamma_o$. This optimization problem can be formulated as maximization of the minimum weighted SINR per data stream:

$$\begin{array}{ll} \text{maximize} & \min\limits_{s=1,\ldots,S} \dfrac{\beta_s^{-1}\left|\mathbf{w}_s^{\mathrm{H}}\widetilde{\mathbf{H}}_{k_s}\mathbf{m}_s\right|^2}{1+\sum_{i=1,i\neq s}^{S}\left|\mathbf{w}_s^{\mathrm{H}}\widetilde{\mathbf{H}}_{k_s}\mathbf{m}_i\right|^2} \\ \text{subject to} & \sum_{s=1}^{S}\left\|\mathbf{m}_s^{[n]}\right\|_2^2 \leq P_n, \quad n=1,\ldots,M \\ & \|\mathbf{w}_s\|_2 = 1, \quad s=1,\ldots,S \end{array} \tag{6}$$

where the variables are $\mathbf{m}_s \in \mathbb{C}^{MN_T}$ and $\mathbf{w}_s \in \mathbb{C}^{N_{R_{k_s}}}$, $s = 1,\ldots,S$. Again, the fixed weights $\beta_s > 0$ can be used to prioritize differently the data streams. The problem above is not jointly convex in variables $\mathbf{m}_s$ and $\mathbf{w}_s$. However, for fixed $\mathbf{m}_s$, (6) has unique solution given by the normalized maximum SINR receiver, i.e.,

$$\mathbf{w}_s = \frac{\tilde{\mathbf{w}}_s}{\|\tilde{\mathbf{w}}_s\|_2}, \tilde{\mathbf{w}}_s^{\mathrm{H}} = \mathbf{m}_s^{\mathrm{H}}\widetilde{\mathbf{H}}_{k_s}^{\mathrm{H}}\left(\sum_{i=1}^{S}\widetilde{\mathbf{H}}_{k_s}\mathbf{m}_i\mathbf{m}_i^{\mathrm{H}}\widetilde{\mathbf{H}}_{k_s}^{\mathrm{H}} + \mathbf{I}\right)^{-1}. \tag{7}$$

Furthermore, for a fixed $\mathbf{w}_s$, (6) is quasiconvex in $\mathbf{m}_s$ [26]. Thus, it can be solved by using the bisection method [14]. Notice that the constraints are also separable, i.e., they act on distinct sets of the variables $\mathbf{m}_s$ and $\mathbf{w}_s$. The above observations suggest using a coordinate ascent method [34] for solving (6). At each iteration the objective is maximized with respect to one set of variables $\mathbf{w}_s$ (or $\mathbf{m}_s$) by considering the other set fixed. This leads to the following algorithm:

Algorithm 1: SINR Optimization Under Per BS Power Constraints

1) Initialize the transmit vectors $\mathbf{m}_s^{(0)}$ such that BS power constraints are satisfied. Let $i = 1$ and go to Step 2.
2) Compute the optimum beamformers $\mathbf{w}_s^{(i)}$, $s = 1,\ldots,S$ given by (7) where $\mathbf{m}_s = \mathbf{m}_s^{(i-1)}$, $s = 1,\ldots,S$.
3) Solve the problem (6) for the variables $\mathbf{m}_s$, $s = 1,\ldots,S$ by fixing $\mathbf{w}_s = \mathbf{w}_s^{(i)}$, $s = 1,\ldots,S$. Denote the solution by $\mathbf{m}_s^{\star}$ and update the transmit beamformers $\mathbf{m}_s^{(i)} = \mathbf{m}_s^{\star}$, $s = 1,\ldots,S$. Test a stopping criterion. If it is not satisfied, let $i = i+1$ and go to Step 2, otherwise STOP. ■

Step 3 of *Algorithm 1* can be solved with any desired accuracy $\epsilon > 0$ by using the following bisection method.

Algorithm 2: SINR Optimization for Fixed Receive Beamformers

1) Initialize $\gamma_{\min} = \mathrm{SINR}_{\min}$ and $\gamma_{\max} = \mathrm{SINR}_{\max}$, where $\mathrm{SINR}_{\min}$ and $\mathrm{SINR}_{\max}$ define the range of relevant SINRs. Let $\epsilon > 0$ be the desired accuracy.
2) Set $\gamma_0 = (\gamma_{\max} + \gamma_{\min})/2$
3) Solve the following feasibility problem

$$\begin{array}{ll} \text{find} & \mathbf{m}_s, \quad s=1,\ldots,S \\ \text{subject to} & \dfrac{\left|\mathbf{w}_s^{\mathrm{H}}\widetilde{\mathbf{H}}_{k_s}\mathbf{m}_s\right|^2}{1+\sum_{i=1,i\neq s}^{S}\left|\mathbf{w}_s^{\mathrm{H}}\widetilde{\mathbf{H}}_{k_s}\mathbf{m}_i\right|^2} \geq \beta_s\gamma_o, \\ & s=1,\ldots,S \\ & \sum_{s=1}^{S}\left\|\mathbf{m}_s^{[n]}\right\|_2^2 \leq P_n, n=1,\ldots,M \end{array} \tag{8}$$

   If the problem is feasible, then set $\gamma_{\min} = \gamma_o$. Otherwise, set $\gamma_{\max} = \gamma_o$.
4) If $(\gamma_{\max} - \gamma_{\min}) > \epsilon/2$ then go to Step 2. Otherwise, return $\mathbf{m}_s^{\star} = \mathbf{m}_s$, $s = 1,\ldots,S$, where $\mathbf{m}_s$ is the last feasible solution of (8) and STOP. ■

Notice that the constraints of the problem (8) can be expressed as generalized inequality with respect to the second-order cone [14], [26]. Therefore, it can be solved by using a second-order cone program (SOCP) solver [30]. By modifying the approach presented in [26, Section IV.B] to accommodate per BS power constraints, the feasibility problem (8) can be reformulated as the following SOCP

$$\begin{array}{ll} \text{find} & \mathbf{m}_s, \quad s=1,\ldots,S \\ \text{s. t.} & \begin{bmatrix} \sqrt{1+\frac{1}{\beta_s\gamma_o}}\mathbf{w}_s^{\mathrm{H}}\widetilde{\mathbf{H}}_{k_s}\mathbf{m}_s \\ \mathbf{M}^{\mathrm{H}}\widetilde{\mathbf{H}}_{k_s}^{\mathrm{H}}\mathbf{w}_s \\ 1 \end{bmatrix} \succeq_{\mathcal{K}} 0, s=1,\ldots,S \\ & \begin{bmatrix} \sqrt{P_n} \\ \mathrm{vec}(\mathbf{M}^{[n]}) \end{bmatrix} \succeq_{\mathcal{K}} 0, \quad n=1,\ldots,M \end{array} \tag{9}$$

where $\mathbf{M} = [\mathbf{m}_1,\ldots,\mathbf{m}_S]$, $\mathbf{M}^{[n]} = [\mathbf{m}_1^{[n]},\ldots,\mathbf{m}_S^{[n]}]$, and $\succeq_{\mathcal{K}}$ denotes the generalized inequality with respect to the second-order cone [14], i.e., for any $x \in \mathbb{R}$ and $\mathbf{y} \in \mathbb{C}^n$, $[x, \mathbf{y}^{\mathrm{T}}]^{\mathrm{T}} \succeq_{\mathcal{K}} 0$ is equivalent to $x \geq \left\|\mathbf{y}\right\|_2$.

Following observation is made about the convergence of *Algorithm 1*. The block coordinate ascent method is guaranteed to converge to the global optimum if the maximization problems solved at each step have unique solutions. The maximization with respect to $\mathbf{w}_s$, $s = 1,\ldots,S$ (i.e., Step 2 of *Algorithm 1*) has a unique solution but the maximization with respect $\mathbf{m}_s$, $s = 1,\ldots,S$ (i.e., Step 3 of *Algorithm 1*) is not guaranteed to have a unique solution in general. Therefore, the global convergence can not be guaranteed.

B. Multiuser Zero Forcing Solution

Iterative block diagonalization (BD) of multiple user channels combined with coordinated TX-RX processing and scheduling between users is a simple but efficient zero-forcing method [31]–[33]:

Algorithm 3: Iterative BD Decomposition

1) Let a scheduling algorithm define the number of streams m_k allocated for each user $k \in \mathcal{A}$. Initialize $\mathbf{F}_k^{(i)}$ matrix to include the first m_k left singular vectors of $\widetilde{\mathbf{H}}_k$. Let $i = 1$.

2) Set $\bar{\mathbf{H}}_k = \mathbf{F}_k^{(i)\mathrm{H}} \widetilde{\mathbf{H}}_k$, define $\mathcal{A}_k = \mathcal{A} \backslash \{k\}$ and $\tilde{\bar{\mathbf{H}}}_k = [\bar{\mathbf{H}}_{\mathcal{A}_k(1)}^{\mathrm{T}} \cdots \bar{\mathbf{H}}_{\mathcal{A}_k(|\mathcal{A}_k|)}^{\mathrm{T}}]^{\mathrm{T}}$.
3) Set $\tilde{\bar{\mathbf{V}}}_k$ to be the orthogonal basis for the null space of $\tilde{\bar{\mathbf{H}}}_k$ so that $\bar{\mathbf{H}}_k \tilde{\bar{\mathbf{V}}}_i = 0$ for $i \neq k$.
4) Perform SVD of $\mathbf{H}'_k = \widetilde{\mathbf{H}}_k \tilde{\bar{\mathbf{V}}}_k = \tilde{\mathbf{U}}'_k \tilde{\mathbf{\Lambda}}'^{\frac{1}{2}}_k \tilde{\mathbf{V}}'^{\mathrm{H}}_k$, and let $\mathbf{U}'_k$ and $\mathbf{V}'_k$ represent the first m_k left and right singular vectors of $\mathbf{H}'_k$, respectively. Set $\mathbf{F}_k^{(i+1)} = \mathbf{U}'_k$ and check the stopping criterion. If it is not satisfied, let $i = i + 1$ and go to Step 2, otherwise STOP. ■

The pre-coding matrix $\mathbf{M}_k$ for user k is now defined as

$$\mathbf{M}_k^{\mathrm{BD}} = \tilde{\bar{\mathbf{V}}}_k \mathbf{V}'_k \mathbf{P}_k^{\frac{1}{2}} = \mathbf{V}_k^{\mathrm{BD}} \mathbf{P}_k^{\frac{1}{2}} \tag{10}$$

where the diagonal matrix $\mathbf{P}_k = \mathrm{diag}\,(p_{k,1}, \ldots, p_{k,m_k})$ controls the powers allocated for each of the m_k eigenmodes (streams). As a result, the, multiple-access interference (MAI) is eliminated between users, i.e., $\mathbf{F}_k^{\mathrm{H}} \widetilde{\mathbf{H}}_k \mathbf{M}_i^{\mathrm{BD}} = \mathbf{0}$ for $i \neq k$ and the channel for user k is diagonalized, $\mathbf{F}_k^{\mathrm{H}} \widetilde{\mathbf{H}}_k \mathbf{M}_k^{\mathrm{BD}} = \mathbf{\Lambda}_k^{\mathrm{BD}\frac{1}{2}} \mathbf{P}_k^{\frac{1}{2}}$, where the diagonal matrix $\mathbf{\Lambda}_k^{\mathrm{BD}} = \mathrm{diag}\,(\lambda_{k,1}, \ldots, \lambda_{k,m_k})$ includes the first m_k eigenvalues of $\tilde{\mathbf{\Lambda}}'_k$ of user k. Note that in such a case the receiver in (3) can be reduced to a simple matched filter.

Suppose now that the system has to keep SINR per data stream $\gamma_{k,i} = \lambda_{k,i} p_{k,i}$ in some fixed ratios, i.e., $\gamma_{k,i}/\beta_{k,i} = \gamma_o$. With ZF processing the problem of maximizing the minimum weighted SINR per data stream under per BS power constraint in (6) is reduced to

$$\begin{aligned} &\text{maximize} && \min_{i=1,\ldots,m_k, k\in\mathcal{A}} \beta_{k,i}^{-1} \lambda_{k,i} p_{k,i} \\ &\text{s. t.} && \sum_{k\in\mathcal{A}} \sum_{i=1}^{m_k} \|\mathbf{v}_{k,i}^{[n]}\|_2^2 p_{k,i} \leq P_n, \quad \forall n \\ &&& p_{k,i} \geq 0, \quad k \in \mathcal{A},\ i = 1, \ldots, m_k \end{aligned} \tag{11}$$

where the variables are $p_{k,i}$, $k \in \mathcal{A}$, $i = 1, \ldots, m_k$ and P_n is the power constraint on the BS n. The weights, $\beta_{k,i} \geq 0,\ \forall\, k, i$, are used to prioritize differently the data streams of different users and it can be chosen based on different criteria, e.g., states of the queues or buffers in case of cross layer optimization schemes. Obviously, when $\beta_{k,i} = 1,\ \forall\, k, i$, the problem reduces to the classical worst SINR maximization problem. It is easy to observe that the objective function of (11) is concave and all the inequality constraints are affine. Thus, the problem (11) is a convex optimization problem, and it can be efficiently solved numerically by using standard optimization software packages, e.g., CVX [30].

IV. Numerical Results

In the simulations, the elements of the channel matrices were modelled as i.i.d. Gaussian random variables and the number of both TX and RX antennas was fixed at 2, $\{N_T, N_{R_k}\} = \{2, 2\}$. For simplicity, the base stations were assumed to have equal maximum power limit P_T, i.e. $P_n = P_T\ \forall\ n$. The impact of the following two power constraints are studied.

- *Sum power constraint*: All M BSs in $\mathcal{S}_k$ have perfect power cooperation in addition to the data cooperation. This provides an unrealistic upper bound, where the pooled maximum available power is always $P_{\mathrm{sum}} = MP_T$, while the antenna array gain from having MN_T TX antennas depends on the RX power differences between BSs.
- *Per BS power constraint*: Available power can be increased up to M times depending on the RX power difference between BSs. Also, the antenna array gain from having MN_T TX antennas depends on the RX power difference.

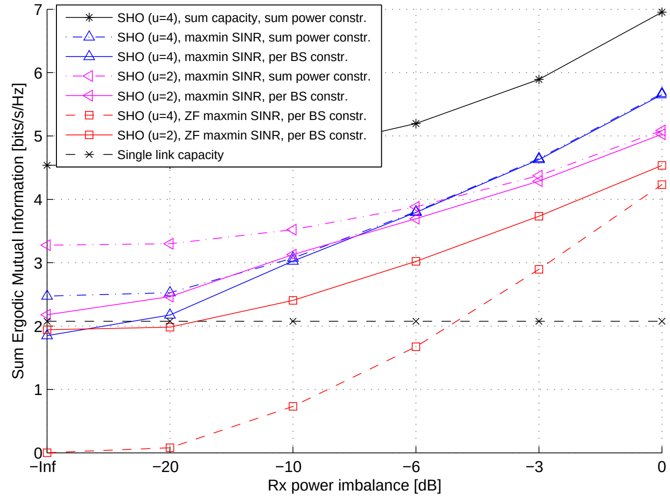

Fig. 1. Ergodic sum of user rates of $\{N_T, N_{R_k}, M\} = \{2, 2, 2\}$ system with equal SINR per stream at 0 dB single link SNR

We study the sum ergodic mutual information for 2-branch SHO achieved using the maximization of the minimum weighted SINR per data stream criterion with different power constraints. The impact of inter-cell interference is omitted for simplicity, i.e., $\mathbf{R}_k = N_0\mathbf{I}$. We consider a case where two or four SHO users, labelled as $u = 2$ or $u = 4$, respectively, are served simultaneously by two base stations in a flat fading scenario at each time instant. A single data stream is assigned to each user. Furthermore, we assume that the users have identical large scale fading coefficients for simplicity, i.e., $a_{\mathcal{S}_k(1),k} = a_{\mathcal{S}_i(1),i}$ and $a_{\mathcal{S}_k(2),k} = a_{\mathcal{S}_i(2),i}\ \forall\ i, k \in \mathcal{A}$.

Figs. 1 and 2 illustrate the ergodic mutual information for different power imbalance values $\alpha = a^2_{\mathcal{S}_k(2),k}/a^2_{\mathcal{S}_k(1),k}$, where $\mathcal{S}_k(1)$ is the BS with the strongest reception at the terminal, and for 0 dB and 20 dB single link SNRs (SNR = $P_T a^2_{\mathcal{S}_k(1)}/N_0$), respectively. The ergodic sum of individual user rates with different power constraints is depicted for the joint Tx-Rx optimization algorithm (Section III-A, labelled as 'maxmin SINR') and the zero forcing method (Section III-B, labelled as 'ZF maxmin SINR'). Equal weighting of data streams $\beta_s = \beta_{k,i} = 1,\ \forall\ s, i, k$ is used for both algorithms. The single user capacity ($u = 1$) without SHO is also included for comparison. Moreover, the sum capacity with sum power constraint is plotted as the absolute upper bound of the scenario.

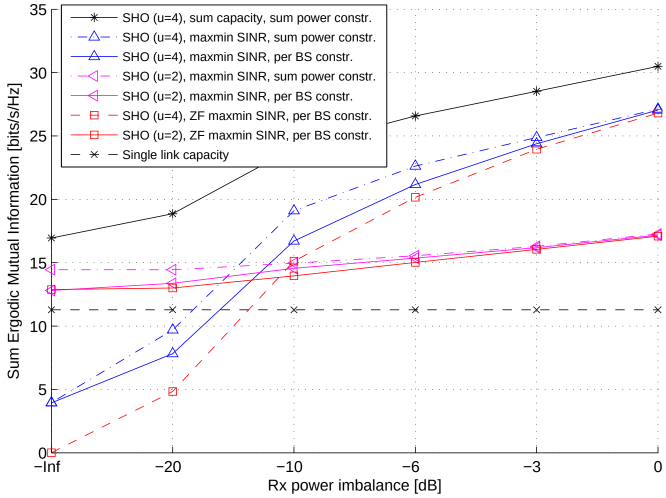

Fig. 2. Ergodic sum of user rates of $\{N_T, N_{R_k}, M\}$ = {2, 2, 2} system with equal SINR per stream at 20 dB single link SNR

Note that the upper bound is not achievable with linear transmission strategy, since the sum capacity achieving schemes require the usage of the nonlinear dirty paper coding based precoder [35]. However, the proposed linear joint Tx-Rx optimization algorithm with equal SINR per data stream constraint is able to achieve more than 80 percent of the sum capacity, with small imbalance between BSs.

The achievable sum rate with equal SINR constraint per stream depends heavily on the imbalance between BSs. As the imbalance approaches infinity, the case with four scheduled users becomes spatially overloaded. In such a case, the achievable SINR γ_s per data stream s with spatial overloading is always sub-unitary, and hence, the maximum achievable rate at high SNR region approaches four, $4 \times \log_2(1+\gamma_s) \leq 4$ [26]. Then, it is obviously more efficient to serve less users at the time, i.e., to divide the users into subgroups and multiplex them in time domain. As the imbalance decreases, i.e., the users are located closer to the cell edge, the case with higher spatial load becomes more advantageous. In other words, full spatial loading ($u = 4$) becomes more efficient than time multiplexing of the data streams with partial loading ($u = 2$). The exact crossing point where the sum rate with the full spatial loading exceeds the sum rate with partial loading depends on the SNR region, as seen from Figs. 1 and 2.

It is seen from the figures that the zero forcing with full spatial loading ($u = 4$) performs rather bad at low SNR range. Also, the achievable sum rate goes to zero as the system becomes spatially overloaded. However, the zero forcing with partial spatial loading ($u = 2$) performs reasonably well at low SNR as compared to the joint Tx-Rx optimization algorithm due to the higher number of spatial degrees of freedom available at both the transmitter and the receiver. At high SNR and with small imbalance between BSs, the performance of the zero forcing method approaches the joint Tx-Rx optimization algorithm.

V. Conclusion

The joint cooperative processing of transmitted signal from several MIMO BS antenna heads was considered for users located within a SHO region. The mathematical framework for the SHO based MIMO system was derived and the joint design of linear transmit and receive beamformers in a MIMO multiuser transmission subject to per BS power constraints was considered. Solution for the maximization of the minimum weighted SINR per data stream criterion was proposed. The proposed joint Tx-Rx optimization algorithm was shown to provide very efficient solutions despite of the fact that the global optimum cannot be always guaranteed due to the non-convexity of the problem. Moreover, a less complex but still efficient allocation method based on zero forcing transmission was provided for the same optimization criterion.

References

[1] S. Shamai and B. Zaidel, “Enhancing the cellular downlink capacity via co-processing at the transmitting end,” in *Proc. IEEE Veh. Technol. Conf.*, Rhodes, Greece, May 2001, pp. 1745 – 1749.

[2] H. Zhang and H. Dai, “Cochannel interference mitigation and cooperative processing in downlink multicell multiuser MIMO networks,” *EURASIP J. Wireless Communications and Networking*, vol. 2004, no. 2, pp. 222–235, Dec. 2004.

[3] N. H. Dawod, I. D. Marsland, and R. H. M. Hafez, “Improved transmit null steering for MIMO-OFDM downlinks with distributed base station antenna arrays,” *IEEE J. Select. Areas Commun.*, vol. 24, no. 3, pp. 419–426, Mar. 2006.

[4] M. K. Karakayali, G. J. Foschini, and R. A. Valenzuela, “Network coordination for spectrally efficient communications in cellular systems,” *IEEE Wireless Commun. Mag.*, vol. 3, no. 14, pp. 56– 61, Aug. 2006.

[5] O. Somekh, O. Simeone, and Y. B.-N. an M. Haimovich, “Distributed multi-cell zero-forcing beamforming in cellular downlink channels,” in *Proc. IEEE Global Telecommun. Conf.*, San Francisco, USA, Nov. 2006.

[6] P. Zhang, X. Tao, J. Zhang, Y. Wang, L. Li, and Y. Wang, “A vision from the future: beyond 3G TDD,” *IEEE Commun. Mag.*, vol. 43, no. 1, pp. 38– 44, Jan. 2005.

[7] X.-H. Yu, G. Chen, M. Chen, and X. Gao, “Toward beyond 3G: The FuTURE project in China,” *IEEE Commun. Mag.*, vol. 43, no. 1, pp. 70– 75, Jan. 2005.

[8] S. Ye and R. S. Blum, “Optimized signaling for MIMO interference systems with feedback,” *IEEE Trans. Signal Processing*, vol. 51, pp. 2839–2847, Nov. 2003.

[9] S. A. Jafar, G. J. Foschini, and A. J. Goldsmith, “Phantomnet: Exploring optimal multicellular multiple antenna systems,” in *Proc. IEEE Veh. Technol. Conf.*, vol. 1, Vancouver, Canada, Sept. 2002, pp. 261 – 265.

[10] S. A. Jafar and A. J. Goldsmith, “Transmitter optimization for multiple antenna cellular systems,” in *Proc. IEEE Int. Symp. Inform. Theory*, vol. 1, Lausanne, Switzerland, June 2002, p. 50.

[11] W. Yu and T. Lan, “Transmitter optimization for the multi-antenna downlink with per-antenna power constraints,” *IEEE Trans. Signal Processing, to appear, also avail. at http://www.comm.utoronto.ca/∼weiyu/publications.html*, Dec. 2006.

[12] W. Yu, “Uplink-downlink duality via minimax duality,” *IEEE Trans. Inform. Theory*, vol. 52, no. 2, pp. 361–374, Feb. 2006.

[13] H. Weingarten, Y. Steinberg, and S. Shamai, “The capacity region of the gaussian multiple-input multiple-output broadcast channel,” *IEEE Trans. Inform. Theory*, vol. 52, no. 9, pp. 3936 – 3964, Sept. 2006.

[14] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge University Press, 2004.

[15] M. S. Lobo, L. Vandenberghe, S. Boyd, and H. Lebret, “Applications of second–order cone programming,” *Linear Algebra and Applications*, vol. 284, pp. 193–228, Nov. 1998.

[16] L. Vandenberghe and S. Boyd, “Semidefinite programming,” *SIAM Review*, vol. 38, no. 1, pp. 49–95, Mar. 1996.

[17] S. Boyd, S. J. Kim, L. Vandenberghe, and A. Hassibi, “A tutorial on geometric programming,” to appear in *Optimization and Engineering*, available online at www.stanford.edu/∼boyd/research.html.

[18] Z.-Q. Luo, “Applications of convex optimization in signal processing and digital communication,” *Mathematical Programming*, vol. 97, Series B, pp. 177–207, 2003.
[19] Z. Q. Luo and W. Yu, “An introduction to convex optimization for communications and signal processing,” *IEEE J. Select. Areas Commun., to appear*, 2006.
[20] M. Chiang, “Geometric programming for communication systems,” *Foundations and Trends in Communications and Information Theory*, vol. 2, no. 1-2, pp. 1–154, July 2005.
[21] D. P. Palomar and S. Barbarossa, “Designing MIMO communication systems: Constellation choice and linear transceiver design,” *IEEE Trans. Signal Processing*, vol. 53, no. 10, pp. 3804–3818, Oct. 2005.
[22] C. W. Tan, D. P. Palomar, and M. Chiang, “Solving nonconvex power control problems in wireless networks: Low SIR regime and distributed algorithms,” in *Proc. IEEE Global Telecommun. Conf.*, vol. 6, St. Louis, MO, Nov. 28 – Dec. 2 2005, pp. 3445–3450.
[23] M. Bengtsson and B. Ottersten, “Optimal and suboptimal transmit beamforming,” in *Handbook of Antennas in Wireless Communications, L. C. Godara, Ed. Boca Raton, FL: CRC Press*, 2001.
[24] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, “Joint Tx-Rx beamforming design for multicarrier MIMO channels: A unified framework for convex optimization,” *IEEE Trans. Signal Processing*, vol. 51, no. 9, pp. 2381–2401, Sept. 2003.
[25] D. P. Palomar, M. Bengtsson, and B. Ottersten, “Minimum BER linear transceivers for MIMO channels via primal decomposition,” *IEEE Trans. Signal Processing*, vol. 53, no. 8, pp. 2866–2882, Aug. 2005.
[26] A. Wiesel, Y. C. Eldar, and S. Shamai, “Linear precoding via conic optimization for fixed MIMO receivers,” *IEEE Trans. Signal Processing*, vol. 54, no. 1, pp. 161–176, Jan. 2006.
[27] H. Holma and A. Toskala, *WCDMA for UMTS*, 3rd ed. John Wiley & Sons, Ltd, 2004.
[28] A. Tölli, M. Codreanu, and M. Juntti, “Soft handover in adaptive MIMO-OFDM cellular system with cooperative processing,” in *Proc. IEEE Int. Symp. Pers., Indoor, Mobile Radio Commun.*, Helsinki, Finland, Sept. 2006.
[29] ——, “Adaptive MIMO-OFDM cellular system with soft handover between distributed base station antennas,” in *Proc. IEEE Global Telecommun. Conf.*, San Francisco, USA, Nov. 2006.
[30] M. Grant, S. Boyd, and Y. Ye, *CVX: Matlab Software for Disciplined Convex Programming*. http://www.stanford.edu/ ∼boyd/cvx/, 2006.
[31] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, “Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels,” *IEEE Trans. Signal Processing*, vol. 52, no. 2, pp. 461–471, Feb. 2004.
[32] Q. H. Spencer and A. L. Swindlehurst, “A hybrid approach to spatial multiplexing in multiuser MIMO downlinks,” *EURASIP J. Wireless Communications and Networking*, vol. 2004, no. 2, pp. 236–247, Dec. 2004.
[33] A. Tölli and M. Juntti, “Efficient user, bit and power allocation for adaptive multiuser MIMO-OFDM with low signalling overhead,” in *Proc. IEEE Int. Conf. Commun.*, Istanbul, Turkey, June 2006.
[34] D. P. Bertsekas, *Nonlinear Programming*, 2nd ed. Belmont, Mass.: Athena Scientific, 2003.
[35] P. Viswanath and D. Tse, “Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality,” *IEEE Trans. Inform. Theory*, vol. 49, no. 8, pp. 1912–1921, Aug. 2003.