Joint Beamforming and Power Control Optimization by Second-Order Cone Programming Approximation

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***Abstract*— In multiuser cognitive radio (CR) network, we address the problem of joint transmit beamforming (BF) and power control (PC) for secondary users (SUs) when they are allowed to transmit simultaneously with primary users (PUs). The objective is to optimize the network sum rate under the interference constraints of PUs. Second-order cone programming approximation (SOCPA) method is proposed as an effective algorithm. Typical network models are approximated to second-order cone programming problems and solved by interior-point method. Finally the network sum rates for different PU and SU numbers are assessed by simulation.**

***Index Terms*—Cognitive radio, beamforming, power control, second-order cone programming.**

I. Introduction

Cognitive radio (CR) techniques have been proposed to implement opportunistic spectrum sharing over the licensed legacy bands to explore the under-utilized spectrum resources or improve the utilization efficiency [1], [2]. Generally speaking, there are three operational models for the CR network, namely interweave, overlay and underlay in [3] and references therein, among which overlay and underlay methods allow the secondary users (SUs) to transmit simultaneously with primary users (PUs). Since the PUs have higher priority than the SUs to use the spectrum resources, one fundamental challenge is to ensure the quality-of-service (QoS) of the PUs [4].

Joint beamforming (BF) and power control (PC) is widely studied for multiuser systems in [4], [10]-[13]. Whitening transform is utilized in [10] to simplify the optimization of capacity, and classic water-filling method is used for BF and PC. The multiuser downlink (DL) problem with individual SINR constraints is discussed in [11], where the goal of designing beamformers is to maximize signal to interference plus noise (SINR). Due to the duality of broadcast (BC) and multiple access (MAC) channels, transmit beamformers of BC are equivalent to receive beamformers of MAC, which optimizes the SINR by MMSE criterion of the uplink (UL). The duality is also used in [12] to maximize the sum rate by joint PC and BF in the multiuser MIMO DL channels, in which PC is optimized by geometric programming in [14]. Joint BF and PC for multiple access channels (MAC) in CR network is discussed in [13]. Zero-forcing based decision feedback equalizer (ZF-DFE) is used as the receive beamformers and a capped multi-level water-filling algorithm is proposed. Joint BF and PC with single and multiple PUs constraints is considered in [4], and direct-channel SVD and projected channel SVD are proposed to solve the problem in MIMO channels.

Moreover, most works about multiuser BF and PC are mainly for BC or MAC channels in cellular network, in which the joint BF and PC algorithm is centralized at the base stations. Most papers consider the maximal total transmission power constraints for all the users, while in [11], [13] the authors constrain maximal transmission power for each user individually. However, there are no work considering interference constraint of single and/or multiple PUs in interference channel.

In this paper, the optimization problem is approximated to a second-order cone programming (SOCP), which can be solved efficiently by interior-point method, or modern softwares (*e.g.* SeDuMi [16]). We set some examples of using second-order cone programming approximation (SOCPA) method to solve the problem in CR network with single or multiple SU/PU. This numerical algorithm is easy to deploy and it can solve such problems efficiently.

The remainder of this paper is organized as follows. In Section II, we introduce the optimization problem. SOCPA method is introduced to solve the problem in Section III. Simulation results are provided in Section IV. Section V concludes the paper.

II. System Model

Consider a CR interference channel model in Fig. 1. There are K SUs and one PU. Each SU-Tx is equipped with N antennas and each receiver has one receive antenna. The objective is to optimize the sum rate by designing beamformers and PC. Multiple PUs will be considered in Section III and IV.

We assume slow flat fading channels. The complex channel attenuations include small-scale fading, path and shadow loss. The noise is assumed to be independent and identically distributed (i.i.d.) complex Gaussian variables with zero mean and unit variance. In this paper, we assume noise power is one, thus, the signal to noise ratio (SNR) is equal to transmit signal power. If the noise power for the receivers are different,

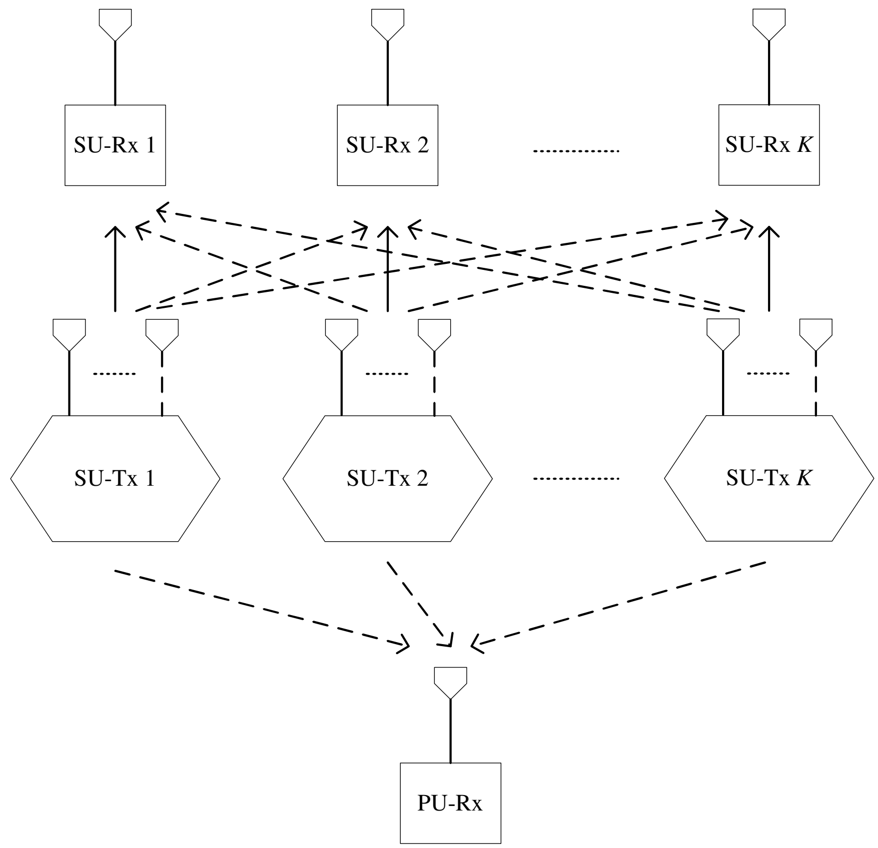

Fig. 1. Cognitive radio network with the K SUs sharing channel with one PU. Solid lines: transmission; dash lines: interference.

the dual method to solve the problem is still effective by using SNR instead.

The objective of the system model is,

$$\sum_k C_k, \tag{1}$$

where C_k is the capacity of the k_{th} SU link of CR network, and has the expression

$$C_k = \log_2(1 + \gamma_k), \tag{2}$$

where γ_k is the SINR of k_{th} SU link, which can be written by

$$\gamma_k = \frac{p_k \left| \boldsymbol{h}_k^H \boldsymbol{v}_k \right|^2}{1 + \sum\limits_{i \neq k} p_i \left| \boldsymbol{h}_{ik}^H \boldsymbol{v}_i \right|^2}, \tag{3}$$

where p_k and $\boldsymbol{v}_k$ ($\|\boldsymbol{v}_k\| = 1$) are the transmission power and beamformer of k_{th} SU-Tx, $\boldsymbol{h}_k^H$ is the channel attenuation of k_{th} SU link, and $\boldsymbol{h}_{ik}^H$ is the interference channel attenuation between different SUs. We assume each SU-Tx has the same maximal transmission power P, *i.e.* $p_k \leq P$. The underlay scenario requests the interference from SUs to PU not larger than the threshold P_t,

$$\sum\nolimits_k p_k \left| \boldsymbol{g}_k^H \boldsymbol{v}_k \right|^2 \leq P_t,$$

where $\boldsymbol{g}_k^H$ is the channel attenuation of k_{th} SU-PU link.

III. SOCPA Suboptimal Solution

SOCP is a class of convex optimization models in engineering design applications [17]. Modern softwares can solve the convex optimization efficiently, *e.g.* SeDuMi can be used to solve SOCP efficiently using interior-point method, and either generates an optimal solution or a certificate showing infeasibility. In the CR network, we can use the SOCPA method and use software to find suboptimal solutions. The software assistant method and a geometric method are considered for single SU and single PU in [5]. However, the two algorithms are exclusively designed for single SU and single PU case, and not valid for more SUs or PUs. We will give some applicable solutions for more nodes in the network.

A. Multiple SUs and Single PUs

In this section we use $\boldsymbol{w}_k$ instead of $\boldsymbol{v}_k$, where $\boldsymbol{w}_k = \sqrt{p_k}\boldsymbol{v}_k$. We make some approximations for the certainty optimization problem in Section II:

1. The interference between different secondary users are zero to simplify the formulation of network capacity, *i.e.* $\boldsymbol{h}_{ik}^H \boldsymbol{w}_i = 0, i \neq k$, $i = 1, \cdots, K$.

2. Use the approximation $\log_2(1 + z^2) \approx |z|$, for the objection function. Although it is a rough approximation, it is effective in low and medium SINR region.

3. Let $\mathrm{Im}\{\boldsymbol{h}_k^H \boldsymbol{w}_k\} = 0$ in order to approximate the objective to the formulation fit for SOCP.

4. The interference threshold constraint is divided into the constraints for each SU, averagely or weighed by α_k, which is decided by corresponding distances from SU-Txs to PU-Rx considering QoS of the SUs, where $\alpha_k = d_k^n / \sum_i d_i^n$, and n is the path loss exponent. Under these approximations, the optimization model is approximated by

$$\max \sum_k \boldsymbol{h}_k^H \boldsymbol{w}_k \tag{4}$$

s.t. $\|\boldsymbol{w}_k\| \leq P$, $\left|\boldsymbol{g}_k^H \boldsymbol{w}_k\right| \leq \sqrt{\alpha_k P_t}$, $\mathrm{Im}\{\boldsymbol{h}_k^H \boldsymbol{w}_k\} = 0$, $\boldsymbol{h}_{ik}^H \boldsymbol{w}_i = 0$, $i \neq k$, $i, k = 1, \cdots, K$. This model can be solved by SeDuMi efficiently (see Section IV).

B. Single SU and Multiple PUs

Make approximations for certainty model with single SU and multiple PU:

1. Let $\mathrm{Im}\{\boldsymbol{h}^H \boldsymbol{w}\} = 0$;

Under this approximation, the optimization model is approximated by

$$\max \boldsymbol{h}^H \boldsymbol{w} \tag{5}$$

s.t.$\|\boldsymbol{w}\| \leq P$,$\left|\boldsymbol{g}_u^H \boldsymbol{w}_u\right| \leq \sqrt{P_t}$, $\mathrm{Im}\{\boldsymbol{h}^H \boldsymbol{w}\} = 0$, $u = 1, \cdots, U$, where $\boldsymbol{g}_u$ is the channel between SU and the u_{th} primary user in this subsection.

It is an option to combine the approximations and constraints in subsection A and B to solve the network with multiple SUs and PUs.

IV. Numerical Simulations

This section will show the simulation results to illustrate the behavior of the proposed algorithm. In the simulations, the small-scaled channel fading are generated by independent circularly symmetric complex Gaussian random vectors with zero mean and unit variance for each element of the vectors. The distances between SU-Tx and corresponding SU-Rx, between SU-Tx and other SU-Rx, and between SU-Tx and PU-Rx are l_1, l_2 and l_3, respectively. They are assumed l, $\sqrt{2}l$ and $\sqrt{3}l$ in the simulation. The same path loss model is used for all channels in the network and the path loss exponent is 4. The noise power and the interference threshold P_t are one for all the simulations.

We simulate three scenarios, which contain single or more SU/PU when all the channel knowledge is perfectly known in the network. It is easy to understand that the network sum rate increases as the SU number increasing, while opposite for the PU number, since more PUs bring more interference constraints for the network. More SUs also bring more interference among the SUs, as a result, the sum rate is lowered. Hence, the network rate for two SUs is less than double rate of single SU.

Furthermore, the noise power and the interference threshold between SUs and PUs are one during the simulations. So the coordinate of maximal transmission power could be treated as the ratio of maximal transmission power to the noise power (interference threshold). Thus, different scenarios with the same ratio of maximal transmission power to the noise power (interference threshold) have the same beamformers and power control solutions, with the same sum rate as well.

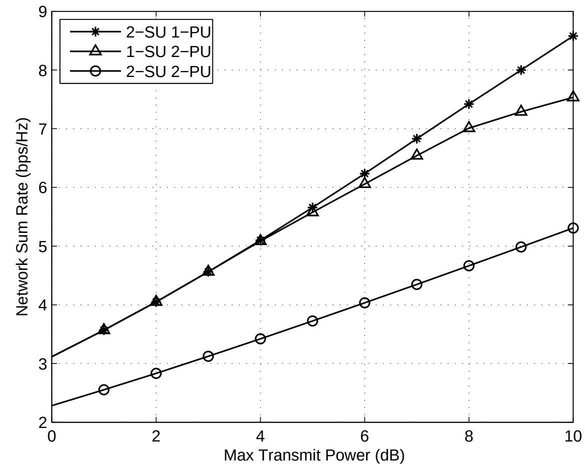

Fig. 2. Network sum rate comparison with different SU and PU numbers. The sum rate increases as SU number increases and as PU number decreases.

V. Conclusion

In this paper we devise an algorithm to optimize network sum rate by designing beamformers and controlling power for each SU of the multiuser CR network. We propose to use interior-point method to solve the problem after approximation of SOCP. It is evaluated by simulation that the algorithm is effective for both multiple SUs and multiple PUs.

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