LETTER

Efficient Subcarrier and Power Allocation Algorithm in OFDMA Uplink System*

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SUMMARY This letter focuses on uplink transmission in OFDMA systems. A subcarrier and power allocation problem is formulated that maximizes the throughput of OFDMA uplink systems while satisfying each user's power constraints. A greedy algorithm known to be the most efficient algorithm for this problem can provide a high quality near-optimal solution, but has the disadvantage of incurring a long computation time. As this problem should be solved in a real-time environment, computation time is a very important performance measure of algorithms. In this letter, a computationally efficient algorithm that provides a nearly identical quality, near-optimal solution as the greedy algorithm but requires less than 10% of the computation time of the greedy algorithm is proposed.
key words: *OFMDA, uplink, subcarrier and power allocation*

1. Introduction

In recent years, wireless communication systems must support reliable and high-rate data transmission to support various services, including digital audio broadcasting, digital video broadcasting and wireless Internet access. Among several candidates, orthogonal frequency division multiple access (OFDMA) is regarded as one of the prime multiple access schemes for broadband wireless networks, and is one of PHY options of the IEEE 802.16 standard [1]. Moreover, OFDMA has also been adopted for a domestic standard of 2.3 GHz wireless broadband (WiBro) service in Korea [2].

In OFDMA systems, each subcarrier is exclusively assigned to only one user, and intra-cell interference is eliminated. This aspect of OFDMA makes decoding at the receiver easier. By splitting each high data-rate stream into a number of lower rate streams, the symbol duration is increased, and it becomes possible to solve the inter symbol interference (ISI) problem, which is a major problem in wideband transmissions over multipath fading channels [3]. Furthermore, in OFDMA systems, different subcarriers can be allocated to different users, so flexibility of resource allocation is increased. In addition, OFDMA enables the exploitation of multiuser diversity in the frequency domain by allocating subcarriers and power to users who are on the peaks of the channel state.

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To exploit these advantages of OFDMA systems, the allocation of subcarriers and power to users is an important issue. Allocating subcarriers and power in OFDM downlink channels has been widely investigated [3]–[7]. In [3], an efficient scheduling algorithm to maximize system throughput while providing fairness among users in the downlink of OFDM systems is proposed. For the downlink, it is an optimal solution that each subcarrier is allocated to the user with the best channel condition and power is allocated by a water-filling algorithm over subcarriers [5], [6]. The optimality of OFDMA is proven from an information theoretical point of view; it is also proven that the optimality of OFDMA holds for any adaptive modulation scheme when the transmission rate can be approximated as a convex function in terms of SINR [7]. However, the optimality of the downlink does not hold for the uplink due to the distributed nature of power constraints. For the uplink case, the frequency-power allocation algorithm is investigated and a practical low-complexity algorithm for a two-user case is proposed in [8]. In [9], a greedy subcarrier allocation algorithm based on the Karush-Kuhn-Tucker condition and an iterative power allocation algorithm based on the water-filling method are proposed for uplink OFDMA systems. However, their algorithm requires a great number of water-filling processes to solve this problem and cannot be used in a real-time environment, as the channel condition varies rapidly in wireless communications.

In this letter, a computationally efficient subcarrier and power allocation algorithm is proposed. The throughput maximization problem in OFDMA uplink channels is considered, as is the power constraints of individual users. The objective here is to maximize the sum of data rate of the system while satisfying the power constraints of each user. Using a Lagrangian relaxation procedure, an efficient subcarrier and power allocation algorithm is proposed. The proposed algorithm was found to produce near optimal solutions with computational efficiency.

This letter is organized as follows: In Sect 2, the system model is described. The OFDMA uplink throughput maximization problem with power constraints is presented in Sect. 3. The proposed subcarrier and power allocation algorithm is presented in Sect. 4. In Sect. 5, the performance of the proposed algorithm is outlined; finally, the conclusion is presented in Sect. 6.

2. System Model

In this letter, an OFDMA uplink system with power constraints of each user is considered. It is assumed that the system has N users and M subcarriers, that each subcarrier has a bandwidth that is much smaller than the coherence bandwidth of the channel, and that the instantaneous channel gains on all subcarriers of all users are known to transmitters. Subcarriers and power allocation are performed repeatedly, and the time interval of allocations is assumed to be short enough so that the channel gains of users remain mostly constant until the next allocation period.

In OFDMA systems, each user is assigned a fraction of the available number of subcarriers, and each user experiences a different channel gain on each subcarrier due to the frequency selective fading. For user i and subcarrier j, the channel transfer function and the total noise power spectral density are denoted as H_{ij} and N_{ij}, respectively. The quality of each user's channel condition g_{ij} can be indicated by

$$g_{ij} = SNR_{ij} = \frac{|H_{ij}|^2}{N_{ij}}$$

which is called channel signal-to-noise-ratio (SNR) function for user i and subcarrier j [5]. In addition, each subcarrier can not be assigned to two or more users simultaneously in an OFDMA systems. The total transmission power of user i can not exceed $\bar{P}_i$.

3. Problem Formulation

In this section, the throughput maximization problem in OFDMA uplink channels with power constraints is presented. Using the Shannon capacity formula for the Gaussian channel, throughput of system T is as follows:

$$T = \sum_{i=1}^{N}\sum_{j=1}^{M} w_{ij}\log(1 + p_{ij}g_{ij}),$$

where p_{ij} is the allocated power of user i to subcarrier j, and w_{ij} is the channel allocation index. The binary variable $w_{ij} = 1$ if the subcarrier j is allocated to user i, and is $w_{ij} = 0$ otherwise.

Mathematically, the throughput maximization problem is formulated as in the following combinatorial optimization problem:

$$\max \sum_{i=1}^{N}\sum_{j=1}^{M} w_{ij}\log(1 + p_{ij}g_{ij}) \tag{1}$$

subject to

$$\sum_{i=1}^{N} w_{ij} \le 1 \quad \text{for all } j, \tag{2}$$

$$\sum_{j=1}^{M} p_{ij} \le \bar{P}_i \quad \text{for all } i, \tag{3}$$

$$\forall w_{ij} \in \{0, 1\}, \tag{4}$$

$$\forall p_{ij} \ge 0. \tag{5}$$

Constraint set (2) ensures that at most one user can be allocated to each subcarrier simultaneously. The power constraint set (3) assures that the total allocated power of user i can not exceed $\bar{P}_i$.

4. Proposed Algorithm

As channel conditions vary rapidly in wireless communication environments, the optimization problem described above should be solved in a short time. However, the greedy algorithm proposed in [9] requires too much computation time to be used in a real-time environment. In this letter, the Lagrangian relaxation procedure is used to develop an efficient heuristic algorithm to solving this optimization problem. By relaxing constraints (3), the relaxed problem (LR) is obtained, as follows:

$$\max \sum_{i=1}^{N}\sum_{j=1}^{M} w_{ij}\log(1 + p_{ij}g_{ij}) - \sum_{i=1}^{N}\mu_i\left(\sum_{j=1}^{M} p_{ij} - \bar{P}_i\right)$$
$$= \sum_{j=1}^{M}\sum_{i=1}^{N}\{(w_{ij}\log(1 + p_{ij}g_{ij}) - \mu_i p_{ij}\} + \sum_{i=1}^{N}\mu_i\bar{P}_i$$

subject to the constraint sets (2), (4), (5), where μ_i are nonnegative Lagrangian multipliers.

This problem is decomposed into the following M subproblems: for each $j^*(1 \le j^* \le M)$,

$$\max T_{j^*} = \sum_{i=1}^{N}\{w_{ij^*}\log(1 + p_{ij^*}g_{ij^*}) - \mu_i p_{ij^*}\} \tag{6}$$

subject to

$$\sum_{i=1}^{N} w_{ij^*} \le 1, \tag{7}$$

$$\forall w_{ij^*} \in \{0, 1\}, \tag{8}$$

$$\forall p_{ij^*} \ge 0.$$

In this sub-problem, at most one w_{ij^*} can have the value of 1 by constraint (7) and (8). Furthermore, the allocated power p_{ij^*} can have a non-zero value only if w_{ij^*} has a value of 1, as the Lagrangian multiplier μ_i is non-negative in the objective function (6).

To solve the sub-problem for subcarrier j^*, it is necessary to select the user and allocate power to that user to maximize (6). Thus, user $\hat{i}$ is selected such that $\hat{i} = \arg\max[\log(1+p_{ij^*}g_{ij^*})-\mu_i p_{ij^*}]$ and $\log(1+p_{ij^*}g_{ij^*})-\mu_i p_{ij^*}$ is maximized when

$$p_{ij*} = \begin{cases} 0 & \text{for } \frac{1}{\mu_i} - \frac{1}{g_{ij^*}} < 0, \\ \frac{1}{\mu_i} - \frac{1}{g_{ij^*}} & \text{for } 0 \le \frac{1}{\mu_i} - \frac{1}{g_{ij^*}} \le \bar{P}_i, \\ \bar{P}_i & \text{for } \bar{P}_i < \frac{1}{\mu_i} - \frac{1}{g_{ij^*}}. \end{cases} \tag{9}$$

The optimal solution of the sub-problem for subcarrier j^* is $w_{ij^*} = 1$, p_{ij^*} has the value described in (9), and other $w_{ij^*}, p_{ij^*} = 0$.

As optimal solutions of sub-problems are easily obtained, it is possible to find an optimal solution of a relaxed problem by combining solutions of M sub-problems. If the optimal solution of the relaxed problem is feasible for the original problem, then it is a viable sub-optimal solution. However, the optimal solution of the relaxed problem is usually not feasible for the original problem, thus the following Lagrangian heuristic is proposed [10], which is a procedure for finding a good feasible solution of the original problem from the optimal solution of the relaxed problem. When the optimal solution of the relaxed problem is not feasible, this heuristic finds the optimal power allocation under the subcarrier allocation given by the optimal solution of the relaxed problem.

[Lagrangian heuristic]

- Step 1. Let $\widehat{p_{ij}}$ and $\widehat{w_{ij}}$ be the optimal solution of the relaxed problem, and $i = 1$.
- Step 2. If $\sum_{j=1}^{M} \widehat{p_{ij}} \leq \bar{P}_i$ for user i, $\widehat{p_{ij}}$ and $\widehat{w_{ij}}$ are feasible solution for the original problem for user i. Otherwise go to Step 3.
- Step 3. For user i such that $\sum_{j=1}^{M} \widehat{p_{ij}} > \bar{P}_i$, assign power to users with $\widehat{w_{ij}} = 1$ through the use of a water-filling algorithm.
- Step 4. If $i = N$, terminate, otherwise $i = i + 1$ and go to Step 2.

In Step 3, the fact that the optimal power to maximize the throughput when a subcarrier allocation is given can be obtained by water-filling algorithm is used [11]. Specifically, optimal solution is

$$p_{ij}^* = [\lambda_i - 1/g_{ij}]^+, \tag{10}$$

where $[\cdot]^* = \max(0, \cdot)$, λ_i satisfies $\sum_{j=1}^{M} p_{ij}^* = \bar{P}_i$ and is regarded as the "water-level" in the water-filling solution (10).

Theoretically, the relaxed problem (LR) should be solved for all nonnegative Lagrangian multipliers μ_i, and the solution found could be used to define the upper bound $\overline{T}$ for the original problem by using the following formula:

$$\overline{T} = T_{LR}(\mu^*) = \min[T_{LR}(\mu)],$$

where $T_{LR}(\mu)$ denotes the objective function value of the relaxed problem (LR). Quickly approaching μ^* is key to developing an efficient Lagrangian relaxation procedure [12]. In this problem, N Lagrangian multipliers exist, and the multiplier updating process is performed by the subgradient method [10], [13]. In the subgradient method, μ_i is updated at each iteration. In this problem, μ_i is updated through the use of the following formula:

$$\mu_i^{k+1} = \max\left[\mu_i^k + s_k\left(\sum_{j=1}^{M} p_{ij}^* - \bar{P}_i\right), 0\right],$$

where k and $k+1$ are the index of two consecutive iterations, s_k in a positive scalar (i.e., a step size) defined by:

$$s_k = \frac{\alpha_k(\overline{T} - T_k^*)}{\sum_{i=1}^{N}(\sum_{j=1}^{M} p_{ij}^* - \bar{P}_i)^2},$$

where p_{ij}^* is the optimal solution of the relaxed problem (LR), T_k^* is the maximum objective function value of the original problem until the kth iteration, and α_k is the value in $0 < \varepsilon_1 \leq \alpha_k < 2 - \varepsilon_2$ with $\varepsilon_2 > 0$.

5. Simulation Results

In this section, numerical results are presented from computer simulations of the subcarriers and power allocation algorithm proposed in this study. In the simulation, a single base station with 1 km radius is considered, and the location of mobile station is uniformly distributed over the cell. The maximum transmit power of each user mobile is normalized to 1.

The channel gain between a mobile and the base station is given as the product of path loss, shadowing and fast fading components. The path loss model is a modified Hata urban propagation model [9], as follows:

$$PL = \begin{cases} 122 + 38\log_{10}(d), & \text{if } d \geq 0.05\,\text{km}, \\ 122 + 38\log_{10}(0.05), & \text{if } d < 0.05\,\text{km}, \end{cases} \tag{11}$$

where d (in km) is the distance between a mobile and the base station. The shadowing component follows a lognormal distribution with a mean value of 0 dB and a standard deviation of 8 dB. A 6-path Rayleigh fading model is used for the fast fading in this simulation. The noise power is −100 dB for this simulation.

Figure 1 shows the throughput versus the number of users when the number of subcarriers is 256. **Proposed** shows the throughput of the proposed algorithm based on the Lagrangian relaxation procedure. **Greedy** shows the throughput of the greedy subcarrier allocation with the water-filling power allocation algorithm proposed in [9]. **MaxCH** shows the throughput of the algorithm proposed in [6], which provides the optimal solution in the downlink. In order to obtain the average throughput of each number of users, 300 trials were performed. The number of multiplier updating processes for the proposed algorithm is limited to 100 iterations. In Fig. 1, the increase of capacity according to the increase of the number of users is due to the multiuser diversity and the increase of the sum of the total power of users. As shown in Fig. 1 the proposed algorithm shows a nearly identical performance to the **Greedy** algorithm, and outperforms the **MaxCH** algorithm.

The computational performances of the Lagrangian relaxation procedure in the proposed algorithm are reported in Table 1. The values in the table are the average value of 100 trials. Table 1 shows that the proposed algorithm requires only 6.71–7.24% of the processing time of the **Greedy** algorithm, as the **Greedy** algorithm requires a computation of $N \times M^2$ numbers of the water-filling methods in order to discover a solution, while the proposed algorithm requires a

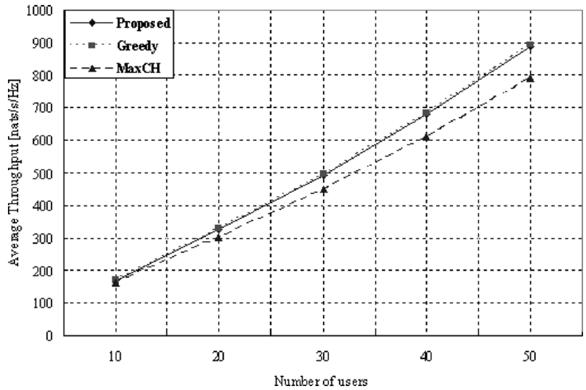

Fig. 1 Average throughput as a function of the number of users when the number of subcarriers is 256.

Table 1 Computational performance of throughput maximization problem (256 subcarriers).

No. of users	10	20	20	40	50
CPU time (s)[a]: **Proposed**[b]	3.030	6.178	9.608	11.728	13.970
CPU time (s)[a]: **Greedy**	41.853	66.215	93.690	144.348	208.123

[a]By MATLAB on Intel Pentium-4 processor operating at 2.4 GHz clock speed with 512 Mbytes RAM.

[b]Processing time after 50 Lagrangian multiplier updating procedures.

computation of $N \times N_{iter}$ numbers of the water-filling methods where N_{iter} denotes the number of multiplier updating processes. Due to computational complexity of the **Greedy** algorithm, it takes an extended amount of time to solve complex problems with the **Greedy** algorithm, especially when the number of subcarriers is large. However, the proposed algorithm required much less processing time compared to the **Greedy** algorithm in order to discover a good solution, even when the complexity of the problem is high.

6. Conclusions

In this letter, subcarriers and power allocation problem which maximize the throughput of OFDMA uplink system while satisfying each user's power constraints were formulated. As the channel conditions vary rapidly in wireless communication environments, subcarriers and power allocation should be performed in a short time. Also in this letter, a computationally efficient algorithm for this problem based on Lagrangian relaxation procedure was developed. The simulation results here show that the proposed algorithm provides comparable throughput with the value of the **Greedy** algorithm and outperforms the **MaxCH** algorithm, which is optimal in the downlink. The proposed algorithm achieves close to optimal solutions and outperforms the **Greedy** algorithm for small-sized problems. In terms of the computational complexity, this algorithm requires at most 7.24% of the processing time of the **Greedy** algorithm, while providing a throughput of more than 99% of the average throughput obtained by the **Greedy** algorithm. In simulations, the computation time of the proposed algorithm is not yet sufficient for use in a real-time environment; however, it is expected that the present approach, based on a Lagrangian relaxation procedure, assists in the development of a real-time algorithm that can address this issue.

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