Distributed Spectrum Sensing for Cognitive Radios by Exploiting Sparsity

Juan-Andres Bazerque and Georgios B. Giannakis
Dept. of ECE, University of Minnesota
Minneapolis, MN 55414, USA
Emails: {bazer002,georgios}@umn.edu

***Abstract*—A cooperative approach to the sensing task of wireless cognitive radios (CRs) is introduced based on a basis expansion model of the power spectral density (PSD) in space and frequency. Joint estimation of the model parameters enables identification of the (un)used frequency bands at arbitrary locations and thus facilitates spatial frequency reuse. The novel scheme capitalizes on the sparsity introduced by the narrowband nature of transmit-PSDs relative to the broad swaths of usable spectrum and the scarcity of position vectors where active radios are located in space. A basis pursuit scheme is developed to exploit this sparsity in the solution and reveal the unknown positions of transmitting CRs. The resultant algorithm accounts for deterministic pathloss as well as random fading propagation and can be implemented via distributed online iterations which solve quadratic programs locally (one per radio). Simulations corroborate that exploiting sparsity in CR sensing reduces spatial spectrum leakage by $15 - 20$dB relative to least-squares (LS) alternatives.**

I. Introduction

Spectrum sensing is a critical prerequisite in envisioned applications of wireless cognitive radios (CRs) which promise to resolve the perceived bandwidth scarcity versus under-utilization dilemma. Detecting the interference level present at each CR transceiver plays an instrumental role in enabling spatial frequency reuse and allowing for dynamic spectrum allocation in a hierarchical access model comprising primary (licensed) and secondary (opportunistic) users [9]. The non-coherent energy detector has been widely used to this end because it is simple and obviates the need for synchronization with unknown transmitted signals, see e.g., [5], [6], [4], [7]. Power information collected locally per CR is fused centrally by an access point in order to decide absence or presence of a primary user per frequency band. These cooperative sensing and detection schemes have been shown to cope with fading propagation effects, reduce the average detection time and improve throughput [5], [6], [4], [7].

The present paper introduces a collaborative sensing scheme whereby receiving CRs cooperate to estimate the distribution of power in space and frequency as well as triangulate the position of the transmitting CRs. The main contribution is a distributed online approach to estimating a map of the power spectral density (PSD) at arbitrary locations in space. This is particularly useful in wide area ad-hoc networks where the power transmitted by primary users reaches only a small subset of CRs. Knowing the spectrum at an any location allows remote CRs to reuse dynamically idle transmission bands. It also enables secondary users to adapt their transmit-power so as to minimally interfere with primary users. In this context, the threshold for deciding occupancy of a frequency band is not set according to the probability of false alarms but through comparing PSD estimates with minimum power levels prescribed by the primary users.

The goal of estimating the power distribution in space and frequency is admittedly very ambitious. The PSD estimate sought however, does not need to be super accurate but precise enough to identify unused bands. This relaxed objective motivates the proposed PSD estimation using a parsimonious basis expansion model. The general setup includes M receiving CRs willing to cooperate in estimating the location of I transmitting CRs as well as the frequency bands used for transmission. Upon constructing a basis expansion model of the PSD map $\Psi(\boldsymbol{x}, f)$, in space $\boldsymbol{x}$ and frequency f, the novel cooperative scheme amounts to estimating the basis expansion coefficients of $\Psi(\boldsymbol{x}, f)$ based on PSD frequency samples f_k collected at receiving CRs located at positions $\boldsymbol{x}_m$.

The rest of the paper is organized as follows. Section II introduces the basis expansion model when the locations $\boldsymbol{x}_i$ of transmitting CRs are known and their transmit-power is affected by deterministic pathloss effects. A centralized algorithm is developed in Section III when CR positions are unknown, case in which the PSD model becomes over-complete and sparsity in the vector of expansion coefficients is exploited to obtain the locations and frequency bands of the transmitting CRs. Random fading effects are dealt with in Section IV by averaging the collected data to obtain a related PSD expansion model in the mean. Based on this average model, an iterative estimator is derived which yields the expansion coefficient vector as new data become available per time slot. A distributed implementation of this estimator is also developed by having CRs share information over a control channel with their one-hop neighbors. The resultant distributed online algorithm for block fading propagation and unknown transmitter locations is provided in Section V.

† Prepared through collaborative participation in the Communications and Networks Consortium sponsored by the U. S. Army Research Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-2-0011. The U. S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation thereon.

II. Cooperative Spectrum Sensing

Let $\Phi_i(f)$ denote the power spectral density (PSD) of the ith radio's transmission. Using a set of *known* bases $\{b_j(f)\}_{j=1}^J$, this PSD is modeled as

$$\Phi_i(f) = \sum_{j=1}^{J} \alpha_{ij} b_j(f) \; , \quad i = 1, \ldots, I \tag{1}$$

where α_{ij} denote the expansion coefficients to be estimated. If $f_{\min}$ ($f_{\max}$) stands for the minimum (maximum) frequency of the system bandwidth $B := f_{\max} - f_{\min}$, consider for specificity that the basis comprises shifted Gaussian bells $b_j(f) = e^{-(f-fj)^2}$ with center frequencies equispaced on the frequency grid; i.e., using Matlab's notation, $f_j = f_{\min} : (B/J) : f_{\max}$, for $j = 1, \ldots, J$.

Consider first a deterministic pathloss model according to which the transmit-power from radio i received at radio m is scaled by the loss coefficient $\gamma_{im} = \exp(-||\boldsymbol{x}_i - \boldsymbol{x}_m||^2)$, where $\boldsymbol{x}_i(\boldsymbol{x}_m)$ is the position vector of the ith transmitter (mth receiver) with respect to a reference coordinate system. The total power received by the mth radio is the superposition of the power emitted from each active transmitting radio plus white noise, i.e.,

$$\Phi_m(f) = \sum_{i=1}^{I} \gamma_{im} \Phi_i(f) + \sigma_m^2 \; , \quad m = 1, \ldots, M \; . \tag{2}$$

Substituting (1) into (2) yields the PSD basis expansion model

$$\Phi_m(f) = \sum_{i=1}^{I} \sum_{j=1}^{J} \alpha_{ij} \gamma_{im} b_j(f) + \sigma_m^2 \tag{3}$$

which is linear in the parameters α_{ij} and σ_m^2. These parameters are to be determined using K frequency samples $\varphi_{mk} = \Phi_m(f_k)$ per receiver m over the grid $f_k = f_{\min} : (B/K) : f_{\max}$, $k = 1, \ldots, K$.

Note that once the parameters α_{ij} are estimated, the PSD (and as a byproduct the interference profile of transmitting or receiving CRs) at any location $\boldsymbol{x}$ is readily available as

$$\Psi(\boldsymbol{x}, f) = \sum_{i=1}^{I} \sum_{j=1}^{J} \alpha_{ij} \gamma_i(\boldsymbol{x}) b_j(f) \; . \tag{4}$$

Remark: So far it has been tacitly assumed that all position vectors (and thus the pathloss coefficients) are known. This may be unrealistic e.g., when the primary transmitting users are part of a legacy network and not willing to cooperatively provide their location to secondary users. This concern will be addressed in the next section.

Given the data φ_{mk}, the unknowns α_{ij} and σ_m^2 can be obtained from (3) using least-squares (LS); i.e.,

$$\min_{\boldsymbol{\alpha} \geq \mathbf{0}, \boldsymbol{\sigma} \geq \mathbf{0}} \sum_{m=1}^{M} \sum_{k=1}^{K} (\varphi_{mk} - \sum_{i=1}^{I} \sum_{j=1}^{J} \alpha_{ij} \gamma_{im} b_j(f_k) - \sigma_m^2)^2 \tag{5}$$

where the vectors $\boldsymbol{\alpha}$ ($\boldsymbol{\sigma}$) collect the naturally nonnegative coefficients α_{ij} (σ_m^2).

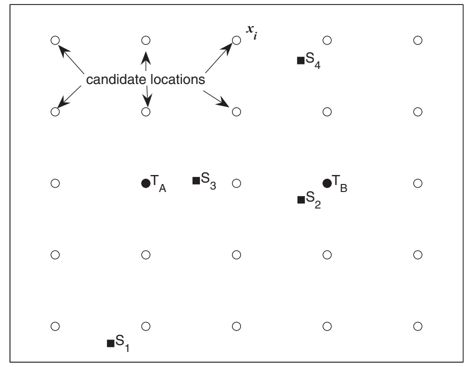

Fig. 1. Virtual network grid with 25 candidate locations and 2 actual transmitters.

III. Estimation via Basis Pursuit

If the position vectors are treated as unknown variables, the model in (3) becomes nonlinear and the LS optimization in (5) is rendered non-convex with multiple local minima. To bypass this challenge, the novel approach adopted here is to introduce a virtual grid of candidate locations as in Fig. 1, where the points $\boldsymbol{x}_i$ are no longer the actual positions of e.g., primary users but known spatial coordinates where transmitting or receiving radios could be present.

With the use of this virtual grid, one solution of (5) with unknown position vectors is possible via exhaustive search, as follows: Assume that only one transmitter is present, and for each candidate location on the grid estimate $\boldsymbol{\alpha}$ using (5). Subsequently, assume that two transmitters are present and for each pair of candidate locations on the grid obtain estimates of $\boldsymbol{\alpha}$ via (5); and so on, until covering the entire grid. Comparing the resultant LS errors and taking into account the model complexity (number of unknowns as in e.g., Akaike's Information Theoretic criterion) it is possible to obtain the unknown parameters in a parsimonious basis expansion.

The exhaustive search is clearly undesirable because it incurs combinatorial complexity in the number of grid points. Innovative results in the area of compressed sensing on the other hand, see e.g., [2] and [3], prompted us to avoid this search through the use of convex algorithms that are particularly suitable when the vector of unknowns is sparse and the locations of the nonzero entries are unknown.

These algorithms share the idea of minimizing the L_1 norm of the unknown vector in order to exploit the sparsity present. The basis pursuit (BP) algorithm [3] in particular, amounts to modifying (5) by superimposing the L_1 term $||\boldsymbol{\alpha}||_1 := \sum_{i=1}^{I} \sum_{j=1}^{J} \alpha_{ij}$ weighted by a tuning parameter λ, as follows

$$\min_{\boldsymbol{\alpha} \geq \mathbf{0}, \; \boldsymbol{\sigma} \geq \mathbf{0}} \sum_{m=1}^{M} \sum_{k=1}^{K} \epsilon_{mk}^2 + \lambda \sum_{i=1}^{I} \sum_{j=1}^{J} \alpha_{ij} \tag{6}$$

$$\epsilon_{mk} := \varphi_{mk} - \sum_{i=1}^{I}\sum_{j=1}^{J} \alpha_{ij}\gamma_{im}b_j(f_k) - \sigma_m^2 \; .$$

Setting $\lambda = 0$ yields the standard LS solution, while increasing λ pushes the solution towards the origin. The choice $\lambda = \hat{\sigma}\sqrt{2\log(IJ)}$ is advocated in [3] to tradeoff lower LS fitting error for higher degree of sparsity in the solution.

The minimization in (6) constitutes a quadratic program. It seeks estimates of α_{ij} when the location of the primary users is unknown. (We stress that $\boldsymbol{x}_i$ *are no longer the true position vectors of the transmitters but points in a prescribed grid of candidate locations.*)

Sparsity in $\boldsymbol{\alpha}$ is manifested because the basis expansion model is parsimonious both in frequency as well as in space. Indeed, relative to the possibly huge (over say 10GHz) system bandwidth, individual transmissions occupy a small fraction (in the order of MHz). Likewise, active radios are located only on a small fraction of the grid points. If the radio located at point $\boldsymbol{x}_i$ is idle, then $\alpha_{ij} = 0$ for all j. But even when a transmitting radio is present at $\boldsymbol{x}_i$, the entries $\alpha_{ij} = 0$ for all the bases $b_j(f)$ with support outside the bandwidth occupied by the ith radio.

It will be shown in Section VI that recovering the sparsity in the solution corresponds to revealing the location of the transmitters and their transmission bands.

IV. Block Fading Propagation

In addition to deterministic path loss, the propagation model in this section will allow for random block fading effects. This will be accomplished by averaging the model in (3) across time slots. To this end, consider replacing each pathloss coefficient γ_{im} in (3) by a random variable $\tilde{\gamma}_{im}$ that is assumed exponentially distributed. Each fading coefficient $\tilde{\gamma}_{im}$ remains constant per time slot n, of duration equal to the channel's coherence time, but is allowed to change independently from slot to slot. The sample mean of $\tilde{\gamma}_{im}$ across slots replaces γ_{im} in Section II and is clearly a function of the distance between transmitting and receiving radios. Compactly written, we assume per slot n that

$$\tilde{\gamma}_{im}^{(n)} \underset{\text{i.i.d.}}{\sim} \exp(1/\gamma_{im}) \; . \tag{7}$$

with $E_{\tilde{\boldsymbol{\gamma}}}(\tilde{\gamma}_{im}) = \gamma_{im}$. Since the fading model is linear in the fading coefficients $\tilde{\gamma}_{im}$, i.e.,

$$\varphi_{mk} = \Phi_m(f_k) = \sum_{i=1}^{I}\sum_{j=1}^{J} \alpha_{ij}\tilde{\gamma}_{im}b_j(f_k) + \sigma_m^2 \tag{8}$$

taking expectation with respect to $\tilde{\boldsymbol{\gamma}}$ yields

$$E_{\tilde{\boldsymbol{\gamma}}}(\varphi_{mk}) = \sum_{i=1}^{I}\sum_{j=1}^{J} \alpha_{ij}\gamma_{im}b_j(f_k) + \sigma_m^2 \; . \tag{9}$$

The next step is to replace $E_{\tilde{\boldsymbol{\gamma}}}(\varphi_{mk})$ by the sample mean $\bar{\varphi}_{mk} = N^{-1}\sum_{n=1}^{N}\varphi_{mk}^{(n)}$. Assuming that the coefficients α_{ij} are invariant during the averaging process, it follows that

$$\bar{\varphi}_{mk} = \sum_{i=1}^{I}\sum_{j=1}^{J} \alpha_{ij}\gamma_{im}b_j(f_k) + \sigma_m^2 \; . \tag{10}$$

Consequently, when block fading effects are accounted for, the coefficients α_{ij} are estimated by solving [cf. (6)]

$$\min_{\boldsymbol{\alpha}\geq\mathbf{0},\ \boldsymbol{\sigma}\geq\mathbf{0}} \sum_{m=1}^{M}\sum_{k=1}^{K} \left(\bar{\epsilon}_{mk}(\boldsymbol{\alpha},\sigma_m^2)\right)^2 + \lambda\sum_{i=1}^{I}\sum_{j=1}^{J}\alpha_{ij} \tag{11}$$

$$\bar{\epsilon}_{mk}(\boldsymbol{\alpha},\sigma_m^2) := \bar{\varphi}_{mk} - \sum_{i=1}^{I}\sum_{j=1}^{J} \alpha_{ij}\gamma_{im}b_j(f_k) - \sigma_m^2 \; . \tag{12}$$

As transmitting radios are switched on and off certain coefficients α_{ij} become nonzero while others vanish. This algorithm can be furnished with tracking capability by simply applying an exponentially weighted moving average window to the data, instead of the sample mean; that is, $\bar{\varphi}_{mk} = \frac{1-\beta}{1-\beta^N}\sum_{n=0}^{N-1}\beta^n\varphi_{mk}^{(n)}$

Furthermore, (11) lends itself naturally to adaptive, i.e., time-recursive implementation, whereby $\boldsymbol{\alpha}$ estimates are updated online as new data become available per time slot.

V. Distributed Online Algorithm

Either batch or adaptive solutions of (11) yield PSD estimates at arbitrary receiving points in space of the aggregate spectrum formed by the superposition of signals emitted from transmitting radios positioned at unknown locations. However, the approach developed so far requires availability of the data φ_{mk} at a designated central unit, e.g., an access point. The goal of this section is to develop a *distributed* solution of (11), whereby radios cooperate to estimate the PSD by exchanging messages with one-hop neighbors over a dedicated control channel.

Although PSD samples are acquired locally per receiving radio, the challenge in obtaining a distributed solution arises because the coefficients α_{ij} are global variables common to all radios. This fact is indeed what requires cooperation among radios as remarked in Section I. Fortunately, it is possible to overcome this challenge by applying recent distributed optimization approaches based on consensus. Specifically, we will adopt the alternating direction method of multipliers [1], define local copies $\boldsymbol{\alpha}^{(m)}$ of $\boldsymbol{\alpha}$ and constrain them to coincide, i.e., consent, as in [8]. Applying these steps to (11) the problem is decoupled as

$$\min_{\boldsymbol{\alpha}\geq 0,\boldsymbol{\sigma}\geq 0} \sum_{m=1}^{M}\sum_{k=1}^{K} \bar{\epsilon}_{mk}^2(\boldsymbol{\alpha}^{(m)},\sigma_m^2) + \frac{\lambda}{M}\sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{j=1}^{J}\alpha_{ij}^{(m)}$$

$$\text{s. to } \alpha_{\text{ij}}^{(\text{m})} = \alpha_{\text{ij}}^{(\text{m}')} \;\forall\; \text{m}' \in \mathcal{N}_{\text{m}} \tag{13}$$

where $\boldsymbol{\alpha}$ collects all the local copies, $\bar{\epsilon}_{mk}$ is defined as in (12) and $\mathcal{N}_m$ is a neighborhood around the mth receiving radio. These neighborhoods define a graph with local control links.

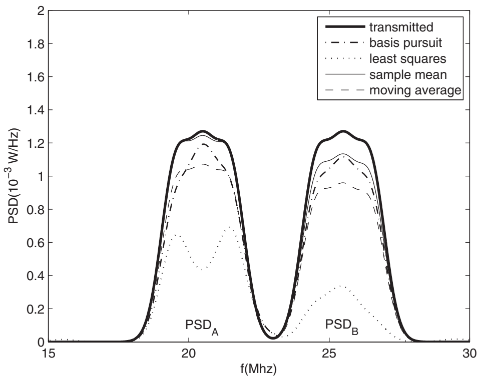

Fig. 2. True PSD transmitted by T_A and T_B and estimates obtained by four methods under consideration.

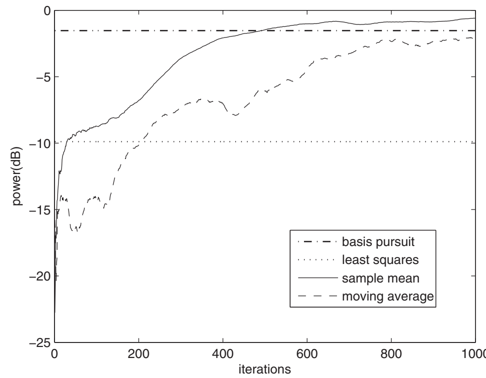

Fig. 3. Power of the estimates obtained with the four methods under consideration normalized to the power of the actually transmitted signals.

Note that if the neighborhoods are selected so that the graph is connected, then (13) is equivalent to (11). The connectivity assures that the coefficients $\boldsymbol{\alpha}^{(m)}$ in the solution of (13) are equal for all m and thus equal to the solution of (11).

The distributed algorithm follows from (13) after writing the augmented Lagrangian and using the method of multipliers [1]. Each radio solves the following problem locally in an iterative fashion (t denotes the iteration index)

$$\begin{aligned} \boldsymbol{\alpha}^{(m)}(t) \;\; = \;\; & \arg \min_{\boldsymbol{\alpha} \geq \mathbf{0},\ \boldsymbol{\sigma} \geq \mathbf{0}} \left\{ \sum_{k=1}^{K} \bar{\epsilon}_{mk}^{2}(\boldsymbol{\alpha}^{(m)}, \sigma_m^2) \right. \\ & + \frac{\lambda}{M} \sum_{i=1}^{I} \sum_{j=1}^{J} \alpha_{ij}^{(m)} \\ & + \sum_{m' \in \mathcal{N}_m} \boldsymbol{\mu}_{mm'}^{T}(t-1)(\boldsymbol{\alpha}^{(m)} - \boldsymbol{\alpha}^{(m')}) \\ & \left. + \sum_{m' \in \mathcal{N}_m} \frac{c}{2} ||\boldsymbol{\alpha}^{(m)} - \boldsymbol{\alpha}^{(m')}||^2 \right\} \end{aligned} \tag{14}$$

and updates its multipliers using a stochastic gradient iteration

$$\boldsymbol{\mu}_{mm'}(t) = \boldsymbol{\mu}_{mm'}(t-1) + c(\boldsymbol{\alpha}^{(m)}(t) - \boldsymbol{\alpha}^{(m')}(t)); \tag{15}$$

subsequently, it communicates the sparse $\boldsymbol{\alpha}^{(m)}(t)$ and $\boldsymbol{\mu}_{mm'}(t)$ iterates with its one-hop neighbors. These three steps are performed per time slot t and repeated in an online loop. During each iteration the data sample mean $\bar{\varphi}_{mk}$ is updated after incorporating each new sample, and a reduced quadratic program is solved locally.

The loop specified by (14) and (15) is the resulting distributed online algorithm for estimating the transmitter locations and the aggregate PSD in lieu of an access point.

VI. Simulations

The network setup used for simulations is shown in Fig. 1. It consists of $M = 4$ receiving radios cooperating to estimate the PSD coefficients and locate transmitters in a grid of $I = 25$ candidate locations. Fig. 1 also depicts two transmitting radios, marked as T_A and T_B. The cooperating radios scan $K = 128$ frequencies from 15 to 30Mhz and adopt a basis expansion model over this band comprising $J = 15$ bases. The PSD of the signals transmitted by T_A and T_B are marked as PSD_A and PSD_B, respectively, and are depicted by the bold solid line in Fig. 2.

At the mth receiving radio the $(K = 128)$-point periodogram of white Gaussian noise is added to the superposition of the PSDs received from both transmitters.

Given these data samples, four algorithms are tested to identify the locations of the transmitters and recover PSD_A and PSD_B. The first one corresponds to the centralized basis pursuit algorithm with deterministic pathloss [cf. (6)]. The second one yields the LS estimate under nonnegativity constrains obtained after setting $\lambda = 0$ [cf. (5)] . The third one relies on sample averaging to cope with random fading and implements the distributed scheme [cf. (13)]; while the last one corresponds again to the distributed scheme but with an exponentially weighted moving average window.

Fig. 2 shows that the four schemes underestimate the actual PSDs. For the *basis pursuit*, *sample mean* and *moving average* algorithms the gap can be explained by the fact that λ pushes the solution towards the origin. This gap can be reduced by discarding the dimensions of the vector $\boldsymbol{\alpha}$ associated with null coefficients in the solution and adjusting it by performing a round of standard LS estimation over the remaining dimensions. Fig. 3 depicts the evolution of iterates across time. Besides additive noise, notice that the sample mean and moving average solutions are affected also by random fading. The sample mean outperforms BP because after sufficiently many iterations, the Lagrange multipliers stabilize and fading and noise are asymptotically averaged out. The moving average scheme on the other hand trades of estimation accuracy for improved tracking capability.

It is worth observing that the nonzero coefficients in all solutions correspond to bases inside the transmission bands, which confirms correct estimation of the PSD support. To this end, the power estimates falling outside this support

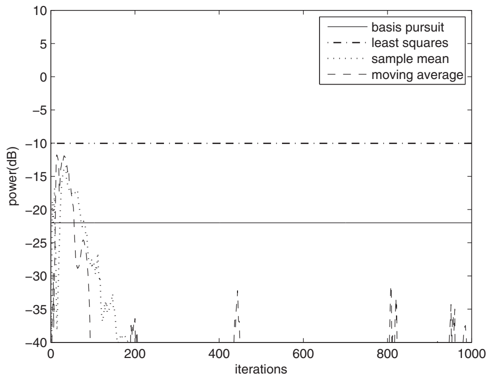

Fig. 4. Aggregate power of spurious estimates outside the transmission bands normalized to the power of the transmitted signals.

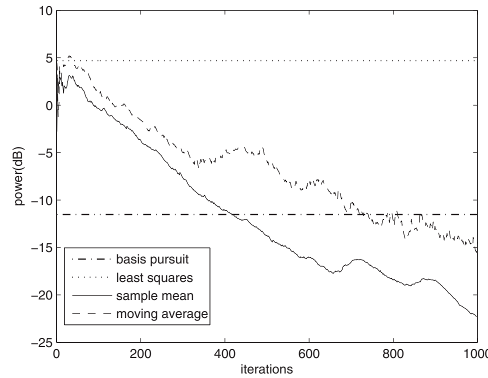

Fig. 5. Aggregate power of spatially spurious estimates normalized to the power of the transmitted signals.

were added and compared to the actual transmission power. Fig. 4 shows how the iterative algorithms set these aggregate estimates to values under -30dB.

However, all solutions entail spurious PSD estimates in space; that is, residual transmission power leaks at points of the grid where no transmitter is present. To capture this leakage, the sum-power of the spurious PSD estimates added over all the idle grid points is used as a figure of merit. Fig. 5 compares this sum-power with the actual transmission power and shows that the iterative algorithms outperform the rest as far as localizing transmitters is concerned. Even though the LS scheme accurately estimates the received signals, it incurs over-fitting errors. Without sparsity constraints, the estimator selects points in the grid that are close to the receiving CRs and for each one yields a basis expansion over frequency that resembles the received signal plus noise. The result is a PSD with many spurious estimates which justifies the use of the term $\lambda||\boldsymbol{\alpha}||_1$ in the optimization problem. This term enforces sparsity in the solution which translates to a parsimonious selection of a transmitters, leading to the identification of the correct positions.

VII. Concluding Summary

The key challenge in developing cognitive wireless transceivers is enabling them to sense the ambient power spectral density at arbitrary locations in space. The present paper addressed this challenging task through a parsimonious basis expansion model of the PSD in frequency and space. This model reduces the sensing task to estimating a sparse vector of unknown parameters. As a byproduct, sparsity also facilitates localization of transmitting radios. The associated estimators rely on the basis pursuit algorithm which here enforces sparsity in the solution to reveal the position and frequency bands of transmitting radios. Once these become available, the model characterizes how power is distributed in frequency and space – the major step enabling spatial frequency reuse.

The novel cooperative sensing approach can be implemented not only in a batch, centralized mode, but also in a distributed online fashion where each radio solves iteratively (per time slot) a reduced-dimension quadratic program locally and exchanges information with its one-hop neighbors to obtain the globally optimum solution via consensus. Simulations confirmed that the iterative algorithm outperforms the centralized one in the presence of fading due to its ability to average out the noise. Additional tests revealed that an exponentially weighted moving average alternative can tradeoff estimation error performance for improved tracking of slowly varying PSDs. Finally, simulations corroborated that exploiting sparsity is well justified in distributed sensing because LS alternatives incur $15-20$dB higher leakage of power across space.

References

[1] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods,* 2nd ed. Belmont, MA: Athena Scientific, 1999.

[2] E. J. Candès, J. Romberg, and T. Tao, "Stable signal recovery from incomplete and inaccurate measurements," *Communications on Pure and Applied Mathematics,* vol. 59 , no.8, pp. 1207-1223, August 2006.

[3] S. S. Chen, D. L. Donoho, and M. A. Saunders, "Atomic decomposition by basis pursuit," *SIAM J. Scientific Computing,* vol. 20, no. 1, pp. 3361, 1999.

[4] G. Ganesan and Y. G. Li, "Cooperative spectrum sensing in cognitive radio networks," in *Proc. IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks (DySPAN'05),* Baltimore, USA, Nov. 8-11, 2005, pp. 137-143.

[5] A. Ghasemi and E. S. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in *Proc. IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks (DySPAN'05),* Baltimore, USA, Nov. 8-11, 2005, pp. 131-136.

[6] S. M. Mishra, A. Sahai and R.W. Brodersen , " Cooperative Sensing among Cognitive Radios," in *Proc. Int. Conf. on Communications* , vol. 4, pp. 1658-1663, Istanbul, Jun. 11-15, 2006 .

[7] Z. Quan, S. Cui, A. Sayed, and H. V. Poor, "Spatial-Spectral Joint Detection for Wideband Spectrum Sensing in Cognitive Radio Networks," in *Proc. Int. Conf. Acoustics, Speech, Sig. Proc.,* Las Vegas, USA, Mar. 30 - Apr. 4, 2008, pp. 2793-2796.

[8] I. D. Schizas, A. Ribeiro and G. B. Giannakis, "Consensus in Ad Hoc WSNs with Noisy Links - Part I: Distributed Estimation of Deterministic Signals," *IEEE Transactions on Signal Processing,* vol. 56, no. 1, pp. 350-364, January 2008.

[9] Q. Zhao and A. Swami, "A Decision-Theoretic Framework for Opportunistic Spectrum Access," in *IEEE Wireless Communications Magazine: Special Issue on Cognitive Wireless Networks,* vol. 14, no. 4, pp. 14-20, August, 2007.