Performance Evaluation of Distributed Compressed Wideband Sensing for Cognitive Radio Networks

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Abstract— In emerging cognitive radio (CR) networks, the first cognitive task preceding any dynamic spectrum management is the sensing and identification of spectrum holes in wireless environments. This paper develops a distributed compressed spectrum sensing approach for (ultra-)wideband CR networks. First, compressive sampling is performed at local CRs to scan the very wide spectrum at practical signal-acquisition complexity. Then, measurements or states from multiple CR detectors are fused to collect spatial diversity gain, which improves the detection quality especially under fading channels. Distributed fusion algorithms are derived to effect collaborative fusion at low communication and computation load scalable to the network size. Multiple performance metrics are identified to evaluate the proposed spectrum sensing techniques, as well as to assess those performance-determining factors in system design. Computer simulations corroborate the effectiveness of the proposed techniques in identifying and locating the spectrum hole opportunities in wireless fading environments.

Keywords: *Performance Evaluation, Cognitive Radio Networks, Dynamic Spectrum Management, Compressed Spectrum Sensing, Distributed Fusion,*

I. INTRODUCTION

Adopting open spectrum access, cognitive radio (CR) communications and networking have emerged as disruptive wireless technologies for alleviating today's spectrum scarcity problem [1]. The first cognitive task preceding any dynamic spectrum management is the sensing and identification of spectrum holes that are temporarily unoccupied by primary communication systems holding the license of the spectrum [2]. Depending on whether the spectrum utilization of the primary systems is low (around 5%), medium (below 20%) or high (above 20%), the front-end radio architecture for sensing can be narrowband, wideband or ultra-wideband based, respectively [3]. The increasingly popular (ultra-)wideband wireless networks not only offer high throughput and user capacity for primary systems, but also purport pronounced dynamic spectrum access opportunities for secondary CR users. Accompanied with these benefits, however, spectrum sensing in the wideband regime also faces considerable technical challenges.

In the wideband regime, a major challenge stems from the high RF signal acquisition costs of current analog-to-digital hardware technology. Very high sampling rates are required by conventional spectral estimation methods which have to operate at or above the Nyquist rate. Meanwhile, due to the timing requirements for rapid sensing, only a limited number of measurements can be acquired from the received signal, which may not provide sufficient statistic when traditional linear signal reconstruction methods are employed.

Furthermore, wireless fading constitutes a major factor of performance degradation to traditional spectrum detection techniques [4], [5]. A CR user may not be able to accurately sense and detect the transmission of a primary system due to several channel fading effects, including the large-scale path loss in power caused by the long distance between the primary transmitter and the secondary receiver, and the small-scale deep fades that are random and unpredictable. When a missed detection arises, the CR may unwittingly transmit over the same channels used by active primary users, causing detrimental interference to legacy services.

To provide reliable spectrum sensing at affordable complexity, this paper presents a distributed compressed sensing framework for wideband communication networks. First, we recognize that the wireless signals in open-spectrum networks are typically *sparse* in the frequency domain. This is due to the low percentage of spectrum occupancy by active radios – a fact motivating dynamic spectrum access. For sparse signals, recent advances in compressive sampling have demonstrated the principle of sub-Nyquist-rate sampling and reliable signal recovery via computationally feasible algorithms [6]–[8]. We develop a compressed sensing technique for detecting wideband signals at reduced signal sampling and acquisition costs.

Next, we deal with the wireless fading effects by performing collaborative fusion among multiple spatially distributed CRs in the network. Measurements or locally estimated state values from multiple CRs are fused to reach a global decision on the spectrum hole opportunities. Because channel fades are independent among CRs, collaborative fusion enables spatial diversity gain, which improves the detection quality especially under fading channels. Research on collaborative sensing has surged recently, primarily for small-size networks probing a single frequency band or operating in flat fading channels, in the presence of a centralized fusion center [9], [10]. A focal contribution of this paper is to develop distributed fusion techniques for multi-hop large networks operating in frequency selective fading channels, without necessarily resorting to a fusion center. Optimization techniques including sub-gradient

based stochastic approximation and consensus filtering [11], are utilized to derive distributed fusion solutions that converge to the globally optimal solutions, using only one-hop local communications among CR neighbors. The computational complexity and communication costs of these distributed fusion techniques are scalable to the network size.

Finally, extensive performance evaluations are provided to assess the proposed spectrum sensing techniques. Relevant performance metrics are identified, including correct detection probability, false alarm probability, sensing time, allowable compression ratio, robustness to noise and quantization errors, sensitivity to channel fading, hardware implementation complexity in acquisition, and speed and cost of signal reconstruction, and so on. Critical system and algorithm design parameters and operating conditions are also elucidated. It is important to assess the impacts of these performance-determining factors on the performance metrics of interest, in order to strike the desired tradeoffs in the system design. Corroborating simulation results are presented to testify the effectiveness of the proposed sensing and fusion techniques in identifying and locating the spectrum hole opportunities in wireless fading environments.

II. Signal Model and Problem Statement

Consider a (ultra-)wide frequency band that hosts both primary communication systems and secondary CR users. The entire wideband channel is divided into M non-overlapping narrowband subchannels centered at $\{f_m\}_{m=0}^{M-1}$, some of which might not be occupied by primary users in a particular geographical region and time. These temporarily idle subchannels are termed spectrum holes and are available for opportunistic spectrum access by secondary users. There are spatially distributed CRs that collaboratively sense the wide band in order to identify those spectrum holes, giving rise to the spectrum sensing and detection problem.

During the detection interval, we assume for simplicity that higher-layer protocols (e.g., medium access control layer) can guarantee that all CRs stay silent such that only the primary users are emitting spectral power. Suppose that there are I active primary users during the detection interval, whose transmitted signals are denoted by $s_i(t)$, $i = 1, \ldots, I$. After propagating through a wireless fading channel, the signal $s_i(t)$ reaches the j-th CR receiver in the form $h_{ij}(t) \star s_i(t)$, where $\star$ denotes the convolution operator and $h_{ij}(t)$ is the channel impulse response that is typically frequency selective over the wide band. We assume that the channels are slowly varying and can be treated as time-invariant during the detection interval. The received signal at CR j is thus given by

$$r_j(t) = \sum_{i=1}^{I} h_{ij}(t) \star s_i(t) + w_j(t) \qquad (1)$$

where the ambient noise $w_j(t)$ is white Gaussian with zero mean and power spectral density (PSD) σ_w^2.

To reflect the discretized signal response on the M subchannels, we take the M-point discrete Fourier transform (DFT) on $r_j(t)$, where M is larger than the channel memory length.

Collecting the frequency-domain samples into an $M \times 1$ vector $\mathbf{r}_f^{(j)}$, we have

$$\mathbf{r}_f^{(j)} = \sum_{i=1}^{I} \mathbf{D}_h^{(ij)} \mathbf{s}_f^{(i)} + \mathbf{w}^{(j)} \qquad (2)$$

where $\mathbf{D}_h^{(ij)} = \mathrm{diag}(\mathbf{h}_f^{(ij)})$ is an $M \times M$ diagonal channel matrix, and $\mathbf{h}_f^{(ij)}$, $\mathbf{s}_f^{(i)}$ and $\mathbf{w}^{(j)}$ are the frequency-domain discrete version of $h_{ij}(t)$, $s_i(t)$ and $w_j(t)$ respectively. This signal model can be expressed in a general form as

$$\mathbf{r}_f^{(j)} = \mathbf{H}_f^{(j)} \bar{\mathbf{s}}_f^{(j)} + \mathbf{w}^{(j)}. \qquad (3)$$

In the absence of the channel state information (CSI) $\{\mathbf{h}_f^{(ij)}\}_i$ at each CR receiver j, it is useful to lump all the transmitted signals as follows ($\mathbf{I}_M$ is an identity matrix):

$$\mathbf{H}_f^{(j)} = \mathbf{H}_f := \mathbf{I}_M; \qquad \bar{\mathbf{s}}_f^{(j)} := \sum_{i=1}^{I} \mathbf{D}_h^{(ij)} \mathbf{s}_f^{(i)} \qquad (4)$$

where $\mathbf{H}_f^{(j)}$ is fixed and $\bar{\mathbf{s}}_f^{(j)}$ represents the received signal component at CR j. When each CR j knows the channels $\{\mathbf{h}_f^{(ij)}\}_i$, one can adopt

$$\mathbf{H}_f^{(j)} := \left[\mathbf{D}_h^{(1j)}, \cdots, \mathbf{D}_h^{(Ij)} \right];$$

$$\bar{\mathbf{s}}_f^{(j)} = \bar{\mathbf{s}}_f := \left[(\mathbf{s}_f^{(1)})^T, \cdots, (\mathbf{s}_f^{(I)})^T \right]^T. \qquad (5)$$

At each CR, *spectrum sensing* boils down to estimating $\bar{\mathbf{s}}_f^{(j)}$ in (3) from $r_j(t)$. In the absence of the CSI, the CR cannot decouple each product $\mathbf{D}_h^{(ij)} \mathbf{s}_f^{(i)} = \mathbf{h}_f^{(ij)} \odot \mathbf{s}_f^{(i)}$ ($\odot$ denotes element-by-element multiplication); hence, $\bar{\mathbf{s}}_f^{(j)}$ to be estimated from (4) entails the channel-dependent aggregated received signal rather than the transmitted signal amplitudes. With CSI, it is possible to estimate the individual sources $\{\mathbf{s}_f^{(i)}\}_{i=1}^{I}$ from $\bar{\mathbf{s}}_f^{(j)}$ in (5). It is of interest to investigate the impact of the channel knowledge on spectrum sensing quality; nevertheless, the sensing techniques for both cases can be the same, based on the general expression in (3).

Depending on the spectrum sharing protocol adopted, the CRs might not be interested in the transmitted (or received) signal strength $\bar{\mathbf{s}}_f^{(j)}$, but simply want to know which of the M subchannels are unoccupied spectrum holes. All CRs avoid transmitting at any occupied subchannels, but dynamically share the spectrum holes among themselves. In this case, the spectrum sensing task is reduced to *spectrum detection*. The goal is to determine the frequency occupancy of primary users by detecting a binary state vector $\mathbf{d} \in \{0,1\}^{M \times 1}$, whose m-th element is defined by

$$d[m] = \begin{cases} 1, & \text{if } f_m \text{ is occupied, i.e., } \exists i : \mathbf{s}_f^{(i)}[m] = s_i(f_m) \neq 0 \\ 0, & \text{if all primary users are silent on } f_m \end{cases}$$
$$(6)$$

Spectrum detection is feasible even in the absence of the channel knowledge. In (4), the elements in $\bar{\mathbf{s}}_f^{(j)}$ indicate whether the M frequency bins are occupied by one or more primary transmitters or not. When $\bar{\mathbf{s}}_f^{(j)}[m] \neq 0$, at least one of the sources $s_i(f)$ is emitting on f_m, and the channel fading $h_{ij}(f_m)$ on this frequency is non-zero. In another words,

occupied subchannels would correspond to non-zero elements in $\bar{\mathbf{s}}_f^{(j)}$, whereas the rest zero elements in $\bar{\mathbf{s}}_f^{(j)}$ reflect idle frequency opportunities for CRs. Exceptions arise when a deep channel fade $h_{ij}(f_m)$ makes it inaccurate to detect a non-zero $\mathbf{s}_f^{(i)}[m]$ from $\bar{\mathbf{s}}_f^{(j)}[m]$, resulting in missed detection.

Hence, the spectrum detection problem can be tackled by finding the (non-)zero elements in the noise-free version $\bar{\mathbf{s}}_f^{(j)}$ of the received signal spectrum $\mathbf{r}_f^{(j)}$. Because detection by one CR receiver is subject to missed detection due to channel fading, it is instrumental for a network of spatially diverse CRs to collaborate during the spectrum detection phase. Multiple CR receivers listen to all channels and make their independent local decisions $\hat{\mathbf{d}}^{(j)}$, and then collaboratively determine the spectrum state vector $\hat{\mathbf{d}}$ by fusing all local decisions $\{\hat{\mathbf{d}}^{(1)}, \hat{\mathbf{d}}^{(2)}, \cdots, \hat{\mathbf{d}}^{(J)}\}$. Collaborative spectrum decision fusion mitigates the channel fading effect by enabling spatial diversity gain, which will be evaluated in this paper.

III. COLLABORATIVE COMPRESSED SPECTRUM SENSING

To achieve high-performance spectrum sensing at practical complexity, this section contributes to develop collaborative sensing schemes that utilize compressive sampling in the temporal-frequency domain and fused detection in the spatial domain. Our schemes consist of two steps: i) compressed spectrum sensing at individual CRs to estimate $\bar{\mathbf{s}}_f^{(j)}$ and make local decisions $\{\hat{\mathbf{d}}^{(j)}\}_j$ at low sampling complexity, and, ii) collaborative, distributed detection across the network to collect spatial diversity gain.

A. Compressed Spectrum Sensing at Individual CRs

Let us now turn to each of the CR receivers j, $j = 1, \ldots, J$. In this subsection we drop the receiver index j for notational simplicity. Locally, the goal is to estimate $\bar{\mathbf{s}}_f$ in (3) given $\mathbf{H}_f$ and $r(t)$. To this end, we develop a compressive sampling approach that reduces the signal-acquisition costs in the wideband regime, in terms of digital sampling and quantization. Our development bears resemblance to the compressed sensing techniques in [13], but with different goals: this work seeks to estimate the spectral shape $\bar{\mathbf{s}}_f$ given the subchannel structure, whereas [12], [13] aims to find the unknown frequency locations of occupied spectrum segments via edge detection.

The first step of compressive sampling is to collect time-domain samples. Motivated by the need to reduce the sampling burden in the wideband regime, we adopt a linear random sampler at each CR to collect a $K \times 1$ time-domain sample vector $\mathbf{x}_t$ from $r(t)$, $K \leq M$, as follows:

$$\mathbf{x}_t = \mathbf{S}^T \mathbf{r}_t \qquad (7)$$

where the $M \times 1$ vector $\mathbf{r}_t$ is the discrete-time representation of $r(t)$ and $\mathbf{S}$ is an $M \times K$ projection matrix. Columns $\{\mathbf{s}_k\}_{k=1}^K$ of $\mathbf{S}$ can be viewed as a set of basis functions or matched filters used for collecting the time-domain samples, while the measurements $\{x_t[k]\}_{k=1}^K$ are in essence the projection of $r(t)$ onto the bases [14]. The model in (7) subsumes all sampling schemes yielding linear measurements. For example, $\mathbf{S} = \mathbf{I}_M$

represents Nyquist-rate uniform sampling, while reduced-rate sampling arises when $K < M$. To distinct, we denote a reduced-rate sampling matrix as $\mathbf{S}_c$. A simple example of $\mathbf{S}_c$ is a selection matrix that randomly retains K columns of the size-M identity matrix $\mathbf{I}_M$, which amounts to collecting samples on the Nyquist sampling grid but with $(K - M)$ random time instants skipped to reduce the average sampling rate.

With the K measurements $\mathbf{x}_t = \mathbf{S}_c^T \mathbf{r}_t$, we now estimate the frequency response $\bar{\mathbf{s}}_f$ in (3). Noting $\mathbf{r}_t = \mathbf{F}_M^{-1} \mathbf{r}_f$, we have

$$\mathbf{x}_t = \mathbf{S}_c^T \mathbf{F}_M^{-1} \mathbf{r}_f = \mathbf{S}_c^T \mathbf{F}_M^{-1} \mathbf{H}_f \bar{\mathbf{s}}_f + \tilde{\mathbf{w}}_f \qquad (8)$$

where $\tilde{\mathbf{w}}_f = \mathbf{S}_c^T \mathbf{F}_M^{-1} \mathbf{w}_f$ is the noise sample vector that remains to be white Gaussian. Because the spectrum utilization by the primary network is low – a fact motivating dynamic spectrum access at the outset, the unknown vector $\bar{\mathbf{s}}_f$ is sparse with only a small number of non-zero elements. The sparsity measure is given by the l-norm $||\bar{\mathbf{s}}_f||_l$, $l \in [0, 2)$, where the l_0-norm indicates exact sparsity [8]. Recent literature has seen the emergence of signal reconstruction techniques developed under the compressive sampling framework [15]–[18]. For example, the Basis Pursuit (BP) technique [18] solves the following linear convex optimization problem:

$$\min_{\bar{\mathbf{s}}_f} ||\bar{\mathbf{s}}_f||_1, \qquad s.t. \quad \mathbf{x}_t = \mathbf{S}_c^T \mathbf{F}_M^{-1} \mathbf{H}_f \bar{\mathbf{s}}_f. \qquad (9)$$

Having obtained $\hat{\bar{\mathbf{s}}}_f$ from (9), each CR receiver can make a local decision on the spectrum state vector $\mathbf{d}$ by comparing each element of $\hat{\bar{\mathbf{s}}}_f$ with a decision threshold η. In the absence of channel knowledge (c.f., (4)), the decision is

$$\hat{\mathbf{d}} = \left(|\hat{\bar{\mathbf{s}}}_f| \geq \eta \right). \qquad (10)$$

Utilizing the channel knowledge in (5), the CR can separate the I transmitted sources $\{\mathbf{s}_f^{(i)}\}_{i=1}^I$ and claim an occupied subchannel as long as there exists one source i with a non-zero element $\mathbf{s}_f^{(i)}[m] \neq 0$, that is ($\oplus$ denotes logical OR operation):

$$\hat{\mathbf{d}}[m] = \oplus_i \left(|\hat{\bar{\mathbf{s}}}_f[iM + m]| \geq \eta \right). \qquad (11)$$

The decision threshold η can be chosen based on a desired level of probability of false alarms P_{fa}, using the well-known Neyman-Pearson hypothesis test rule.

B. Collaborative, Distributed Spectrum Detection Fusion

Several fusion schemes can be devised depending on the available information to the fusion process. We consider:

i) in the absence of channel knowledge CSI, the fusion decision on $\mathbf{d}$ can be made based on the local detection decisions $\{\hat{\mathbf{d}}^{(j)}\}_j$ or the local sensing estimates $\{\hat{\bar{\mathbf{s}}}_f^{(j)}\}_j$, which we term as *detection state fusion* and *estimation state fusion* respectively;

ii) With CSI, the fusion decision can be made based on the measurements $\{\mathbf{x}_t^{(j)}\}_j$, which we term as *measurement fusion*; alternatively, we can perform *detection state fusion with CSI* and *estimation state fusion with CSI* as counterparts of those in i).

In all these fusion scenarios, it is important to design distributed and decentralized fusion rules that are scalable to the

network size J. Next, we present the distributed fusion algorithms for the above collaborative spectrum sensing scenarios, using the centralized fusion rules as references.

1) Detection State Fusion: In the centralized case, a dedicated base station or one of the CR nodes serves as a fusion center and collect all the local decisions $\{\hat{\mathbf{d}}^{(j)}\}_{j=1}^{J}$ made at the J CRs. The globally fused spectrum decision is thus given by the majority vote, that is,

$$\hat{\mathbf{d}} = \left(\frac{1}{J} \sum_{j=1}^{J} \hat{\mathbf{d}}^{(j)} > \gamma_{th} \right). \tag{12}$$

The threshold $\gamma_{th} \in [1/J, 1]$ is chosen to reflect how conservative the network is in protecting the primary users. In the most conservative case, detection is declared as long as there exists a single CR which locally declares detection, corresponding to setting $\gamma_{th} = 1/J$. Another viable choice is $\gamma_{th} = 1/2$, which reflects the majority voting rule. In reaching (12), each CR only needs to send the binary vector $\hat{\mathbf{d}}^{(j)}$ to the fusion center, which amounts to JM bits for storage and transmission over the entire network.

In the absence of a fusion center, our goal is to devise a distributed fusion rule that only requires local information exchange among one-hop CR neighbors, while still reaching the globally optimal sensing decisions for a multi-hop network. To this end, we note from (12) that detection decision fusion amounts to averaging the local decisions $\{\hat{\mathbf{d}}^{(j)}\}_j$ and then comparing the averaged value on each subchannel with γ_{th}. It is thus well motivated to adopt the average-consensus technique [19], [20], which is an algorithm design technique for distributed computing over a large network.

To represent an average consensus problem, Let $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ be an undirected connected graph with node set $\mathcal{N} = \{1, \ldots, J\}$ and edge set $\mathcal{E}$, where each edge $(j, k) \in \mathcal{E}$ is an unordered pair of distinct nodes. Let $\mathbf{c}_j(0) := \hat{\mathbf{d}}^{(j)}$ be a real vector associated with CR node j at time $t = 0$. The (distributed) average consensus problem is to compute the average $(1/J) \sum_{j=1}^{J} \mathbf{c}_j(0)$ at every node, via local communication and computation on the graph. Thus, node j carries out its update, at each step, based on its local state and communication with its neighbors $\mathcal{N}_j = \{k | (j, k) \in \mathcal{E}\}$.

We focus on a particular class of iterative algorithms for average consensus, which has been widely used in distributed computing [20]. Each node updates itself by adding a weighted sum of the local discrepancies, i.e., the differences between neighboring node values and its own [20]:

$$\mathbf{c}_j(t+1) = \mathbf{c}_j(t) + \sum_{k \in \mathcal{N}_j} \mathbf{W}_{jk} \left(\mathbf{c}_k(t) - \mathbf{c}_j(t) \right), \tag{13}$$
$$j = 1, \ldots, J; \ t = 0, 1, \ldots.$$

Here $\mathbf{W}_{jk}$ is a weight associated with the edge (j, k). These weights are algorithm parameters, and for simplicity can be parameterized by scalars $\{w_{jk}\}_{j,k}$ as $\mathbf{W}_{jk} = w_{jk}\mathbf{I}$. Since we associate weights with undirected edges, we have $\mathbf{W}_{jk} =$

$\mathbf{W}_{kj}$. With properly designed weights, we can guarantee that

$$\lim_{t \to \infty} \mathbf{c}_j(t) = \frac{1}{J} \sum_{k=1}^{J} \mathbf{c}_k(0) = \frac{1}{J} \sum_{k=1}^{J} \hat{\mathbf{d}}^{(k)}, \quad \forall j = 1, \ldots, J. \tag{14}$$

Thus, through local one-hop communications, each CR obtains the averaged statistic of the entire multi-hop network. Subsequently, each CR can make the fusion decision on $\mathbf{d}$ straightforwardly by comparing $\mathbf{c}_j(t)$ with γ_{th} at a sufficiently large t, in the absence of a centralized fusion center.

2) Measurement Fusion: In the centralized case, the fusion center collects all the measurements $\{\mathbf{x}_t^{(j)}\}_{j=1}^{J}$ in (8). The fusion problem becomes estimating the common transmission vector $\bar{\mathbf{s}}_f$ in (5). The CSI $\{\mathbf{H}_f^{(j)}\}_j$ from all CRs also need to be made available to the fusion center. Extending the sparse signal recovery algorithm in (9), we can perform centralized fusion to improve the estimation quality on $\bar{\mathbf{s}}_f$, using the following fusion rule:

$$\min_{\bar{\mathbf{s}}_f} \quad ||\bar{\mathbf{s}}_f||_1 \tag{15}$$
$$s.t. \quad \mathbf{x}_t^{(j)} = (\mathbf{S}_c^{(j)})^T \mathbf{F}_M^{-1} \mathbf{H}_f^{(j)} \bar{\mathbf{s}}_f, \quad j = 1, \ldots, J.$$

The same algorithms for solving (9) apply to (15) straightforwardly, simply by stacking all the linear constraints in (15) into a single linear constraint of an enlarged size. On the other hand, this fusion formula is costly to implement, since the fusion center needs to know, in addition to the measurements, the sampling matrices $\{\mathbf{S}_c^{(j)}\}_j$ and channel matrices $\{\mathbf{H}_f^{(j)}\}_j$ from all CRs, not mention the computational load.

We design a distributed version of (15) via the primal-dual approach in convex optimization [11]. To do so, we write the Lagrangian associated with (15) as

$$\mathcal{L}(\bar{\mathbf{s}}_f; \boldsymbol{\lambda}) = ||\bar{\mathbf{s}}_f||_1 + \sum_j \boldsymbol{\lambda}_j^T \left((\mathbf{S}_c^{(j)})^T \mathbf{F}_M^{-1} \mathbf{H}_f^{(j)} \bar{\mathbf{s}}_f - \mathbf{x}_t^{(j)} \right). \tag{16}$$

The first-order derivative of $\mathcal{L}(\bar{\mathbf{s}}_f; \boldsymbol{\lambda})$ with respect to $\bar{\mathbf{s}}_f$ is given by

$$\frac{\partial \mathcal{L}(\bar{\mathbf{s}}_f; \boldsymbol{\lambda})}{\partial \bar{\mathbf{s}}_f} = \text{sgn}(\bar{\mathbf{s}}_f) + \sum_j \left((\mathbf{S}_c^{(j)})^T \mathbf{F}_M^{-1} \mathbf{H}_f^{(j)} \right)^T \boldsymbol{\lambda}_j. \tag{17}$$

Since the subgradient of the dual function can be computed efficiently, the use of the subgradient method is well grounded. Via subgradient search, low-complexity stochastic approximation can be implemented to iteratively update the multipliers $\{\boldsymbol{\lambda}_j\}_j$ and the spectral estimate $\bar{\mathbf{s}}_f$ [11]. CRs need to broadcast the updated vector $((\mathbf{S}_c^{(j)})^T \mathbf{F}_M^{-1} \mathbf{H}_f^{(j)})^T \boldsymbol{\lambda}_j$ to their one-hop neighbors at each iteration, but can avoid exchanging the channel and sampling matrices $\mathbf{H}_f^{(j)}$ and $\mathbf{S}_c^{(j)}$. Under some mild conditions, all CRs converge to the steady state estimate $\hat{\bar{\mathbf{s}}}_f$ which is globally optimal, as in (15).

3) Estimation State Fusion with CSI: When CRs acquire the CSI $\{\mathbf{h}_f^{(ij)}\}_{i,j}$, the signal model in (5) applies, and the individual primary transmission signals $\{\mathbf{s}_f^{(i)}\}_i$ (a.k.a. estimation state vectors) can be estimated locally from all CR receivers. In this presence of a centralized fusion center (FC),

both the local state estimates $\{\hat{\bar{\mathbf{s}}}_f^{(j)}\}$ and the CSI $\{\mathbf{h}_f^{(ij)}\}_{i,j}$ are used for state fusion. The decision statistic $\hat{\bar{\mathbf{s}}}_f$ in the optimal centralized fusion rule is given by a weighted linear combination of $\{\hat{\bar{\mathbf{s}}}_f^{(j)}\}_j$, where the weights are proportional to the corresponding channel gains $\mathbf{h}_f^{(ij)}$. Mathematically, $\hat{\bar{\mathbf{s}}}_f^{(\text{FC})} = \alpha \sum_j \mathbf{H}_f^{(j)} \hat{\bar{\mathbf{s}}}_f^{(j)}$, where α is a positive scaling factor. Spectrum hole detection $\hat{\mathbf{d}}$ is then performed based on the fused estimate $\{\hat{\bar{\mathbf{s}}}_f^{(\text{FC})}\}$ according to (11).

To design a distributed fusion algorithm in the absence of the fusion center, we adopt the consensus-averaging algorithm and modify it to perform weighted averaging. The local vector $\mathbf{c}_j(0)$ in (13) is now replaced by the weighted local state estimate $\mathbf{c}_j(0) = \mathbf{H}_f^{(j)} \hat{\bar{\mathbf{s}}}_f^{(j)}$. The rest procedure of distributed fusion follows straightforwardly. Upon convergence, each CR reaches the globally optimal fused state estimate (subject to scaling):

$$\lim_{t \to \infty} \mathbf{c}_j(t) = \frac{1}{J} \sum_k \mathbf{H}_f^{(k)} \hat{\bar{\mathbf{s}}}_f^{(k)} = \frac{1}{\alpha J} \hat{\bar{\mathbf{s}}}_f^{(\text{FC})}, \qquad j = 1, \dots, J. \tag{18}$$

IV. Performance Evaluation

This section aims to identify those performance-determining parameters in the network and in our algorithm design, and also provide a comprehensive evaluation of their impacts on several key performance metrics of the CR network.

A. Simulation Setup and Performance Metrics

We consider a wide band of interest that is partitioned into $M = 30$ equal-bandwidth subchannels. Primary users randomly occupy some of the subchannels, with an average spectrum occupancy ratio of 20%. The wideband channel experiences frequency-selective fading, which is modeled as a multipath channel with N_p time-delayed taps and independent Rayleigh fading gains on these taps.

The signal to noise ratio (SNR) is defined as the energy of the wideband signal over the entire spectrum of interest, scaled by the power density of the ambient noise. The compression ratio K/M reflects the reduced number of samples used with reference to the number M needed in full-rate Nyquist sampling. When a quantizer is employed, each sample in $\mathbf{x}_t$ is quantized into ENOB bits, and the impact of quantization errors is compared to the case of unquantized samples.

For the spectral hole detection problem, critical performance metrics of interest include the sensing time, the probability of detection P_d and the probability of false alarms P_{fa}, which we average over all subchannels as follows:

$$P_d = \mathrm{E} \left\{ \frac{\mathbf{d}^T(\mathbf{d} \neq \hat{\mathbf{d}})}{\mathbf{1}^T \mathbf{d}} \right\}, \qquad P_{fa} = \mathrm{E} \left\{ \frac{(\mathbf{1} - \mathbf{d})^T(\mathbf{d} \neq \hat{\mathbf{d}})}{M - \mathbf{1}^T \mathbf{d}} \right\}$$

where $\mathbf{1}$ denotes the all-one vector. We choose the decision threshold η in local detection (10) and (11) such that the probability of false alarms in a single-CR case is fixed at 0.1.
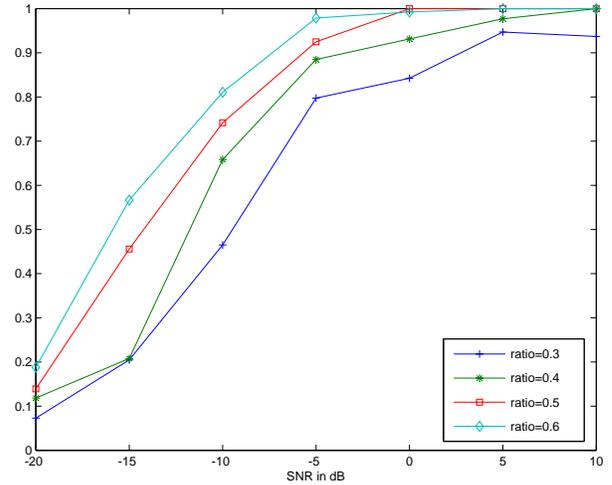

Fig. 1. Probability of detection at various compression ratios K/M.

The choice of the spectrum sensing scheme reflects the tradeoffs between protecting primary users and offering transmission opportunities to secondary users. When a missed detection occurs, secondary users might take an incorrect spectrum opportunity and cause collisions to the primary transmissions; hence, a high P_d is crucial in protecting the primary users. Meanwhile, the secondary CR network gains spectrum access opportunities when the subchannels are idle and no false alarms are declared; hence, a metric for the sum opportunistic throughput capacity of the secondary network is given by $1 - P_{fa}$. A longer sensing time may improve the receiver operating characteristics (ROC) in terms of P_d versus P_{fa}, but it also delays the response time of the network. Specifically, during the sensing time, standby secondary CRs cannot take new opportunities, and an active CR might not be prompt enough to evacuate the channel if a primary user turns on.

When spectral shape estimation is concerned, the perform metric is the normalized root mean-square estimation error (MSE) measured by $\mathrm{E}\{\|\hat{\bar{\mathbf{s}}}_f - \bar{\mathbf{s}}_f\|_2 / \|\bar{\mathbf{s}}_f\|_2\}$.

B. Compression-related Performance Evaluation

1) Effect of Compression and its Robustness to Noise: One of the important features in our spectrum sensing design is the use of compressive sampling to alleviate the sampling burden and energy consumption of CRs in the wideband regime. Compression, on the other hand, incurs performance degradation, especially in the presence of ambient noise and channel fading.

Fig. 1 depicts the probability of detection P_d versus SNR, for various compression ratios K/M. Encouragingly, the compressed sensing scheme is able to detect the primary users at very strong compression, when the number of samples used (K) is much less than that required by Nyquist rate sampling (M). When K/M increases, the robustness to noise improved, so does the detection performance.

2) Effect of Finite-bit Quantization: Fig. 2 depicts the estimation performance versus SNR for various values of the effective number of quantization bits (ENOB), when the

compression is strong (33%) and medium (50%) respectively. Apparently, there is a critical number of quantization bits ENOBo, below which the compressed sensing algorithm would not function. In our test, this critical point is ENOBo=4 bits/sample, which is within the suitable range for practical ADCs to reach high-speed processing. Meanwhile, the performance improvement by increasing the ENOB exhibits diminishing returns, and saturates when ENOB is beyond 8 to 16 bits. In addition, the following observations can be made:

- When SNR is small, the compression ratio is the determining factor of the estimation performance. The MSE flattens out and does not improve as ENOB increases.
- When SNR is large, increasing ENOB improves the estimation performance.
- When compression is stronger (i.e. smaller K/M), more SNR regions become insensitive to quantization bits.

These observations allude that the degrading effect of quantization errors is more pronounced in higher-performance regions, but become less obvious when the estimation performance has already been degraded by stronger compression or smaller SNR.

3) Effect of Compression Samplers: The sensing performance is also affected by the random samplers $\mathbf{S}_c$ adopted in (7). Well-known random samplers include the random Gaussian sampler, random Bernoulli samplers [6], and non-uniform periodic samplers [7]. The choice of the random samplers reflects the tradeoff in recovery accuracy, storage and sampling energy consumption. The quality of the random samplers depends on the distribution of the input data to be sampled, and can be evaluated by the *uniform uncertainty principle* (UUP) [6] and the *mutual coherence* measure defined by [15]

$$MC(\mathbf{S}_c) = \max_{1 \leq m, k \leq M, m \neq k} \left| \left[\mathbf{S}_c \mathbf{S}_c^T \right]_{(m,k)} \right|.$$

4) Effect of the Reconstruction Algorithms: A number of recovery methods have been developed to retrieve highly sparse signals. These algorithms offer different tradeoffs in recovery complexity, performance, robustness to noise, as well as the allowable compression ratios given the sparsity of the original signal. Some of the recovery techniques are simple to implement, but may require a large number of samples in order to reach a desired performance level. In this case, the energy saving in computation is offset by the sampling cost. It is thus important to properly balance between performance and complexity in order to improve the overall recovery efficiency.

Fig. 3 compares four representative algorithms: Basis Pursuit in (9) [18], orthogonal matching pursuit (OMP) and tree-based OMP (TOMP) [16], and re-weighted l_1 minimization [17]. The OMP and TOMP are greedy search algorithms that are fast in computation, and thus take small running time to converge to the steady state solutions. However, their recovery accuracy is not as good, and in general need to collect more measurements in order to reach a comparable MSE performance as BP and reweighted l_1 algorithms. The
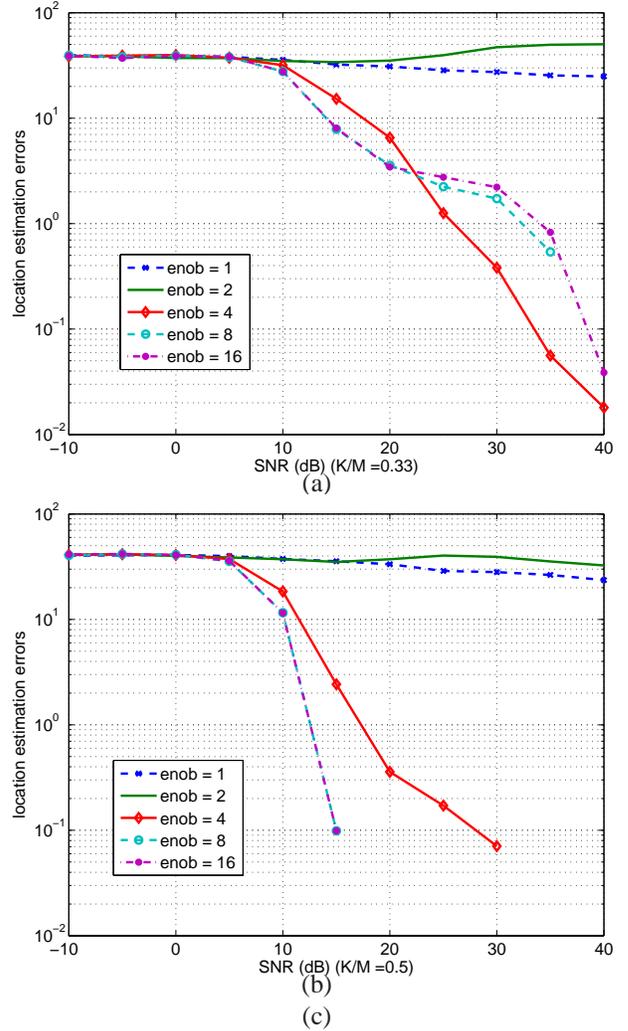

Fig. 2. spectral shape estimation MSE for various quantization bits: (a) $K/M = 33\%$, (b) $K/M = 50\%$.

BP and re-weighted l_1 algorithms, on the other hand, represent more accurate solutions at the expense of heavier computation burdens. It is of interest to investigate the overall energy consumption of these algorithms, taking into account of both computation and signal acquisition costs.

C. Fusion-related Performance Evaluation

1) Diversity Gain enabled by CR Collaboration: Wireless fading considerably degrades the probability of detection P_d for point-to-point links. When a primary user transmission is not detected, the CRs may cause undesired interference to the primary user. CR collaboration enables diversity gain, which we quantify via simulations in Fig. 4. The simple detection state fusion scheme is used for illustration, in the absence of any channel knowledge. It can be shown that the detection performance improves as the number of collaborating CRs J increases. By comparing the results for different compression ratio K/M, it is also observed that compression can offset some of the spatial diversity gain enabled by collaboration.

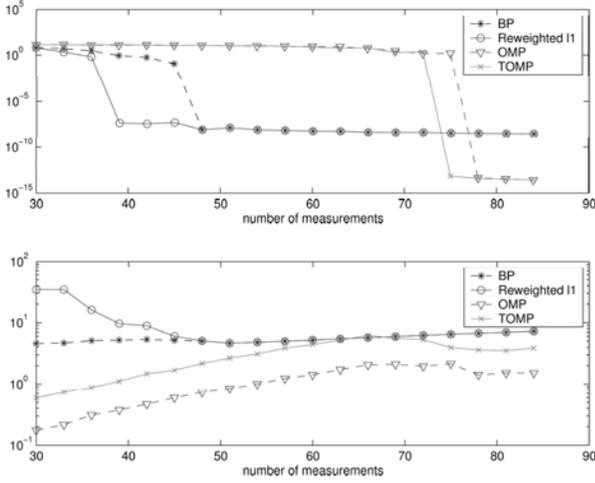Here, the decision threshold η in local CRs is chosen to

Fig. 3. Comparisons of sparse signal recovery methods: (upper) MSE vs. K, (lower) running time vs. K. The problem size M is fixed.

attain a fixed probability of false alarms at individual CRs. With collaboration, the probability of false alarms of the network is improved as J increases. Therefore, when the curves are plotted for a fixed P_{fa} after network collaboration, the performance gain of collaboration in terms of P_d is expected to be more notable.

It is possible to analyze the spatial diversity gain of user collaboration. For frequency flat fading channels, some preliminary analysis has been done for fixed CR topology in [9]. We anticipate that the diversity gain, when measured by the MSE performance at high SNR region, would equal to the number of collaborating CRs J that are spatially independent. Analysis on this issue will be investigated in future work.

2) Comparison of Different Distributed Fusion Types: In Section III-B, we have discussed several types of centralized and distributed fusion techniques. They can be categorized as: i) measurement fusion vs. state fusion, where the state can be the output of spectral hole detection vs. spectral shape estimation; ii) CSI-free vs. CSI-dependent; iii) consensus-based vs. primal-dual subgradient based distributed algorithms.

In general, measurement fusion can be more accurate than state fusion in the centralized fusion case, at the expense of higher communication overhead in terms of bandwidth and power consumed. On the other hand, distributed fusion schemes require information exchanges among neighboring CRs in an iterative manner. When the communication bandwidth for information exchanges is constrained, or when the running time for iterations is limited in real-time processing, there might not be adequate resources for a measurement fusion algorithm to reach its steady state, thus resulting in performance loss. Under practical resource constraints, the spectrum sensing performance of measurement fusion versus state fusion needs to be investigated in order to justify their implementation costs.

Both consensus-based algorithms and primal-dual subgradient algorithms can reach the globally optimal fusion perfor-
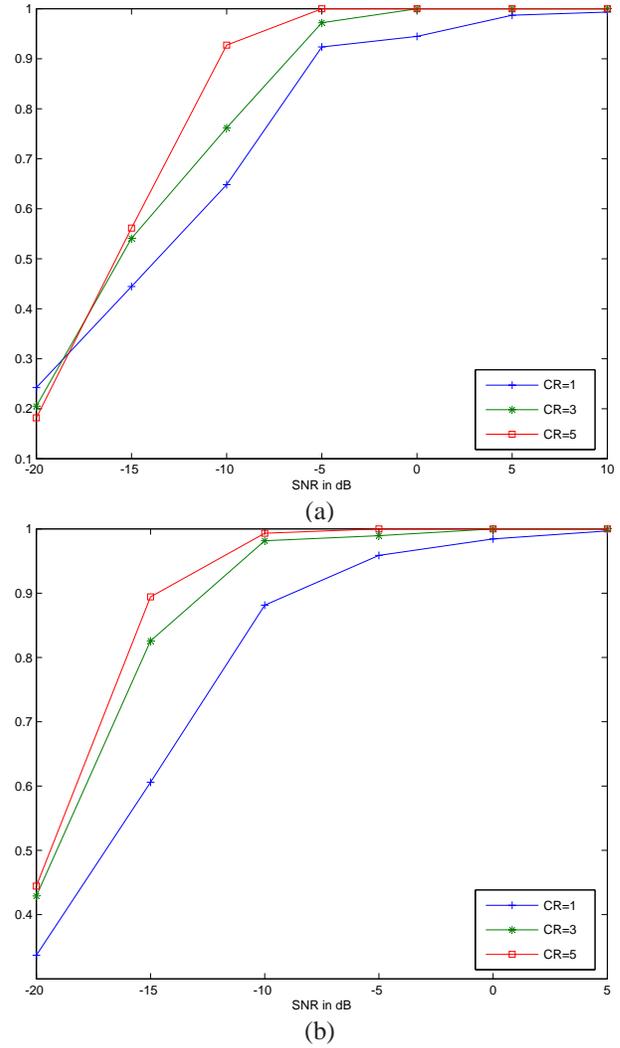

(a)

(b)

Fig. 4. Probability of detection for various number of collaborating CRs: (a) moderate compression $K/M = 60\%$, (b) little compression $K/M = 90\%$.

mance at steady state, using only distributed computation and local communications among one-hop neighbors. On the other hand, their convergence rates may vary depending on different operating scenarios. There are other options of distributed fusion that have suboptimal performance compared with optimal centralized fusion, but may offer less complexity and faster convergence. It is of interest to compare our proposed distributed algorithms with other fusion options.

In terms of the sensing time, distributed collaborative sensing presents an intricate tradeoff. On one hand, the cooperative gain reduces the number of samples required to attain the desired P_d, thus reducing the sensing period of individual participating CRs. On the other hand, the iterative procedures used in distributed sensing algorithms consume processing time for information exchanges and fusion convergence, which prolong the overall sensing time. As the number of participating CRs J increases, both the cooperative gain and the cooperative costs increase. This tradeoff suggests us to judiciously select the number of participating CRs.

3) Value of Channel Knowledge: Acquisition of the CSI requires channel estimation, which may be costly or even difficult in multiuser wireless systems. Hence, the cost of channel estimation needs to be justified by the sensing performance gain it offers in collaborative fusion. Furthermore, the sensitivity of the fusion decisions to channel estimation errors need to be quantified in order to properly allocate system resources to the channel estimation task.

V. Summary

This paper discusses distributed fusion and compressive sampling techniques for wideband spectrum sensing in cognitive networks. Multiple performance metrics are identified to evaluate the intricate tradeoffs between guaranteeing the quality-of-service of primary systems and enhancing the transmission opportunities and throughput of secondary CR users. These performance metrics provide important guidelines on balancing the desired cooperative gain against the cooperation costs, which boils down to proper selections of system design parameters including the number of collaborative CRs, the signal acquisition components (compression ratio and quantization bits in ADCs), the acquisition and use of channel knowledge, the fusion mechanisms and distributed algorithms, and the information propagation and transport protocols in multi-hop networks.

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