Yonina C. Eldar, Zhi-Quan Luo,
Wing-Kin Ma, Daniel P. Palomar,
and Nicholas D. Sidiropoulos

Convex Optimization in Signal Processing

In recent years, we have witnessed technical breakthroughs in a wide variety of topics where the key to success is the use of convex optimization. In fact, convex optimization has now emerged as a major signal processing tool that has made a significant impact on numerous problems previously considered intractable. Considering the foundational nature and potential impact of convex optimization in signal processing, we have put together this special issue that aims to provide tutorials of convex optimization techniques (including available software) and various successful signal processing applications. Our goal is not only to contribute to the diffusion of recent developments in this research area within the signal processing community, but also to spur further advances in and applications of convex optimization for signal processing.

There was an enthusiastic response to our initial call for papers. More than 40 white papers were received, representing a broad mix of classical and contemporary signal processing topics where convex optimization has made a significant impact. Aiming for balanced coverage while respecting page limitations, we were only able to accommodate eight articles in this special issue. Collectively, these eight articles introduce convex optimization techniques, give insights into how convex optimization can make a difference, and showcase some notable successes.

The first three articles in this special issue introduce various convex optimization methodologies that are especially useful in signal processing applications. The article by Luo et al. introduces semidefinite programming and the semidefinite relaxation (SDR) technique, and highlights their uses in a wide variety of problems, ranging from multiple-input, multiple-output (MIMO) detection, to transmit shimming in magnetic resonance imaging, and on to sensor network localization. This article also gives an overview of key theoretical results on SDR from a signal processing perspective.

The article by Scutari et al. introduces game theoretic techniques for signal processing applications. In recent years, game theory has made headway in distributed signal processing, communications, and networking, e.g., to address fundamental issues in peer-to-peer wireless and emerging cognitive radio networks. This article starts at the confluence of convex optimization and game theory and takes us on to a fascinating (yet crisp and rigorous) tour-de-force of variational inequality theory and methods—a powerful framework that allows tackling not only classical optimization and equilibrium problems, but also pertinent extensions that arise in the aforementioned application areas and beyond.

The article by Mattingley and Boyd reviews the present state of the art in real-time convex optimization-based signal processing. The authors describe how to leverage the enormous speed increases in computing hardware, advanced programming, and modern algorithms to solve moderately sized convex optimization problems in microsecond- or millisecond-time scales, and with strict deadlines.

The next five articles in this special issue showcase some notable successes of convex optimization in signal processing applications. The article by Gershman et al. presents a comprehensive tutorial survey on the use of convex optimization techniques to solve a wide variety of beamforming problems in wireless communications. The review focuses on three main classes of beamforming: receive beamforming, transmit beamforming, and distributed beamforming for relay networks. The authors discuss the importance of robust designs based on different criteria, such as worst-case and probabilistic analysis, and they illustrate the importance of convex optimization techniques in solving the resulting problem formulations.

Sparse signal reconstruction is another important application of convex optimization. The currently predominant algorithmic approach for solving this type of problem is based on mixed one- and two-norm optimization, which yields a convex problem. The article by Zibulevsky and Elad offers a concise review of recent developments towards acceleration of an important class of problem-specific solutions known as iterative shrinkage algorithms (ITAs), which is significantly more efficient than the generic convex optimization solvers. Design of finite impulse response (FIR) filters is one of the classic problems in digital signal processing. Effective filter design requires judicious tradeoffs between conflicting properties of the filter.

The article by Davidson presents FIR filter design from the perspective of convex optimization. It shows how this approach can enrich the art of designing FIR filters, through both the direct design of filters and the efficient computation of appropriate tradeoffs between conflicting design criteria. In addition, it illustrates, with some simple examples, how new user interfaces to general-purpose convex optimization software have placed these tools at our fingertips.

Dynamic spectrum access is an emerging area where convex optimization plays

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positions where the Reynolds number exceeds a critical value. The acoustic volume velocities can be obtained for the glottal excitation and for the mouth and nostril radiation. The output volume currents act through radiation impedances and encounter atmospheric pressure. The sound pressure in front of the speaker's mouth is determined as the superposition of the radiation of pistons (mouth and nostril) set in a spherical baffle (the head). All controls are related to the dynamic physiology. Initial implementations are exceedingly primitive. But, by appealing to an articulatory domain for parameterization, we are able to focus on the speech-producing mechanism, rather than on the sound output itself.

Preliminary experiments with such a "mimic" suggest that information rates in the range of 1,000–2,000 b/s may preserve quality and personal characteristics. But, so far deep studies of the fluid flow approach have not been made (hampered in part by the fact that even a "stripped down" model runs over 100 times real time on a mainframe computer, mainly to compute solutions of the Navier-Stokes fluid flow equations).

Because this discussion has focused primarily on applied commercial voice transmission, it has not touched on a variety of related topics that partake of common fundamental components. Automatic speech recognition (What was said?) and talker verification (Who said it?) are cases in point, and are being brought into commercial telecom services. The continuing interests in formant analysis/synthesis seek automatic extraction of the time-varying eigen frequencies of the vocal system. These contribute the prominent maxima in the short-time amplitude spectrum and, perceptually, promise even more parsimonious description of speech information. All these factors underlie the transmission techniques emphasized here.

EPILOGUE

So, practical speech compression has advanced a distance, but likely has not reached the limit of efficiency. Implied is even the possibility for obtaining fundamental speech coding parameters at the neural level. That is, just *think* what you want to say! And, there are ambitious studies commencing in this sector. The future is certain to prove interesting!

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AUTHOR

James L. Flanagan (jlf@caip.rutgers.edu) is a Professor Emeritus at Rutgers University.

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an important role in system design. The article by Zhang et al. gives a broad overview of the spectrum sharing approach for cognitive radio networks and describes in detail various convex optimization formulations and solutions for the design of cognitive radio systems.

Finally, the article by Jiang and Li focuses on the applications of convex optimization to discriminative training in speech and language processing. For many widely used statistical models, discriminative training for speech processing normally leads to nonconvex optimization problems. This article shows how convex relaxation techniques (such as linear programming relaxation or SDR) can be used in this context.

In closing, we would like to thank all of our colleagues who have contributed to this special issue, including the authors of submitted papers. We also thank the reviewers for their quality work, and the editorial board for their support, without which this special issue would not have been possible.

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