9.3-4

Approximations of Multiobjective Optimization for Dynamic Spectrum Allocation in Wireless Sensor Networks

Sang-Seon Byun[1] and Ilangko Balasingham[1, 2]

[1]Dept. Electronics and Telecommunications
Norwegian University of Science and Technology
[2]Interventional Center
Oslo University Hospital, Rikshospitalet

Abstract—**This paper considers the problem of the centralized spectrum allocation in wireless sensor networks towards the following goals: (i) *maximizing fairness,* (ii) *maximizing spectrum utilization,* (iii) *reflecting the priority among sensor data, and* (iv) *avoiding unnecessary spectrum handoff.* We cast this problem into a *biobjective mixed integer nonconvex nonlinear programming* that is absolutely intractable to solve at least globally without any aid of conversion and approximation. We tackle this intractability with *convexification*, *scalarization*, and *rounding* method that yield good approximate integer solutions.**

I. INTRODUCTION

In this paper, we consider the deployment of dynamic spectrum allocation (DSA) to wireless sensor networks (WSNs) in order to realize cognitive radio [1] WSNs, where sensors (unlicensed users) are allowed to access to spectrum resources that are not occupied by licensed users, with the following principles:

- High utilization and fair allocation of idle spectrum bands: Scarce spectrum resources should be allocated as fairly as possible and maximally utilized; in addition, it is necessary to prioritize the transmissions. This can be achieved by maximizing *proportional fairness* with demanding weights [2].
- Avoiding unnecessary spectrum handoff: In cognitive radio, *Spectrum handoff* occurs 1) when licensed user is detected or 2) current spectrum condition becomes worse; unlicensed users move to the “best” matched available spectrum band. According to [3], it is shown that frequency or timing synchronization consume a certain amount of power, which means that unnecessary spectrum handoff should be eliminated.
- Centralized spectrum allocation: In centralized algorithms for DSA, a centralized authority (e.g., base station or dedicated coordinator) detects and identifies spectrum opportunities and allocates the identified spectra to sensors [4]. On the other hand, in distributed algorithms, each sensor itself should detect the spectrum opportunities and determine an optimal strategy to maximize its benefits, which is infeasible since it requires full functionalities of cognitive radio in each sensor. Thus, in a moderate size of WSN where the sensors are distributed within a cell or segment boundary (e.g., a home automation system in a house), the centralized algorithm is preferred to the distributed scheme.

We formulate the problem with the abovementioned principles into a biobjective (and bicriteria) mixed integer nonlinear nonconvex programming (BO-MINLP) that is, however, known as at least globally intractable without any aid of modification or approximation.

Our approaches to tackle this intractability are summarized as follows: (i) Converting the problem into a quasi-equivalent convex form through *arithmetic-geometric mean approximation* (AGMA) and logarithmic change of decision variables [5]; (ii) Relaxing integer constraints, i.e., *NLP-relaxation*; (iii) Collapsing the biobjective problem into a single objective one using *scalarization* based on *weighted Chevyshev norm* [6] by which we can maintain the convexity and achieve NLP-relaxed solutions that satisfies weak Pareto optimality; (iv) Rounding the NLP-relaxed solutions to obtain approximate integer solutions.

II. PROBLEM FORMULATIONS AND APPROACHES

We assume that the available spectrum resources are expressed as a number of *spectrum units*, and the bandwidth of each spectrum unit is fixed.

Parameters

- V: Set of sensors that request spectrum resources
- S: Set of idle spectrum units
- L_{is}(binary): It means that currently sensor i is synchronized with the spectrum unit s.
- w_i: The weight of sensor i. The higher weight (priority) a sensor has, the more spectrum bands will be allocated
- P_i^{max}: Maximal transmission power available at sensor i.
- $INSR_i^{max}$: $1/SINR_i^{min}$ where $SINR_i^{min}$ is the minimal $SINR$ demanded by sensor i.

Decision variables

- x_{is}(binary): It indicates that sensor i occupies the spectrum unit s
- p_{is}: Transmission power for sensor i in the spectrum unit s.
- $INSR_{is}$: Defined as

$$INSR_{is} = \frac{\sum_{j \in V \setminus \{i\}} x_{js} p_{js} G_{jk}^{s} + \sigma^2}{p_{is} G_{ik}^{s}} . \quad (1)$$

Then the problem is formulated in BO-MINLP as

Objectives

$$\text{Maximize } f_1 = \sum_{i \in V} w_i \ln\left(\sum_{s \in S} x_{is}\right), \quad (2)$$

$$\text{Maximize } f_2 = \sum_{s \in S} \sum_{i \in V} L_{is} x_{is} , \quad (3)$$

subject to

This research work was funded by the SAMPOS project (176875/S10) of the Research Council of Norway.

978-1-4244-4316-1/10/$25.00 ©2010 IEEE

$$x_{is} INSR_{is} \le INSR_i^{\max} \text{ for all } i \in V \text{ and } s \in S \,, \tag{4}$$

$$\sum_{s \in S} x_{is} p_{is} \le P_i^{\max} \text{ for all } i \in V \,. \tag{5}$$

If we let β_{is} = ln(x_{is}) and γ_{is} = ln(p_{is}) for all $i \in V$ and $s \in S$ where $-\infty \le \beta_{is} \le 0$ and $-\infty \le \gamma_{is} \le \ln(P_i^{max})$, then f_1 is convexified by an equivalent log-sum-exp function form and AGMA as

$$\text{Maximize } Cv(f_1^*) = \sum_{i \in V} w_i \ln\left(\prod_{s \in S} \left(\frac{e^{\beta_{is}}}{\alpha_{is}} \right)^{\alpha_{is}} \right) \tag{6}$$

where

$$\alpha_{is} = \frac{e^{\beta_{is}}}{\sum_{s \in S} e^{\beta_{is}}} \text{, for all } i \in V \text{ and } s \in S \,, \tag{7}$$

and α_{is} is converged by *condensation algorithm* [5]. Similarly, f_2 is converted into an equivalent convex form:

$$\text{Minimize } Cv(f_2) = \sum_{s \in S} \sum_{i \in V} L_{is} e^{-\beta_{is}} \,. \tag{8}$$

Then these two convexified objective functions are collapsed into a single function using weighted Chebyshev norm. We denote $Cv(f_1^*)$ and $Cv(f_2)$ as g_1 and g_2 respectively. Consequently, the original problem is fully convexified as

Objective

$$\text{Minimize } z, \tag{9}$$

subject to

$$\ln\left(\left(\sum_{j \in V \setminus \{i\}} e^{(\beta_{is} + \beta_{js} + \gamma_{js} - \gamma_{is})} G_{jk}^s + e^{(\beta_{is} - \gamma_{is})} \sigma^2 \right) \left(G_{ik}^s \right)^{-1} \left(INSR_i^{\max} \right)^{-1} \right) \le 0 \,, \tag{10}$$

$$\ln\left(\sum_{s \in S} e^{(\beta_{is} + \gamma_{is})} \left(P_i^{\max} \right)^{-1} \right) \le 0 \,, \tag{11}$$

$$\delta_1 \times \left(\frac{ -\sum_{i \in V} w_i \ln\left(\prod_{s \in S} \left(\frac{e^{\beta_{is}}}{\alpha_{is}} \right)^{\alpha_{is}} \right) - g_1^* }{ g_{1w} - g_1^* } \right) \le z \,, \tag{12}$$

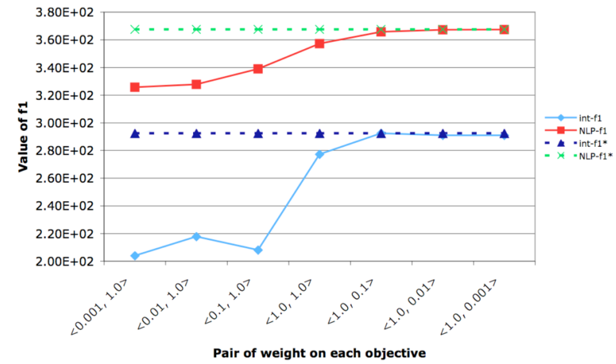
$$\delta_2 \times \left(\frac{ \sum_{s \in S} \sum_{i \in V} L_{is} e^{-\beta_{is}} - g_2^* }{ g_{2w} - g_2^* } \right) \le z \,. \tag{13}$$

where $\delta_m > 0$ is the weight of function g_m, g^*_m is the optimal value when only g_m is minimized, and g_{mw} is the value of g_m only $g_{k(\neq m)}$ is minimized. Therefore, prior to solving the convexified biobjective problem, we should compute the optimal value of each convexified f_1 and f_2; the global solution of all these problems can be computed using a general NLP solving method such as *interior point method* [7].

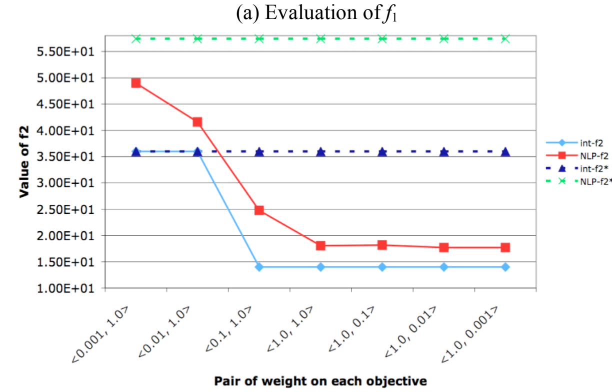
Regardless of the convexity and linearity, a problem with integer constraints is in general very hard to solve. In this paper we apply a simple rounding method introduced in [5] for finding approximate integer solutions. We observe that the rounding method works well for our problems since all the objective functions and constraints involve exponential function.

III. NUMERICAL EXPERIMENTS

For the experiments, we consider a simple sensor topology of 15 sensors in a 20m × 20m rectangular area, and let $|S|$ = 70, σ^2=0.5μW, P_i^{max} = 10W, $INSR_i^{max}$ = -10dB, and $G_{ij} = d_{ij}^{-\upsilon}$ where d_{ij} is the distance between sensor i and j, and υ = 4.



(a) Evaluation of f_1



(b) Evaluation of f_2

Fig. 1. Evaluation of f1 and f2 solved by the scalarization method. The label of X-axis on each graph corresponds to the tuple <δ_1, δ_2>.

We evaluate the solutions determined by the scalarization method with varying the weight on each objective function. The graphs in Figure 1 illustrate that the integer solutions are quite close to the NLP solutions with the factor of less than 1, and the improvement in one objective function penalizes the other, and balanced solutions can be found by tuning the weights.

REFERENCES

[1] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Area. Commun.*, vol. 23, no. 2, Feb. 2005.

[2] M. Pioro and D. Medhi, *Routing, flow, and capacity design in communication and computer networks*, Morgan Kaufmann, 2004.

[3] E. Coersmeier et al., "Combining cognitive radio and software radio approach for low complexity receiver architecture," *Proc. Inter. Sym. Advanced Radio Tech.*, Feb. 2007.

[4] I. Katzela and M. Naghshineh, "Channel assignment schemes for cellular mobile telecommunication systems: A comprehensive survey," *IEEE Personal Commun.*, vol 3, pp. 10-31, June 1996.

[5] S. Boyd et al., "A tutorial on geometric programming," *Optimization and Engineering*, vol. 8, no. 1, pp. 67-127, Mar. 2007.

[6] G. Eichfelder, *Adaptive scalarization methods in multiobjective optimization*, Springer, 2008.

[7] A. Wachter and L. T. Biegler, "On the implementation of a primal-dual interior point filter line search algorithm for large-scale nonlinear programming," *Mathematical Programming*, vol. 106, no. 1, pp. 25-57, Jan. 2006.