

LETTER

An Efficient Rate and Power Allocation Algorithm for Multiuser OFDM Systems*

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SUMMARY In this paper, we propose an efficient rate and power allocation scheme for multiuser OFDM systems to minimize the total transmit power under the given QoS requirements. We deduce the optimal solution of transmit power minimization problem and develop a suboptimal algorithm with low complexity based on the theoretical analysis. Because of the avoidance of iterative procedure, it is less complex than the existing schemes. The simulation results show that our proposal outperforms the existing schemes and it is very close to the optimal solution.

key words: OFDM, multiaccess communication, adaptive resource allocation

1. Introduction

Orthogonal frequency-division multiplexing (OFDM) technology has been proposed for the next generation wireless communication systems due to its capability in combating frequency selective fading and intersymbol interference (ISI) [1]. Since the channel fading for different users are independent, it is unlikely that a subcarrier will be in deep fade for all the users. By adaptively allocating subcarriers and selecting modulation levels, multiuser OFDM can take advantage of channel diversity among users in different locations. Hence spectral efficiency can be improved, or equivalently, transmit power can be reduced [2].

Recently there are have been numerous papers studying the resource allocation in multiuser OFDM systems. Reference [3] developed a transmit power adaptation method that maximizes the total data rate from the view of information theory. The authors formulated the data rate maximization problem by allowing the multiple users to share the same subcarrier. The conclusion indicates that the subcarrier should be assigned to only one user who has the best channel gain. However, it did not investigate the bit loading issue. Reference [4] proposed an optimal, computationally efficient, integer-bit and power allocation algorithm to maximize the throughput under the bit error rate constraints. However, it is effective only if the channel conditions change very slowly. In [5], dynamic subcarrier and power allocation was performed to maximize the minimum

capacity of all the users under the total transmit power constraints. This algorithm is from the view of fairness, but the system throughput is not optimal and the data rate allocation is not flexible. In [6], the authors proposed a dynamic allocation scheme to minimize the overall transmit power. Although this scheme can achieve very good performance, it is extremely complicated and not suitable for the real time applications.

In this letter, we analyze the resource allocation of downlink in multiuser OFDM systems with adaptive modulation, where the subcarrier, data rate and power are assigned to the users based on the instantaneous channel conditions. Our objective is to minimize the overall transmit power by allocating the rate and the power level of users based on the QoS requirements. We formulate the multiuser rate and power allocation problem and propose a suboptimal algorithm based on the theoretical analysis to perform the resource allocation efficiently. Because of the avoidance of iterative procedure, it is less complex than the existing schemes.

2. Rate and Power Allocation

2.1 Theoretical Analysis

Assume that the multiuser OFDM system has N users sharing M subcarriers with the QoS requirement C_t . Let $\mathbf{P} = \{p_1, p_2, \dots, p_M\}$ denotes the power allocation to each subcarriers and $\mathbf{K} = \{k_1, k_2, \dots, k_M\}$ denotes the subcarrier allocation to each users. Here, $k_j = i$ means that the j th subcarrier is assigned to the i th user. Also, denote by $h_{i,j}$ the channel gain of the j th subcarrier as seen by the i th user. Furthermore, denote by c_j the number of bits assigned to the j th subcarrier and $\mathbf{C} = \{c_1, c_2, \dots, c_M\}$ denotes a rate allocation scheme. Mathematically, the resource allocation problem in multiuser OFDM systems can be formulated as an optimization model for minimization of the total transmit power

$$\begin{aligned} \min_{\mathbf{C}, \mathbf{P}, \mathbf{K}} \quad & \sum_{j=1}^M p_j \\ \text{s.t.} \quad & \sum_{j=1}^M c_j \geq C_t, \end{aligned} \quad (1)$$

In order to maintain the required QoS at the receiver, the transmit power allocated to the j th subcarrier by the i th

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user must satisfy $p_j = \frac{f(c_j)}{h_{k_j,j}^2}$. Here, denote by $f(c_j)$ the required power in a subcarrier for reliable reception of c_j information bits/symbol when the channel gain is equal to unity. In order to make the problem tractable, we further require that $f(c_j)$ is a convex and increasing function. Almost all popular coding and modulation schemes satisfy this condition.

It is important to note that our problem is formulated to minimize the overall transmit power for the given QoS requirements. The same solution can be applied to maximize the throughput of the system for the given overall transmit power [6].

The goal of the power and rate allocation algorithm is then to find the best assignment of c_j , p_j and k_j so that the total transmit power over all subcarriers is minimized. To make the problem tractable, we relax the requirement $c_j \in \{0, 1, 2, \dots\}$ to allow c_j to be a real number. The optimization problem in (1) can be formulated as an unconstrained optimization problem through the Lagrange multiplier λ

$$L(\mathbf{C}, \lambda) = \sum_{j=1}^M \frac{f(c_j)}{h_{k_j,j}^2} - \lambda \left(\sum_{j=1}^M c_j - C_t \right). \quad (2)$$

If we find a specific λ such that $\sum_{j=1}^M c_j = C_t$, the optimal solution to the rate and power allocation algorithm for multiuser OFDM can be obtained. Taking the first-order derivative in (2) with respect to c_j , we obtain,

$$c_j = [f'^{-1}(\lambda h_{k_j,j}^2)]^+, \quad (3)$$

where $[x]^+ \triangleq \max\{x, 0\}$ since the assigned rates have to be non-negative and $f^{-1}(x)$ is the inverse function of $f(x)$.

Consider a system that employs M-ary quadrature amplitude modulation (MQAM), the required power for supporting c_j bits/symbol at the given BER P_e is

$$f(c_j) = \frac{N_0}{3} \left[Q^{-1} \left(\frac{P_e}{4} \right) \right]^2 (2^{c_j} - 1), \quad (4)$$

where N_0 is the single-sided power spectral density level and

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-t^2/2) dt. \quad (5)$$

Let $\Phi \triangleq \frac{N_0}{3} [Q^{-1}(\frac{P_e}{4})]^2$, we get $f'(c_j) = \Phi 2^{c_j}$ and $f'^{-1}(y) = \log_2(\frac{y}{\Phi})$. From (3), we have

$$c_j = \left\lceil \log_2 \left(\frac{\lambda h_{k_j,j}^2}{\Phi} \right) \right\rceil. \quad (6)$$

Since λ satisfies $\sum_{j=1}^M c_j = C_t$, we can get the expression of λ . From (6), we have

$$c_j = \frac{1}{M} \left\lceil C_t + \log_2 \left(\prod_{l=1}^M \frac{h_{k_j,j}^2}{h_{k_l,l}^2} \right) \right\rceil. \quad (7)$$

Taking the minimum power $p_j = \frac{f(c_j)}{h_{k_j,j}^2}$, the transmit power should be allocated to the j th subcarrier as

$$p_j = \frac{\Phi}{\ln 2} \left[2^{\frac{C_t}{M}} \left(\prod_{l=1}^M h_{k_l,l}^2 \right)^{-\frac{1}{M}} - h_{k_j,j}^{-2} \right]^+. \quad (8)$$

This solution gives a lower bound to the total transmit power. However, we cannot use the results in (1) immediately because some c_j may not be integer, which is not consistent with practise.

To solve this problem, we propose a suboptimal scheme where the rate and power allocation follows essentially the solution to the lower bound in (1).

2.2 Suboptimal Allocation Scheme

According to the above analysis, we develop a suboptimal rate and power allocation algorithm where the subcarrier is assigned to the user with the best channel gain, and then the data rate on each subcarrier follows essentially the solution to the lower bound in (1). Since the data rate must be integer, we modify c_j for the optimization problem in (1) to c_j^* , where c_j is rounded to c_j^* . Then quantizing c_j may not satisfy the QoS requirements in (1). Since the $f(c_j)$ is a convex and increasing function (i.e., $f(c+1) - f(c)$ is increasing in c), to deal with the part $\sum_{j=1}^M c_j \neq C_t$, we increase or decrease the users' rate with the maximum or minimum c_j^* respectively. We denote the total transmit power obtained using this suboptimal scheme by P_T . It is easy to see that $\underline{P}_T \leq P_T^* \leq P_T$, where \underline{P}_T is the minimum power in the problem (1), and P_T^* is the minimum power achieved by optimal algorithm without the relaxation. This result can also be supported by simulation results later, where our scheme achieves almost the same performance as the optimal one.

Let $L = \sum_{l=1}^M \log_2(h_{k_l,l}^2)$. Then the basic structure of the suboptimal algorithm can be described as follows:

Algorithm: Suboptimal Rate and Power Allocation Algorithm in Multiuser OFDM Systems

1. Determine the subcarrier allocation:
 $k_j = \arg \max_i h_{ij}^2$, $i = 1, 2, \dots, N$, $j = 1, 2, \dots, M$
2. Determine the data rate assignment:
 $c_j = \lceil \frac{1}{M}(C_t - L) + \log_2(h_{k_j,j}^2) \rceil^+$
3. Round c_j to c_j^* :
 $c_j^* = \begin{cases} \lfloor c_j \rfloor, & c_j < \lfloor c_j \rfloor + 0.5 \\ \lfloor c_j \rfloor + 1, & c_j \geq \lfloor c_j \rfloor + 0.5 \end{cases}$
4. **If** $(\sum_{j=1}^M c_j > C_t)$
then $\hat{j} = \arg \max_j c_j^*$; $c_j^* = c_j^* - 1$
- If** $(\sum_{j=1}^M c_j < C_t)$

then $\hat{j} = \arg \min_j c_j^*$; $c_j^* = c_j^* + 1$

5. Determine the power allocation to the j th subcarrier:

$$p_j^* = \frac{\Phi}{\ln 2} [2^{\frac{c_j^*}{M}} L^{\frac{-1}{M}} - h_{k,j}^{-2}]^+$$

3. Simulation Results

To evaluate the performance of the proposed scheme, we have simulated 1000 sets of five-path frequency selective Rayleigh fading channels with an exponential power delay profile. We consider a four user OFDM system with 256 subcarriers over a 5 MHz band. The single-sided power spectral density level N_0 is equal to unity. Assume that the average subcarrier channel gain $E|h_{i,j}|$ is equal to unity for all i and j .

For comparison purpose, we consider three other multiuser subcarrier allocation methods and two adaptive rate assignment schemes. To ensure a fair comparison, we use the optimal per-bit rate allocation (OBA) and equal rate allocation (EBA) algorithm for each user on the assigned subcarriers and the subcarrier allocation algorithms are OFDM-TDMA scheme, OFDM-FDMA scheme and OFDM Interleaved-FDMA scheme. Note that these three schemes are static, which predetermine subcarrier allocation independent of the channel gains of the users. The results of theoretical optimization solution and the suboptimal allocation algorithm are also displayed.

Both Fig. 1 and Fig. 2 show the average bit SNR needed to achieve a BER at $P_e = 10^{-4}$ for a four-user system versus the root mean square (RMS) delay spread (for definition, see for example [7]) for different multiuser OFDM schemes. We observe in Fig. 1 that the proposal is 4–5 dB better than the static schemes with OBA, which are in turn 6–10 dB better than that with EBA. We also find that when OBA is used, the OFDM interleaved-FDMA scheme and the OFDM-FDMA scheme have very similar performance, and both of them outperform the OFDM-TDMA scheme. Figure 2 indicates that the proposed suboptimal scheme is never more than 0.2 dB from the theoretical optimization solution and have very similar performance with the per-bit algorithm.

Figure 3 shows the average bit SNR needed to achieve the same BER versus the number of users when the RMS delay spread is 100 ns. We find that the saving in the required bit SNR achieved by the proposed scheme increases with the number of the users. This is because we make use of multiuser diversity. The more user, the more the effect of multiuser diversity can be achieved.

While these three figures above show the improvement in the required bit SNR, the results can perhaps be more easily understood using the more familiar bit SNR versus BER curves. The results are plotted in Fig. 4 for a four-user system with the RMS delay spread of 100 ns. We find that the proposal has at least 3–4 dB advantage over all the other schemes.

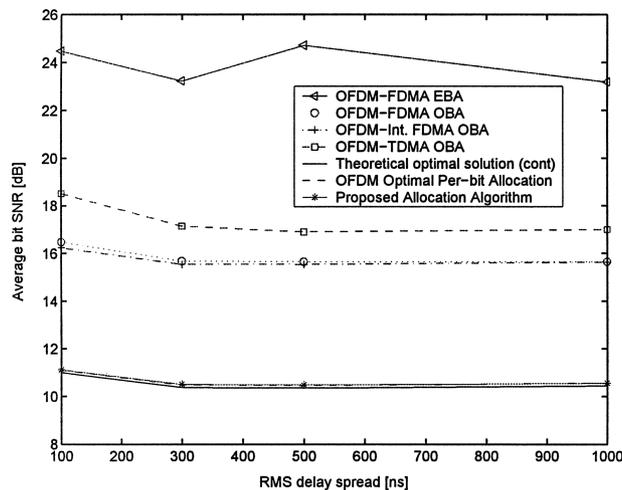


Fig. 1 Average bit SNR of different schemes in various RMS delay spread.

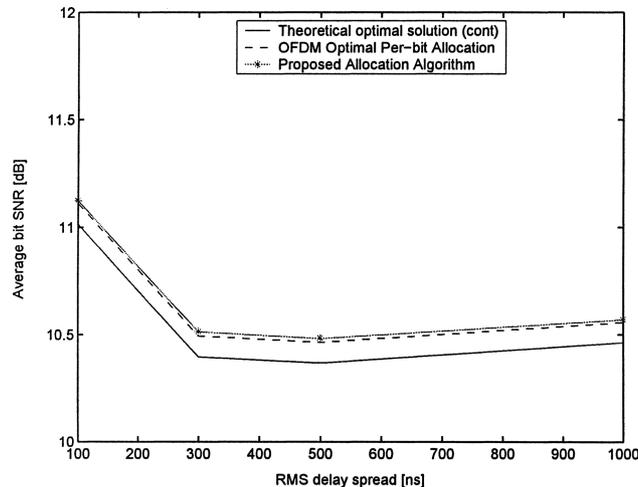


Fig. 2 Average bit SNR of different schemes in various RMS delay spread.

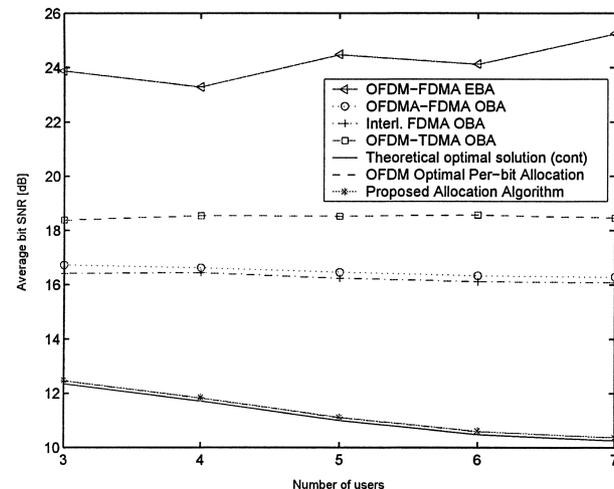


Fig. 3 Average bit SNR of different schemes versus number of users.

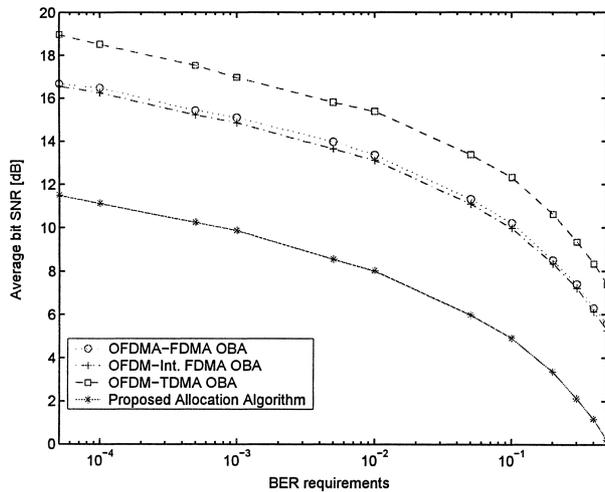


Fig. 4 Average bit SNR of different schemes in various bit error rate requirement.

4. Conclusions

Resource allocation in multiuser OFDM systems with adaptive modulation is essential for the system design. We solve this problem by formulating it into the optimization model, where we minimize the system total transmit power under the given QoS requirements. We deduce the optimal solution and propose a new efficient rate and power allocation algorithm, which is less complex than the existing schemes because of the avoidance of iterative procedure. Simulation results show that the proposal outperforms other static

TDMA or FDMA techniques and never more than 0.2 dB away from the optimal solution.

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References

- [1] B. Yang, K.B. Letaief, S. Cheng, and Z. Cao, "Channel estimation for OFDM transmission in multipath fading channels based on parametric channel modeling," *IEEE Trans. Commun.*, vol.49, no.3, pp.467–479, March 2001.
- [2] H. Minn and V.K. Bhargava, "An investigation into time-domain approach for OFDM channel estimation," *IEEE Trans. Broadcast.*, vol.46, no.4, pp.240–248, Dec. 2000.
- [3] J. Jang and K.B. Lee, "Transmit power adaptation for multiuser OFDM system," *IEEE J. Sel. Areas Commun.*, vol.21, no.2, pp.171–178, Feb. 2003.
- [4] B.S. Krongold, K. Ramchandran, and D.L. Jones, "Computationally efficient optimal power allocation algorithms for multicarrier communication systems," *IEEE Trans. Commun.*, vol.48, no.1, pp.23–27, Jan. 2000.
- [5] W. Rhee and J.M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," *Proc. IEEE Vehicular Technology Conf. (VTC 2000)*, vol.2, pp.1085–1089, May 2000.
- [6] C.Y. Wong, R.S. Cheng, K.B. Letaief, and R.D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Sel. Areas Commun.*, vol.17, no.10, pp.1747–1758, Oct. 1999.
- [7] T.S. Rappaport, *Wireless Communications: Principles and Practice*, Prentice-Hall, Englewood Cliffs, NJ, 1996.