A Survey on Sub-Nyquist Sampling

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ABSTRACT

This paper gives a survey of sub-Nyquist sampling. Sub-Nyquist sampking is of great interest in the environment that sampling by Nyquist rate is infeasible due to either hardware or software limitation. The survey summarizes a number of recent important researches on sub-Nyquist sampling (compressive sensing). In addition to the basics of sub-Nyquist sampleing, including the fundamental theory and the way it works, numerous compressive sensing recovery algorithms are also presented. Some selected compressive sensing applications are also shown in the survey.

Keywords

Sub-Nyquist Sampling, Compressed Sensing, Compressive Sampling.

1. INTRODUCTION

In 1949, Dr. Shannon published the paper "Communication in the Presence of Noise," which establishes the foundation of information theory [1-2]. The sampling theorem given in the paper is as follows:

Theorem 1 [1]: If a function $f(x)$ contains no frequencies higher than ω_{max} (in radians per second), it is completely determined by giving its ordinates at a series of points spaced $T = \pi/\omega_{max}$ seconds apart.

The reconstruction formula complementing the above sampling theorem is

$$f(x) = \sum_{k \in Z} f(kT)\,\mathrm{sinc}(x/T - k), \tag{1}$$

where the equidistant samples of $f(x)$ may be interpreted as coefficients of some basis functions obtained by appropriate shifting and rescaling of the sinc-function: $\mathrm{sinc}(x)=\sin(\pi x)/(\pi x)$. Dr. Shannon's paper shows that eq. (1) is exact if $f(x)$ is bandlimited to $\omega_{max} \leqq \pi/T$, which is called Nyquist rate, a term that was coined by Shannon in recognition of Nyquist's important contributions in the communication theory [3].

Recently, sub-Nyquist sampling has attracted a lot of attention of both mathematicians and computer scientists. Sub-Nyquist sampling, also known as compressive sampling or compressed sensing, refers to the problem of recovering signals by samples much fewer than suggested by Nyquist rate. Compressive sensing is of interest in the scenario where sampling by Nyquist rate is either not feasible or not efficient. For example, in sensor networks, the number of sensors may be limited. Also, measurements might be considerably expensive as in certain imaging processes via neutron scattering. Or the sampling rate is bounded by hardware limitation so sampling as fast as Nyquist rate is not achievable. These circumstances bring up important questions. Is an accurate recovery possible if we have only samples much less than Nyquist rate suggests? How can one approximate based on this information?

The above sub-Nyquist sampling, or compressive sensing, problem is modeled as finding the sparsest solution to an underdetermined system of linear equations [5-7]. Followed by the formulation in [8], let the vector $u \in \mathbb{R}^m$ denote the recorded values of a signal $f(t)$. The goal of compressive sensing is to find or approximate u from information $y = (\langle u, \varphi_1 \rangle, \cdots \langle u, \varphi_n \rangle) \in \mathbb{R}^n$, where n is much smaller than m. To make this possible, compressive sensing relies on two principles: *sparsity*, which relates to the signals of interest, and *incoherence*, which relates to the sensing modality.

It is known that many natural signals are sparse or compressible as they have concise representations when expressed in the proper domain Ψ. This fact expresses the idea of sparsity that the "information rate" of a continuous time signal may be much smaller than suggested by its bandwidth, or that a discrete-time signal depends on a number of degrees of freedom that is much smaller than its finite length. Also, incoherence is as important as sparsity so the sensing waveforms have an extremely dense representation in Ψ.

Based on the above two properties, the sparsest solution of compressive sensing problem is given by

$$(\mathrm{P}_0) \quad \min_{v \in \mathbb{R}^m} \|v\|_0 \quad \text{subject to } \Phi v = y, \tag{2}$$

where Φ is the coding/sampling matrices with rows $\varphi_j \in \mathbb{R}^m$. However, such minimization problem is NP-hard [11, 27] since it requires exhaustive searches over all subsets of columns of Φ. Fortunately, a number of studies [9-12] have shown that P_0 is equivalent to l_1-norm minimization problem, which can be recast as a linear program. Several upper bounds of sparsity and coherence under different conditions are given in [7-8, 13].

This paper intends to conduct a survey on the above sub-Nyquist sampling techniques from the fundamental theory, signal recovery, to recent applications. The rest of this survey is organized as follows. Section 2 introduces numerous important foundation of compressive sensing. Section 3 briefly describes a number of signal recovery techniques for compressive sensing. Innovative applications of compressive sensing in all kinds of areas are presented in Section 4 and Section 5 concludes the paper.

2. COMPRESSIVE SENSING BASICS

2.1 Sparsity

Sparsity is important in compressive sensing as it determines how efficient one can acquire signals nonadaptively. The commonest definition of sparsity used in compressive sensing is as follows. Let the vector $u \in \mathbb{R}^m$ denote the recorded values of a signal $f(t)$, which is expanded in an orthonormal basis $\Psi = [\psi_1 \psi_2 \cdots \psi_m]$ as follows:

$$v = \sum_{i=1}^{m} x_i \psi_i, \quad (3)$$

where x is the coefficient sequence of v, $x_i = \langle v, \psi_i \rangle$. Let Ψ denote the $m \times m$ matrix with $\psi_1 \cdots \psi_m$ as columns. If a signal has a sparse expansion, discarding small coefficients will not have much perceptual loss. A signal is called K-sparse if its expansion has at most K nonzero entries. This principle is what underlies most modern lossy coders such as JPEG-2000 [14] and many others.

2.2 Coherence

Let the pair (Φ, Ψ) be the orthobases of $\mathbb{R}^m$, where Φ is the sensing modality and Ψ is the signal model. The definition of coherence between the sensing basis Φ and the representation basis Ψ given in [15] is

$$\mu(\Phi, \Psi) = \sqrt{m} \cdot \max_{1 \le k, j \le m} \left| \langle \varphi_k, \psi_j \rangle \right| \quad (4)$$

This coherence stands for the largest correlation between any two elements of Φ and Ψ. The coherence will be large is they contain correlated elements; otherwise, it is small. For μ to be close to its minimum value of 1, each of the measurement vectors (rows of Φ) must be "spread out" in the Ψ domain. [15] proves that a K-sparse signal can be reconstructed from K*log(m) measurements in any domain where the test vectors are "flat," i.e., the coherence is O(1).

2.3 Random Sampling

The concept of sparsity and incoherence quantizes the compressibility of a signal. A signal is more compressible if it has a larger sparsity in some representation domain Ψ that is less coherent to the sensing domain Φ. However, such compression is obviously infeasible to perform on most sensing devices as transforming from a dense domain to a sparse domain can be computationally expensive.

Fortunately, it turns out random matrices are largely incoherent with any fixed basis Ψ [4]. With high probability, a uniform randomly selected orthobasis Φ, which can be done by otrhonomalizing m vectors sampled independently and uniformly on the unit sphere, has a coherence of $\sqrt{2 \log m}$ between any fixed basis Ψ. Also, random waveforms with independent identically distributed (i.i.d.) entries, e.g., Guassian or Bernoulli random entries, will also exhibit a very low coherence with any fixed representation Ψ [4]. The general rule to make measurements to be incoherent in the sparsity domain Ψ is to make Φ unstructured with respect to Ψ. Taking random measurements is in some sense an optional strategy for acquiring sparse signals [15]. It requires a near-minimal number of measurements [6, 7, 13, 16, 17] and all of the constants appearing in the analysis are small [18].

The following theorem proposed in [15] proves the above observation.

Theorem 2 [15]: Fix $u \in \mathbb{R}^m$ and suppose that the coefficient sequence x of u in the basis Ψ is K-sparse. Select n measurements in the Φ domain uniformly at random. Then if

$$n \ge C \cdot \mu^2(\Phi, \Psi) \cdot K \cdot \log m \quad (5)$$

for some positive constants C, the sparsest solution to recover u is exact with overwhelming probability.

In other words, if a signal is known to be K-sparse in some domains Ψ, by randomly selecting the samples in the domain Φ, the minimum number of selected samples n is $O(C \cdot \mu^2(\Phi, \Psi) \cdot K \cdot \log m)$. This theorem suggests a practical acquisition strategy that moves the compression step from sensors to the post-processing computers. Sensors can simply sample nonadaptively in an incoherent domain, which would essentially acquire the signal in a compressed form. All that is needed is a decoder, usually a non-power-constrained computer, to "decompress" the data. If the signal happens to be sufficiently sparse, exact recovery occurs.

However, this brings up another potential issue that the quality of recovery now depends on the quality of random number generators [1]. A naïve way to select n measurements in the Φ domain uniformly at random is to first collect enough samples and then drop redundant ones. Such approach is not preferred as it wastes power on sensing and wastes space on storing samples. Another way is to randomly generate the interval to sample such that the samples are equivalent to being selected uniformly or normally. This approach allows sensors to sample only desired measurements and thus sampling rate is indeed reduced. Such approach, however, raises a potential problem that the quality of the simulated uniform random variables may not be the same as previous one, and therefore it might degrade the quality of recovered signal. There are also some random sampling schemes that encode a number of samples to a single measurement, such as random linear sampling or Bernoulli random sampling.

3. CS RECOVERY ALGORITHMS

Now that previous section has depicted the fundamental theory of compressed sensing, we next discuss several compressed sensing recovery algorithms (data "decompress" algorithms) in this section.

3.1 l_1-norm Minimization

Since [4], several studies [6, 7, 18] have demonstrated the effectiveness of of l_1-norm minimization for recovering sparse signals from a limited number of measurements. Followed the notation of P_0 given in Section 1, the problem of l_1-norm minimization can be formulated as follows:

$$(P_1) \quad \min_{v \in \mathbb{R}^m} \|v\|_1 \text{ subject to } \Phi v = y. \quad (6)$$

It is known that the problem (P_1) can be solved by linear programming technique. Several algorithms, such as fixed point continuation (FPC) [19], interior point methods, and homotopy methods, are proposed specifically for solving the above l_1-norm minimization problem. Recovery algorithms based on l_1-norm minimization are mostly accurate and robust for reconstructing sparse signals while some of them may take long time to obtain the sparsest solution.

Based on l_1-norm minimization recovery algorithm, it has been shown in [20] that exact reconstruction occurs for K-sparse signal when K<(1+1/M)/2, where M is the coherent parameter of the

[1] Note that to the best of the author's knowledge, there is no reference regarding this issues, while all of this is learnt by the author during the work on the course project of compressive sensing framework on TinyOS.

measurement matrices Φ and the signal model Ψ. This allows one to build rather simple deterministic matrices Φ with $K \approx \sqrt{n}$. Some later work [21-22] further proves the existence of matrices Φ with $K \approx n/\log(m/n)$, which is substantially larger than $\sqrt{n}$.

Fig. 1 shows an example of l_1 recovery shown in [4]. Fig. 1(a) is the original sparse real valued signal. Fig. 1(b) is the reconstruction from 60 (complex valued) Fourier coefficients by l_1-norm minimization. It turns out the reconstruction is exact.

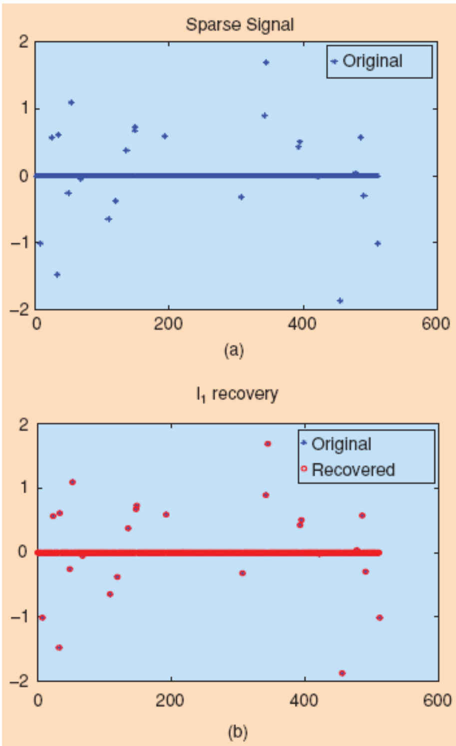


Fig. 1 l_1 recovery example. [4]

3.2 1-Bit Compressive Sensing

A recent work further shows that, by preserving only the sign information of the random measurements, the signal can still be recovered through a modified l_1-norm minimization algorithm [23]. The 1-bit compressive sensing problem is studied so that compressive sensing can still work with 1-bit quantizers, which are extremely inexpensive and fast hardware device.

Their proposed reconstruction algorithm is based on FPC algorithm [19] with two modifications. They first modify the computation of the gradient descent step in [19] such that it computers the gradient of one-sided quadratic penalty projected on the unit sphere $\|x\|_2 = 1$. Secondly, they introduce a renormalization step after each iteration of the algorithm to enforce the constraint that the solution lays on the unit sphere. The authors point out that these two modifications are similar to the ones introduced in [24] to stabilize the reconstruction of sparse signal from their zero crossings. The similarity is because both sign measurements and zero crossings information eliminate amplitude information from the signal. The major difference, however, is measurements of zero crossings are signal-dependent, while compressive measurements are signal-independent. Fig. 2 given in [23] shows the simulated reconstruction quality of their proposed recovery algorithm and the LARS algorithm with LASSO modification [25]. Note that the constant K stands for K-sparse signal.

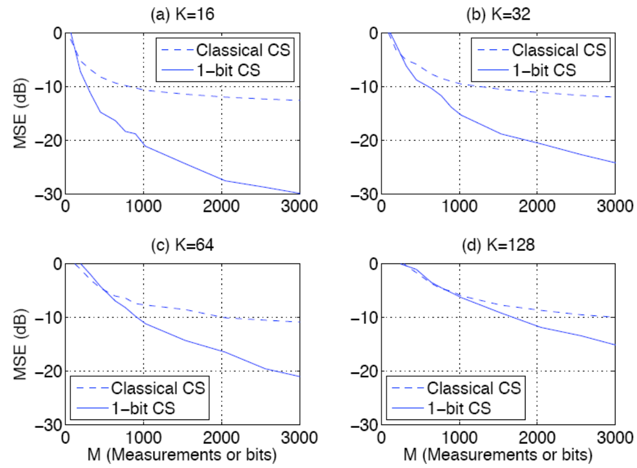


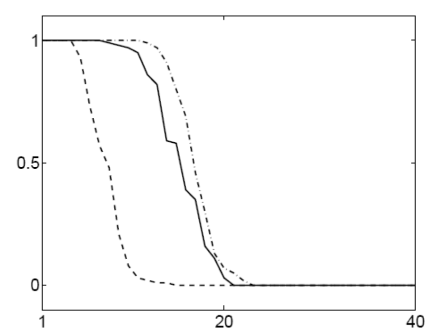
Fig. 2 Reconstruction quality from 1-bit measurements. [23]

3.3 Matching Pursuit

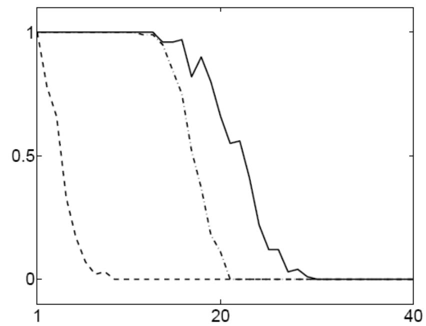
The concept of matching pursuit recovery [26-27] is to think of signal recovery as a problem dual to sparse approximation. Since the signal has only K nonzero components, the measurement vector is a linear combination of m columns from the sensing domain Φ.

Therefore, sparse approximation algorithms can be used for recovering sparse signals. To identify the ideal signal, we need to determine which columns of Φ participate in the measurement vector. The idea behind the algorithm is to pick columns in a greedy fashion. At each iteration, we choose the column of Φ that is most strongly correlated with the remaining part of the measurement vector. Then the contribution of such column is subtracted off from the measurement vector and then iterate on the residual measurement vector.

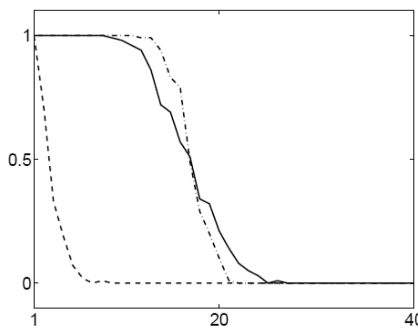
The key advantage of this algorithm is the complexity is largely reduced. Based on the analysis in [28], the time complexity of their proposed algorithm is $O(mNd)$, where m is the sprsity of the signal, N is the number of measurements, and d is the dimension of the signal. According to [29], the algorithms to solve l_1-norm minimization problem with a dense and unstructured measurement matrix takes $O(N^2d^{3/2})$. Fig. 3 as given in [30] shows the recovery success rate and computation time comparison of three different types of recovery algorithms.



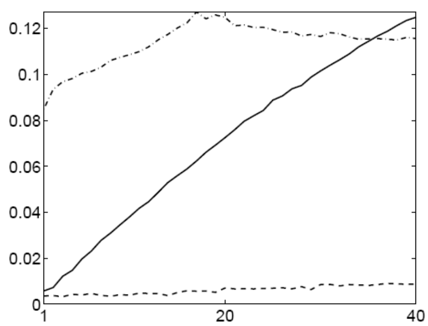
(a) Success rate vs. sparsity $M = 1, \ldots, 40$ for $D = 100$, $N = 40$, $|c_k| = 1$ for $k \in T$, FFT.



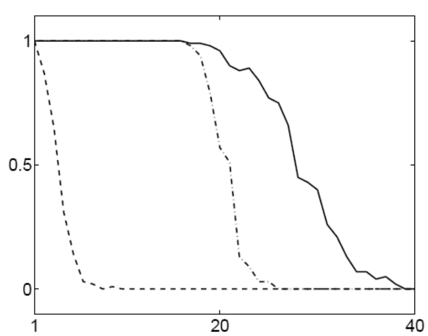
(b) Success rate vs. sparsity $M = 1, \ldots, 40$ for $D = 100$, $N = 40$, FFT.



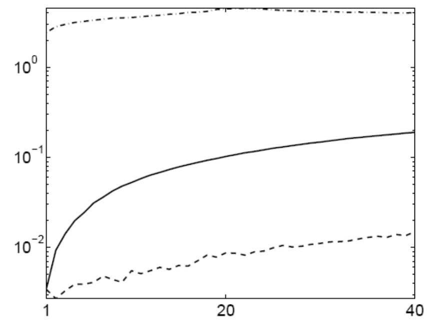
(c) Success rate vs. sparsity $M = 1, \ldots, 40$ for $D = 100$, $N = 40$, NFFT.



(d) Computation time in seconds vs. sparsity $M = 1, \ldots, 40$ for $D = 100$, $N = 40$, NFFT.



(e) Success rate vs. sparsity $M = 1, \ldots, 40$ for $D = 1000$, $N = 80$, FFT.



(f) Computation time in seconds vs. sparsity $M = 1, \ldots, 40$ for $D = 1000$, $N = 80$, FFT.

Fig. 3 Success rate and computation time of Thresholding [31] (dashed), Matching Pursuit (solid), and l_1-norm minimization (dash-dot) with respect to an increasing number M of non-zero Fourier coefficients. The number of samples N and the dimension D remain fixed; 100 runs have been conducted for each setting.[28]

3.4 Other Recovery Algorithms

There are still many lots of recovery algorithms that are not commonly used. This subsection briefly goes through three of them. Readers interested in these algorithms are highly recommended to further study the reference mentioned in each algorithm.

3.4.1 Iterative Thresholding

Iterative thrsholding algorithm [31] has been compared in matching pursuit algorithm above. The theory of iterative thresholding is analogous to l_1-norm minimization. Two basic steps of the algorithm is to first from guessing x_k, backproject to get $\omega_k \approx \Phi^T \Phi x_k$. Secondly, by thresholding (pruning) ω_k to get x_{k+1}. The algorithm works since for sparse signals, ω_k will be big on the active set and small elsewhere. As shown in Fig. 3, iterative thresholding always takes least time while does not achieve the same quality as the other two.

3.4.2 Total-Variation Minimization

Another recovery algorithm is called total-variation (TC) minimization [32], which is a classical image denoising technique. This method is a popular formulation from variational image processing. The sparsity in TV minimization problem is re-defined as the number of "jumps" in the image. It is modeled as a convex optimization problem as follows:

$$\min_x \mathrm{TV}(x) \approx \|\nabla x\|_{l1} \quad \text{subject to } \Phi x = y. \tag{7}$$

Similar to problem (P_1), this problem can also be solved by interior point methods or some types of first-order gradient projection. It recovers the signal accurately and is robust to noise while under certain cases, it can be slow.

3.4.3 l_1 Filtering

It is obvious that all of the above algorithms are for the most part "static": they focus on finding the solution for a fixed set of measurements. A recent work proposes a method for quickly updating the solution to some l_1-norm minimization problems as new measurements are added [33]. They proposed the "l_1 filter," which can be implemented using standard techniques from numerical linear algebra. Their proposed scheme is homotopy based where they add new measurements in the system and instead of solving updated problem directly, a series of simple intermediate problems that lead to the desired solution is solved. Fig. 4 shows the average number of iterations per new measurement, which turns out that their proposed scheme is fairly efficient to update the solution.

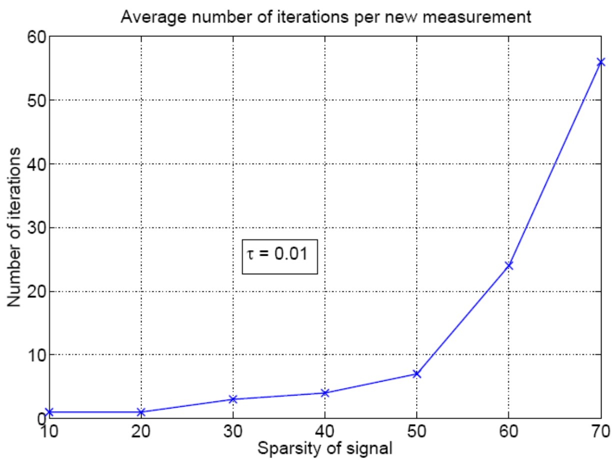


Fig. 4 Average number of homotopy steps with one new measurement at different sparsity levels. (n=256, m-150)

4. APPLICATION OF CS

As the fundamental theory is mostly settled, the number of compressed sensing based application grows rapidly in recent years. The area of application spans from sensor networks, image processing, medial imaging, compressive radar, astronomy, communications, remote sensing, to robotics and control. This section selects and presents a number of most recent and most innovative (from author's point of view) application.

4.1 A camera's worth a single pixel

In 2008, Rice University unveiled the prototype of a 1-pixel camera. The idea of such camera is instead of taking several million samples (pixels) at one time, they exploit the idea of compressive sensing to randomly take a 1-pixel measurement of a picture in a given time period. Based on the compressed sensing theory, as long as these measurements are taken randomly, an approximation can be made once measurements are more than the number suggested by the sparsity of the signal. In other words, this 1-pixel camera does not sample only in the spatial domain, while it measures both temporal and spatial domains so that the number of pixels taken at a time moment is reduced.

Fig. 5 shows the architecture of this 1-pixel camera. The camera uses a so-called digital micromirror device to randomly alter where the light hitting the single-pixel originates from within the camera's field of view as it builds the image. This random shuffling of where a small part of the image originates serves to compress the image, and therefore something done in digital cameras now using microprocessor power.

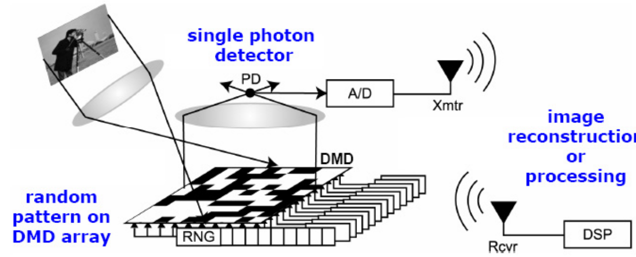
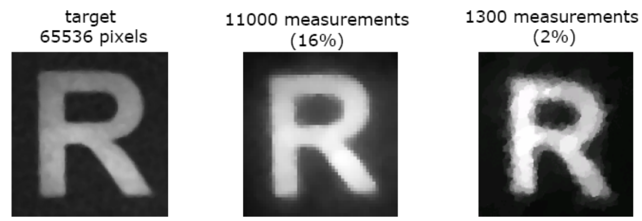
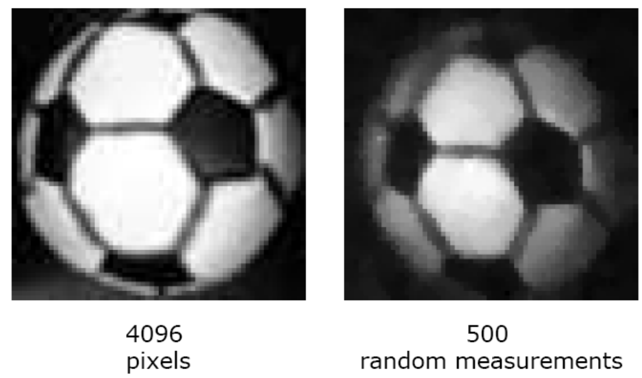


Fig. 5 Architecture of 1-pixel camera [34]

Fig. 6 shows the recovery results of 1-pixel camera by taking different number of measurements.



(a) first image acquisition



(b) second image acquisition

Fig. 6 1-pixel camera: recovery results

In addition to this work, there are also many other imaging applications. For example, compressive camera arrays where sampling rate scales logarithmically in both number of pixels and number of cameras. Compressive radar [35-37] and sonar with greatly simplified receivers explores radar/sonar networks. Also, compressive DNA microarrays [38-40] exploit sparsity in presentation of organisms to array, which is smaller, more agile arrays for bio-sensing.

4.2 Distributed CS

In sensor networks, there are intra-sensor and inter-sensor correlation, so it is possible to exploit these to jointly compress. The idea of distributed compressed sensing is to measure separately at sensors but reconstruct jointly. The obvious advantage of such design is zero collaboration, trivially scalable, robust, low complexity, and universal encoding. Fig. 7 shows the experiment result comparing the performance of transform coding, compressed sensing, and distributed compressed sensing [41].

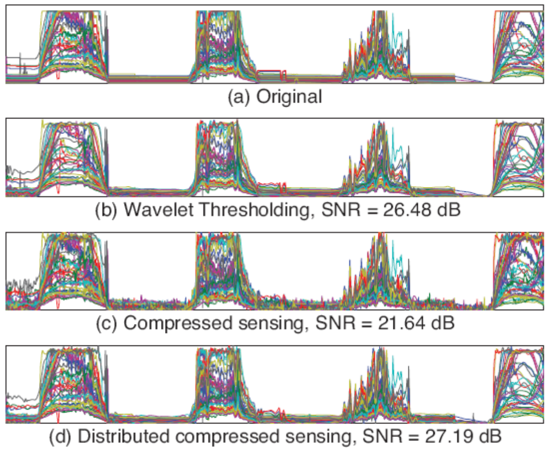


Fig. 7 Reconstruction of light intensity signals from 48 sensors with length N = 1024. (a) Original signals; (b) wavelet thresholding using 100 coefficients per sensor, average SNR = 26.48dB; (c) separate reconstruction of each signal using CS from M = 400 random projections per sensor, average SNR = 21.64dB; (c) joint reconstruction of the signal ensemble using DCS from M = 400 random projections per sensor, average SNR = 27.19dB.

4.3 Medical and Scientific Imaging

Compressive sensing is also recently initially extended to the medical and scientific imaging applications. [42-44] are works on magnetic resonance imaging. Fig. 8 shows the result given in [42] comparing the MRI quality of compressed sensing. [45] is the work on compressed sensing based astronomy application. Fig. 9 gives the result of astronomy photo compressed by compressive sensing.

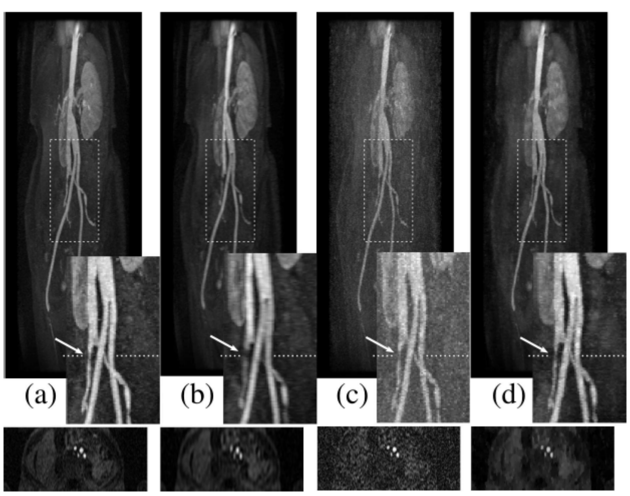


Fig. 8 Reconstruction from 5-fold accelerated acquisition of first-pass contrast enhanced abdominal angiography. (a) Reconstruction from a complete data set. (b) LR (c) ZF-w/dc (d) CS reconstruction from random undersampling. The patient has a aorto-bifemoral bypass graft. This is meant to carry blood from the aorta to the lower extremities. There is a high-grade stenosis in the native right common illiac artery, which is indicated by the arrows. In figure parts (a) and (d) flow across the stenosis is visible, but it is not on (b) and (c).

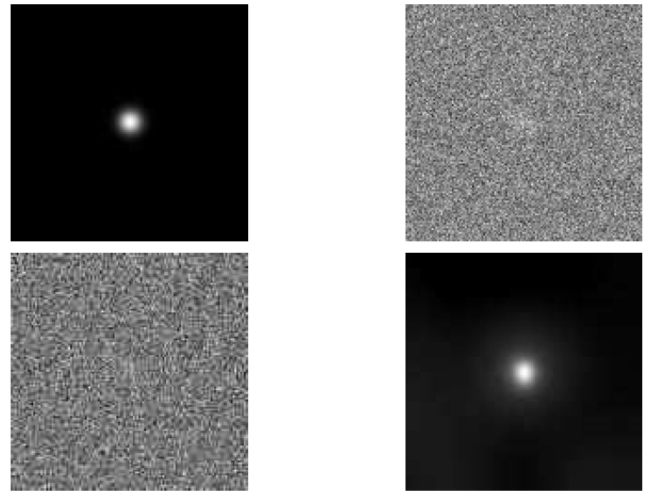


Fig. 9 **Top - left** : Input image of size 128×128. **Top - right** : First noisy input data x1. White Gaussian noise is added with SNR = .26dB. **Bottom-left** : Estimate from the average of 10 images compressed by JPEG with a compression rate ρ = 0.25. **Bottom-right** : Estimate from 10 pictures compressed by CS with a compression rate ρ = 0.25.

5. CONCLUSION AND FUTURE WORK

In this paper, we describe the fundamental theory of compressed sensing. A number of commonly used recovery algorithms are also presented. Several recent application based on compressive sensing are demonstrated as well. Although it seems that the basics of compressed sensing are mostly settled, there are still open problems worth further investigation.

First of all, the connection between compressed sensing and coding and machine learning is not known yet. Also, current theory assumes only single-signal. A multi-signal compressed sensing such as sparsity-based array processing, beamforming, and localization is still not explored. In addition, so far, most of the recovery algorithms can still work only for static measurements, a fast and real-time recovery algorithm is certainly helpful in developing more applications. Last but not least, based on this new theory, the development of new sensors, such as camera arrays, microscopes and telescopes, and imagers (MRI, radar, and sonar) is inevitable. One can expect more new applications and more efficient sensing algorithms on the basis of compressed sensing theory will be developed rapidly in the near future.

6. ACKNOWLEDGMENTS

A Special thank to the DSP group in Rice University for their great paper collection page of compressed sensing [45]. Most of the papers described in this survey are found in their website. Readers interested in more detail are encouraged to further explore their website for more literatures.

7. REFERENCES

[1] C. E. Shannon, “Communication in the presence of noise,” in *Proc. IRE*, vol. 37, pp. 1021, 1949.

[2] C.E. Shannon, “Classic paper: Communication in the presence of noise,” *Proc. IEEE*, vol. 86, no. 2, pp. 447-457, 1998.

[3] H. Nyquist, “Certain topics in telegraph transmission theory,” *Trans Amer. Inst. Elect. Eng.*, vol. 47, pp. 617-644, 1928.

[4] E. J. Candes and M. B. Wakin, “An introduction to compressive sampling,” *IEEE Signal Processing Magazine*, 2008.

[5] E. Candes, J. Romberg, T. Tao, “Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information,” *IEEE Trans. Information Theory*, vol. 52, no. 2, pp. 489-509, Feb 2006.

[6] E. Candes and T. Tao, “Near optimal signal recovery from random projections: universal encoding strategies,” *IEEE Trans. Information Theory*, vol. 52, no. 12, pp. 5406-5425, Dec. 2006.

[7] D. Donoho, “Compressed sensing,” *IEEE Trans. Information Theory*, vol. 52, no. 4, pp. 1289-1306, Apr. 2006.

[8] B. S. Kashin and V. N. Temlyakoc, “A remark on compressed sensing,” *Mathematical Notes*, vol. 82, no. 5-6, pp. 748-755, Nov. 2007.

[9] D. L. Donoho and X. Hus, “Uncertainty principles and ideal atomic decomposition,” *IEEE Trans on Information Theory*, vol. 47, pp. 2845-2862, 2001.

[10] M. Elad and A. M. Bruckstein, “A generalized uncertainty principle and sparse representation in paris of $\mathbf{R}^N$ bases,” *IEEE Trans. on Information Theory*, vol. 48, pp. 2558-2567, 2002.

[11] R. Gribonval and M. Nielsen, “Sparse representations in unions of bases,” *IEEE Trans. on Information Theory*, vol. 49, pp. 3320-3325, 2003.

[12] J. A. Tropp, “Greed is good: algorithmic results for sparse approximation,” technical report, the University of Texas at Austin, 2003.

[13] E. Candes and T. Tao, “Decoding by linear programming,” *IEEE Trans. on Information Theory*, vol. 51, no. 12, pp. 4203-4215, Dec. 2005.

[14] D. S. Taubman and M. W. Marcellin, *JPEG 2000: Image Compression Fundamentals, Standards and Practice*. Norwell, MA: Kluwer, 2001.

[15] E. Candes and J. Romberg, “Sparsity and incoherence in compressive sampling,” *Inverse Problem*, vol. 23, no. 3, pp. 969-985, 2007.

[16] R. G. Baraniuk, M. Davenport, R. DeVore, and M. Wakin, “A simple proof of the restricted isometry property for random matrices,” *Constructive Approximation*, vol. 28, no. 3, pp. 253-263, Dec. 2008.

[17] D. L. Donoho, “For most large underdetermined systems of linear equations the minimal l_1-norm solution is also the sparsest solution,” *Comm. Pure Appl. Math.*, vol. 59, no. 6, pp. 797-829, 2006.

[18] D. L. Donoho and J. Tanner, “Neighborliness of randomly-projected simplices in high dimensions,” *Proc. Natl. Acad. Sci.*, vol. 102, no. 27, pp. 9452-9457, 2005.

[19] E. T. Hale, W. Yin, and Y. Zhang, “A fixed-point continuation method for l_1-regularized minimization with applications to compressed sensing,” *Rice University, CAAM Tech. Report*, TR07-07, Jul. 2007.

[20] D. L. Donoho, M. Elad, and V.N. Temlyakov, “Stable recovery of sparse overcomplete representations in the presence of noise,” *IEEE Tran. Information Theory*, vol.52, pp.6-18, 2006.

[21] R. DeVore, “Optimal computation,” *ICM Proceedings*, Madrid 1, pp. 187-215, 2006.

[22] E. Candes, “Compressive sampling,” *ICM Proceedings*, Madrid 3, pp. 1433-1452, 2006.

[23] P. Boufounos and R.G. Baraniuk, “1-bit compressive sensing,” in *Proc. of Conference on Information Sciences and Systems*, Mar. 2008.

[24] P. Boufounos and R.G. Baraniuk, “Reconstructing sparse signals from their zero crossings,” in *Proc. of IEEE International Conference on Acoustics, Speech, and Signal Processing 2008* (ICASSP ’08).

[25] B. Efron, T. Hastie, I. Jognstone, and R. Tibshirani, “Least angle regression,” *Annals of Statistics*, vol. 32, no. 2, pp. 407-499, Apr. 2004.

[26] Y. C. Pati, R. Rezaiifar, and P.S. Krishnaprasad, “Orthogonal matching pursuit: recursive function approximation with applications to wavelet decomposition,” in *Proc. 27th Annu. Asilomar Conference Signals, Systems, and Computers*, vol. 1, pp. 40-44, Nov. 1993.

[27] G. Davis, S. Mallat, and M. Avellaneda, “Greedy adaptive approximation,” *Constr. Approximiation*, vol. 13, pp. 57-98, 1997.

[28] J. A. Tropp and A. C. Gilbert, “Signal recovery from random measurements via orthogonal matching pursuit,” *IEEE Trans. Information Theory*, vol. 53, no. 12, Dec. 2007.

[29] Y.E. Nesterov and A.S. Nemirovski, “Interior Point Polynomial Algoeithms in Convex Programming,” *SIAM*, 1994.

[30] S. Kunis and H. Rauhut, “Random Sampling of Sparse Trigonometric Polynomials, II. Orthogonal Matching Pursuit versus Basis Pursuit,” *Foundations of Computational Mathematics*, vol. 8, no. 6, Nov. 2008.

[31] M. Fornasier and H. Rauhut, “Iterative thresholding algorithms,” *Applied and Computational Harmonic Analysis*, vol. 25, no. 2, pp. 187-208, Sep. 2008.

[32] A. Chambolle1 and P.-L. Lions, “Image recovery via total variation minimization and related problems,” *Numerische Mathematik*, vol. 76, no. 2, pp. 167-188, Apr. 1997.

[33] M. S. Asif and J. Romberg, “Streaming measurements in compressive sensing: L1 filtering,” *Asilomar Conference on Signals, Systems and Computers*, Oct. 2008.

[34] M. Duarte, M. Davenport, D. Takhar, J. Laska, T. Sun, K. Kelly, and R. Baraniuk, “Single-pixel imaging via compressive sampling,” *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 83 - 91, Mar. 2008

[35] R. Baraniuk and P. Steeghs, “Compressive radar imaging,” *IEEE Radar Conference*, Apr. 2007

[36] L. Potter, P. Schniter, and J. Ziniel, "Sparse reconstruction for RADAR," *SPIE Algorithms for Synthetic Aperture Radar Imagery XV*, 2008.

[37] S. R. Bhaskaran, L. Davis, A. Grant, S. Hanly, P. Tune, "Downlink Scheduling Using Compressed Sensing," *Information Theory Workshop (ITW)*, 2009.

[38] Mona Sheikh, Olgica Milenkovic, and Richard Baraniuk, "Compressed sensing DNA microarrays," *Rice ECE Department Technical Report TREE 0706*, May 2007.

[39] W. Dai, M. Sheikh, O. Milenkovic, and R. Baraniuk, "Compressive Sensing DNA Microarrays," *EURASIP Journal on Bioinformatics and Systems Biology*, 2009.

[40] N. Shental, A. Amir, O. Zuk, "Rare-Allele Detection Using Compressed Se(que)nsing," *CoRR*, 2009.

[41] M. F. Duarte, M. B. Wakin, D. Baron, and R. G. Baraniuk, "Universal Distributed Sensing via Random Projections," in *Proc. of IPSN 2006*.

[42] M. Lustig, D. Donoho, and J. M. Pauly, "Sparse MRI: The application of compressed sensing for rapid MR imaging," *Magnetic Resonance in Medicine*, vol. 58, no. 6, pp. 1182-1195, Dec. 2007

[43] C. Qiu, W. Lu and N. Vaswani, "Real-time Dynamic MR Image Reconstruction using Kalman Filtered Compressed Sensing," in *Proc. of IEEE Int. Conf. on Acoustics, Speech, and Signal Processing (ICASSP)*, Taipei, Taiwan, Apr. 2009

[44] H. Jung, J. C. Ye, "Performance evalution of accelerated functional MRI acquisition using compressed sensing," in *Proc. IEEE International Symposium on Biomedical Imaging (ISBI)*, pp. 702-705, Jun. 2009.

[45] Compressive Sensing Resources, http://www.compressedsensing.com/