

LETTER

A Semidefinite Relaxation Approach to Spreading Sequence Estimation for DS-SS Signals

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SUMMARY In non-cooperative scenarios, the estimation of direct sequence spread spectrum (DS-SS) signals has to be done in a blind manner. In this letter, we consider the spreading sequence estimation problem for DS-SS signals. First, the maximum likelihood estimate (MLE) of spreading sequence is derived, then a semidefinite relaxation (SDR) approach is proposed to cope with the exponential complexity of performing MLE. Simulation results demonstrate that the proposed approach provides significant performance improvements compared to existing methods, especially in the case of low numbers of data samples and low signal-to-noise ratio (SNR) situations.

key words: direct sequence spread spectrum signal, semidefinite relaxation, maximum likelihood estimate, spreading sequence estimation

1. Introduction

Direct sequence spread spectrum (DS-SS) signals have been widely used for military and civil communications for a long time due to their low probability of intercept properties. In conventional cooperative applications, the transmitted information can be demodulated with prior knowledge of the spreading sequence at the receiver. However, in non-cooperative applications such as eavesdropping, spectrum surveillance and source localization, the spreading sequence is unknown to the receiver, and recovering information has to be done in a blind manner.

The blind estimation of DS-SS signals has received significant attention in the past years [1]–[7]. An eigenanalysis estimator [1], [2] was developed to estimate the spreading sequence at low signal-to-noise ratio (SNR), and some other modifications have been proposed in [3]–[6]. To cope with the high computational complexity of the eigenanalysis estimator, based on maximum likelihood function, a fast algorithm with a slight performance degradation was proposed in [7]. However, those works [1]–[7] are all focused on the short-code DS-SS signals where the spreading sequence period L equals the symbol duration G , and the results cannot be applied to the long-code DS-SS signals where $L > G$ [8], such as the tracking and data relay satellite system (TDRSS).

Blind recovery of long-code DS-SS signals has not been widely reported. Recently, under the restriction of L being an integer multiple of G , two spreading sequence esti-

mation methods have been proposed in [9], [10]. However, this restriction maybe too strict in applications because L may not be divisible by G in practical DS-SS communication systems, such as TDRSS. The segmentation estimator [8] and the dominant mode despreading (DMDS) estimator [11] are two methods to deal with the long-code DS-SS signals without the above restriction. To achieve good performances at low SNR, both segmentation and DMDS estimators require a large number of data samples so that the spreading sequence can be continuously observed many times. However, it is impractical for the burst-mode or high velocity Doppler shift scenarios, where the received DS-SS signals have short observation time. Hence, it becomes challenging to estimate the spreading sequence accurately from few data samples at low SNR. Furthermore, both estimators lead to significant performance loss compared to the eigenanalysis estimators [1], [2] when they are applied to the short-code DS-SS signals.

In this letter, to combat the performance degradation of existing estimators [8], [11] in the case of low numbers of data samples and low SNR, we propose a novel approach to spreading sequence estimation both for long-code as well as for short-code DS-SS signals based on semidefinite relaxation (SDR) [12], [13], called the SDR estimator hereafter. First, we derive the maximum likelihood estimate (MLE) of spreading sequence under the assumption of Gaussian distributed noise. Then, due to the exponentially increasing computational complexity of implementing MLE, we propose a SDR approach, which leads to an approximate MLE algorithm with an affordable worst-case complexity. The major advantage of our proposed estimator is that it provides good estimation performance from few data samples at low SNR, and therefore our proposed scheme is appealing in practical non-cooperative context. Moreover, the proposed estimator also shows good performance when applied to the short-code DS-SS signals. Simulation results demonstrate that the proposed SDR estimator provides significant performance improvements compared to the segmentation and DMDS estimators in the case of low numbers of data samples and low SNR, for both short-code and long-code DS-SS signals.

Notation: Uppercase and lowercase boldface denote matrices and vectors, respectively. $\mathbf{M}_{i,j}$ represents the (i, j) th entry of \mathbf{M} and \mathbf{v}_i the i th entry of \mathbf{v} . The symbols $(\cdot)^T$, $\text{Tr}(\cdot)$, $\|\cdot\|$ and $\mathbf{0}_L$ denote the transpose, trace, Frobenius norm, and the zeros matrix of size $L \times L$, respectively.

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2. Signal Model

Consider an intercepted long-code DS-SS signal with processing gain G . After chip-matched filtering and chip-rate sampling, the corrupted long-code DS-SS signal by noise at the receiver filter can be modeled as $y(n)$, $n = 0, 1, \dots, N-1$

$$y(n) = As(n) + v(n) \quad (1)$$

$$s(n) = \sum_{\bar{m}=0}^{\bar{M}-1} b(\bar{m})q(n - \bar{m}G) \sum_{m=0}^{M-1} c(n - mL) \quad (2)$$

where $A > 0$ is the received signal amplitude, and $v(n)$ is zero-mean additive white Gaussian noise with variance σ^2 . $\{c(l)\}_{l=0}^{L-1}$ and $\{b(\bar{m})\}_{\bar{m}=0}^{\bar{M}-1}$ represent the spreading sequence with length of L and the transmitted symbol sequence with symbol duration of G , respectively. $\{b(\bar{m})\}$ is an independent and identically distributed (i.i.d.) equiprobable $\{\pm 1\}$ sequence, and $c(l) \in \{\pm 1\}$ for $l = 0, \dots, L-1$. $M = N/G$ is the sample size in terms of the number of spreading sequence period, and $\bar{M} = \lceil N/G \rceil$ is the number of symbols, where $\lceil x \rceil$ denotes the largest integer not smaller than x . $q(n)$ denotes the rectangular function with magnitude equal to one for $n = 0, 1, \dots, G-1$. The SNR is defined as $\rho = A^2/\sigma^2$. Equation (1) can be expressed concisely as the following vector form

$$\mathbf{y} = A\mathbf{s}(\mathbf{x}) + \mathbf{v} \quad (3)$$

where $\mathbf{x} = [\mathbf{c}^T \ \mathbf{b}^T]^T$, $\mathbf{c} = [c(0), \dots, c(L-1)]^T$ and $\mathbf{b} = [b(0), \dots, b(\bar{M}-1)]^T$. The signal vector $\mathbf{s}(\mathbf{x})$ is expressed as a function of \mathbf{x} . We note that Eq. (2) and Eq. (3) reduce to a short-code DS-SS signal when $L = G$.

As the DMDS estimator requires L and G are coprime, only the constraint $L \geq G$ is required in proposed approach. To simplify the analysis, we assume that L and G are known apriori or estimated and the signal has been synchronized [2], [3].

3. Proposed Approach

In this section, first, we derive the joint MLE of spreading sequence and symbol sequence, then, a semidefinite relaxation approach is proposed to cope with the exponential complexity of performing MLE.

3.1 Maximum Likelihood Estimate

The conditional probability density function of the observation \mathbf{y} under the Gaussian noise is given by

$$p[\mathbf{y}|\mathbf{x}, A, \sigma^2] = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left\{-\frac{\|\mathbf{y} - A\mathbf{s}(\mathbf{x})\|^2}{2\sigma^2}\right\} \quad (4)$$

and the negative log-likelihood function has the following form

$$-\log p(\mathbf{x}, A, \sigma^2) = \frac{N}{2} \log(2\pi\sigma^2) + \frac{\|\mathbf{y} - A\mathbf{s}(\mathbf{x})\|^2}{2\sigma^2} \quad (5)$$

For fixed \mathbf{x} , the minimum with respect to A and σ^2 is readily derived as

$$\hat{A} = \frac{\mathbf{s}(\mathbf{x})^T \mathbf{y}}{\|\mathbf{s}(\mathbf{x})\|^2} = \frac{\mathbf{s}(\mathbf{x})^T \mathbf{y}}{N}, \quad \hat{\sigma}^2 = \frac{1}{N} \|\mathbf{y} - \hat{A}\mathbf{s}(\mathbf{x})\|^2 \quad (6)$$

Substituting Eq. (6) into Eq. (4) shows that the MLE of \mathbf{x} is obtained by solving the following problem

$$\begin{aligned} \max \quad & \mathbf{s}(\mathbf{x})^T \mathbf{y} \\ \text{s.t.} \quad & \mathbf{x}_i \in \{1, -1\}, \forall i \in \{1, \dots, L + \bar{M}\} \end{aligned} \quad (7)$$

According to the analysis above, we note that performing the joint MLE of spreading sequence \mathbf{c} and symbol sequence \mathbf{b} is necessary to solve the combinatorial optimization problem in Eq. (7), which can be solved using an exhaustive search. However, the complexity of the exhaustive search is generically exponential, due to NP-hardness. In what follows, we show that problem Eq. (7) can be approximately solved by relaxing some of its constraints, which features polynomial worst-case complexity $O((L + \bar{M})^3)$.

3.2 Semidefinite Relaxation (SDR Estimator)

We define observation matrix $\mathbf{Q} \in \mathbb{R}^{L \times \bar{M}}$, where the $(\text{mod}(n, L) + 1)$ th row and $(\lfloor n/G \rfloor + 1)$ th column element of \mathbf{Q} is $y(n)$, and the other elements are assumed to be zeros. $\lfloor x \rfloor$ and $\text{mod}(n, L)$ denote the largest integer not larger than x and the modulo n of L , respectively. For example, we assume $L = 7, G = 3, N = 12$, then \mathbf{Q} has the following structure form

$$\mathbf{Q} = \begin{bmatrix} y(0) & 0 & y(7) & 0 \\ y(1) & 0 & y(8) & 0 \\ y(2) & 0 & 0 & y(9) \\ 0 & y(3) & 0 & y(10) \\ 0 & y(4) & 0 & y(11) \\ 0 & y(5) & 0 & 0 \\ 0 & 0 & y(6) & 0 \end{bmatrix} \quad (8)$$

We then build the following symmetric matrix $\bar{\mathbf{Q}} \in \mathbb{R}^{(L+\bar{M}) \times (L+\bar{M})}$

$$\bar{\mathbf{Q}} = \begin{bmatrix} \mathbf{0}_L & \mathbf{Q} \\ \mathbf{Q}^T & \mathbf{0}_{\bar{M}} \end{bmatrix} \quad (9)$$

From Eq. (2) and Eq. (9), it's easy to verify that

$$\mathbf{s}(\mathbf{x})^T \mathbf{y} = \frac{1}{2} \mathbf{x}^T \bar{\mathbf{Q}} \mathbf{x} \quad (10)$$

Using the fact $\mathbf{x}^T \bar{\mathbf{Q}} \mathbf{x} = \text{Tr}(\bar{\mathbf{Q}} \mathbf{x} \mathbf{x}^T)$, we can equivalently rewrite the problem Eq. (7) under Eq. (10) as

$$\begin{aligned} \min \quad & \text{Tr}(-\bar{\mathbf{Q}} \mathbf{X}) \\ \text{s.t.} \quad & \mathbf{X} = \mathbf{x} \mathbf{x}^T, \mathbf{X}_{i,i} \in \{1, -1\}, \forall i \in \{1, \dots, L + \bar{M}\} \end{aligned}$$

$$(11)$$

The constraint $\mathbf{X} = \mathbf{x}\mathbf{x}^T$ implies that \mathbf{X} is positive semidefinite (PSD) and $\text{rank}(\mathbf{X}) = 1$, which leads to a non-convex optimization problem. Relaxing the rank-one constraint, we obtain the following semidefinite programming (SDP) problem

$$\begin{aligned} \min \quad & \text{Tr}(-\bar{\mathbf{Q}}\mathbf{X}) \\ \text{s.t.} \quad & \mathbf{X} \succeq 0, \mathbf{X}_{i,i} = \{1, -1\}, \forall i \in \{1, \dots, L + \bar{M}\} \end{aligned} \quad (12)$$

where $\mathbf{X} \succeq 0$ means that \mathbf{X} is PSD.

The SDP problem Eq. (12) is a convex optimization problem which does not suffer from local maxima, and can be solved using an efficient optimization algorithm based on interior-point methods, such as DSDP5 [14]. Based on the optimal solution \mathbf{X}_{opt} of Eq. (12), an approximate rank-one solution $\tilde{\mathbf{x}}$ to the original problem Eq. (7) can be easily generated using the following randomization procedure [12].

1) Compute the eigenvector $\mathbf{u} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_{L+\bar{M}}]^T$ that corresponds to the largest eigenvalue of \mathbf{X}_{opt} .

2) Generate a fixed number D (typically 10-30) of i.i.d. $(L+\bar{M})$ -dimensional binary vector samples $\tilde{\mathbf{z}}_d, d = 1, \dots, D$, whose i th entry ($i = 1, \dots, L + \bar{M}$) follows the distribution

$$\begin{aligned} p\{\mathbf{z}_i = +1\} &= (1 + \mathbf{u}_i)/2 \\ p\{\mathbf{z}_i = -1\} &= (1 - \mathbf{u}_i)/2 \end{aligned} \quad (13)$$

3) Pick $\tilde{\mathbf{x}} := \arg \min_d (-\tilde{\mathbf{z}}_d^T \bar{\mathbf{Q}} \tilde{\mathbf{z}}_d)$.

After the steps above, the spreading sequence and symbol sequence estimations are readily obtained, which are given by

$$\begin{aligned} \hat{\mathbf{c}} &= [\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \dots, \tilde{\mathbf{x}}_L]^T \\ \hat{\mathbf{s}} &= [\tilde{\mathbf{x}}_{L+1}, \tilde{\mathbf{x}}_{L+2}, \dots, \tilde{\mathbf{x}}_{L+\bar{M}}]^T \end{aligned} \quad (14)$$

Due to the diagonal structure of the constraints, the proposed SDR estimator has the polynomial worst-case complexity $O((L + \bar{M})^3)$ [12], [13]. Let $L_0 (L_0 < G)$ denotes the duration of short-time segments in the process of the segmentation estimator, the computation complexity of the segmentation and eigenanalysis estimators are $O(L_0^3 + ML_0^2)$ and $O(L^3 + ML^2)$ [8], respectively. Since the complexity of the DMDS estimator comes mainly from the calculations of the eigendecomposition and the autocorrelation matrix of the frequency-channelized signal [11], the computation complexity is approximately $O(L^3 + ML^3)$. Note that $\bar{M} \approx LM/G$. Therefore, the complexity of proposed estimator is higher than that of both segmentation and eigenanalysis estimators, but lower than that of the DMDS estimator.

4. Simulations

In this section, simulation results are presented to demonstrate the performance gains of our SDR estimator compared to existing methods. Performance evaluation is conducted

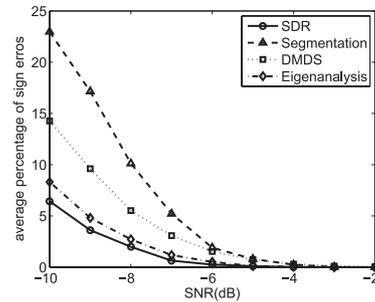


Fig. 1 Performance for the short-code DS-SS signal under different SNR, $L = G = 30, M = 30$.

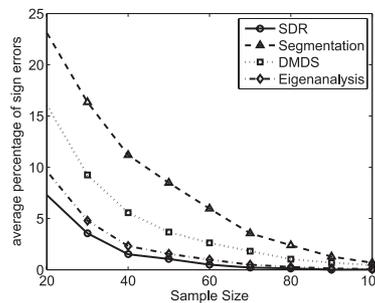


Fig. 2 Performance for the short-code DS-SS signal under different M , $L = G = 30, \rho = -9$ dB.

using average percentage of sign errors of estimated spreading sequence over 500 Monte Carlo runs. In each run, the spreading sequence \mathbf{c} and symbol sequence \mathbf{b} are generated randomly, and we assume $D = 20$ in the randomization procedure.

In the first experiment, we examine the capability of the proposed SDR estimator for short-code DS-SS signal. The eigenanalysis estimator [1], [2], which is specifically developed for the short-code DS-SS signals, is considered as a benchmark estimator for comparison. Figure 1 demonstrates the performance comparison against the SNR for a short-code DS-SS signal with $L = G = 30$ and $M = 30$, and the performance comparison against the sample size M with $\rho = -9$ dB is shown in Fig. 2. It can be seen that for the short-code DS-SS signal, a slight performance improvement over eigenanalysis estimator is provided by SDR estimator in the case of low numbers of data samples and low SNR region, while the performance of both segmentation and DMDS estimators degrades dramatically.

In the second experiment, we investigate the performance of the proposed SDR estimator for long-code DS-SS signal. We consider a long-code DS-SS signal with $L = 127$ and $G = 30$. Figure 3 shows the performance of various estimators with $M = 30$ under different SNR. We can observe that the SDR estimator significantly outperforms the segmentation and DMDS estimators, especially in the low SNR region. Then, we fix the SNR at $\rho = -9$ dB, the performance versus sample size M is shown in Fig. 4. We can see that SDR estimator exhibits much better performance than the other two estimators in the case of low numbers of data

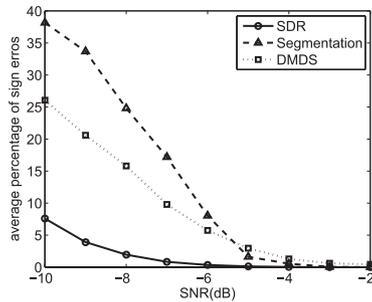


Fig. 3 Performance for the long-code DS-SS signal under different SNR, $L = 127$, $G = 30$, $M = 30$.

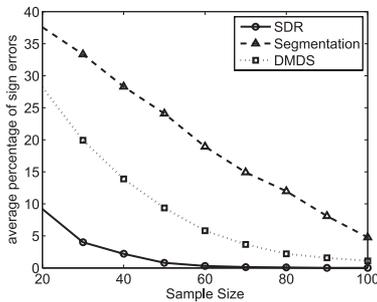


Fig. 4 Performance for the long-code DS-SS signal under different M , $L = 127$, $G = 30$, $\rho = -9$ dB.

samples.

To evaluate the performance of the proposed estimator under multipath fading channels, the signal is assumed to undergo three resolvable paths with the following impulse response:

$$h(n) = 1.2 - 0.8\delta(n-1) + 0.6\delta(n-2) \quad (15)$$

The above channel model has been adopted in [1]. Fixing the SNR at $\rho = -9$ dB, Fig. 5 and Fig. 6 demonstrate the performance comparison against the sample size for the short-code and long-code DS-SS signals used in the previous experiments, respectively. Compared with Fig. 2 and Fig. 4, it can be seen that multipath fading channel leads to performance degradations for all estimators. However, the proposed estimator still significantly outperforms the existing estimators and provides good estimation performance with a minor increase in sample size threshold.

5. Conclusion

In this letter, we have proposed a SDR approach to spreading sequence estimation both for long-code as well as for short-code DS-SS signals. The proposed approach provides significant performance improvements over existing estimators, especially in the case of low numbers of data samples and low SNR situations.

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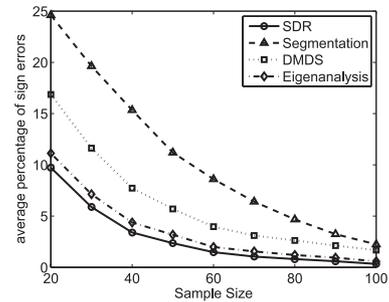


Fig. 5 Performance for the short-code DS-SS signal under multipath fading channel.

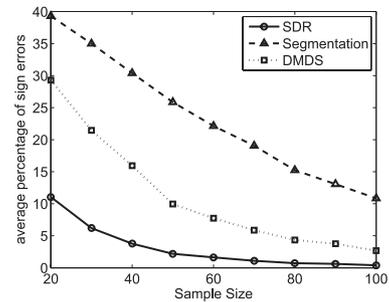


Fig. 6 Performance for the long-code DS-SS signal under multipath fading channel.

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