

Reduced Complexity Demodulation and Equalization Scheme for Differential Impulse Radio UWB Systems with ISI

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Abstract—In this paper, we consider the demodulation and equalization problem of differential Impulse Radio (IR) Ultra-WideBand (UWB) Systems with Inter-Symbol-Interference (ISI). The differential IR UWB systems have been extensively discussed recently [1], [2], [3], [4], [5], [6], [7]. The advantage of differential IR UWB systems include simple receiver frontend structure. One challenge in the demodulation and equalization of such systems with ISI is that the systems have a rather complex model. The input and output signals of the systems follow a second-order Volterra model [7]. Furthermore, the noise at the output is data dependent. In this paper, we propose a reduced-complexity joint demodulation and equalization algorithm. The algorithm is based on reformulating the nearest neighborhood decoding problem into a mixed quadratic programming and utilizing a semi-definite relaxation. The numerical results show that the proposed demodulation and equalization algorithm has low computational complexity, and at the same time, has almost the same error probability performance compared with the maximal likelihood decoding algorithm.

I. INTRODUCTION

Ultra-WideBand (UWB) communication systems have attracted much attention recently. The UWB communications have the advantages of robustness due to multi-path diversity, low possibilities of intercept and high location estimation accuracy. UWB systems are favorable choices for short range high bit rate communications or medium-to-long range low bit rate communications. For example, UWB systems have been considered for video communications in Wireless Personal Area Networks (WPAN). In this case, the transmission rates can be as high as 400M bits per second. UWB communication systems have also been considered for Wireless Sensor Networks (WSN) as a low-power and low-cost solution. The FCC (US Federal Communications Commission) has recently approved the use of UWB communications and allocated a spectrum range of 7.5 GHz for UWB communications.

A communication system is considered to be a UWB system, if the system's bandwidth spans more than 1.5 GHz, or 25% of the center frequency. The UWB systems transmit data by sending pulses, each with very small time duration. For one transmitted pulse, a large number of replicas of the same pulse are received at the receiver side due to multi-path. The number of resolvable multi-paths can be as high as more than 100 as shown in [8]. As a consequence, multi-path diversities are automatically achieved. However, accurate channel estimation can be quite complex and difficult.

The existing approaches for UWB communications include, Direct-Sequence (DS) UWB, Multi-Band (MB) UWB, and low-complexity non-coherent Impulse Radio (IR) UWB systems. The DS-UWB systems use direct sequence spreading technique to convert information signals into wide-band signals, [9], [10]. Under the condition that the channel estimation is accurate, the RAKE receiver is the optimal demodulation scheme. However, the channel estimation for UWB channels is difficult and complex. Without the information about the correct RAKE weights, the systems suffer a performance loss by using sub-optimal RAKE structures (for example, equal weight combining).

MB-UWB systems are recently proposed and discussed in [11], [12], [13]. The MB-UWB systems use the Orthogonal Frequency Division Multiplexing (OFDM) technology. The advantage of MB-UWB systems include higher achievable bit rates, flexibility in spectrum occupation, good coexistence with narrow band communications. The disadvantages include complex architectures, and high power consumption. The third class of UWB systems is the non-coherent IR UWB systems. In such systems, complete channel estimation is not required. Therefore, the channel estimation constraint is greatly relaxed.

In this paper, we consider a low-complexity non-coherent IR UWB system - the differential IR UWB system proposed in [4]. In the differential IR UWB systems, the transmitted information is differentially encoded. At the receiver side, a low-complexity Autocorrelation (AcR) receiver is adopted. The decoding decision variables are autocorrelations

$$\int_{t_0}^{t_1} r(t)r(t+\delta)dt, \quad (1)$$

where, $r(t)$ is the received signal, and δ is the time difference between two consecutive pulses. The integral can be implemented either in the analog domain or in the digital domain. In both cases, the decoder architecture is largely simplified.

One problem of the AcR receiver is that the transmitted messages and the receiver decoding decision variables follow a nonlinear second-order Volterra model, especially when Inter-Symbol-Interference (ISI) is present in the systems [7]. The maximal-likelihood sequential decoders can be adopted, however their computational complexities generally grow exponentially with the length of delay spread.

In this paper, we propose a reduced-complexity demodulation and equalization algorithm. The algorithm is based

on a reformulation of the nearest neighborhood decoding problem into a mixed quadratic programming and a Semi-Definite Programming (SDP) relaxation. The computational complexity of the proposed algorithm grows only polynomially with respect to the block length and is independent of the length of delay spread. We show by simulation results that the performance loss caused by the proposed sub-optimal demodulation algorithm is negligible.

SDP relaxation has been previously adopted to solve decoding problems and combinatorial optimization problems. In [14], an approximation algorithm for maximum cut problem based on SDP relaxation has been proposed. Detection algorithms for MIMO channels based on SDP relaxation have also been proposed in [15], [16], [17], [18], [19]. For interested readers, a review of SDP optimization can be found in [20].

The rest of this paper is organized as follows. In Section II, we describe the system model. We present the proposed demodulation and equalization algorithm in Section III. Numerical results are presented in Section IV. Conclusions are presented in Section V.

Notation: We use the symbol \mathcal{S} to denote the set of symmetric matrices. Matrices are denoted by upper bold face letters and column vectors are denoted by lower bold face letters. We use $\mathbf{A} \succeq 0$ to denote that the matrix \mathbf{A} is positive semi-definite. The symbol \otimes is used to denote the Kronecker product. We use $\mathbf{A}_{i,j}$ to denote the element of the matrix \mathbf{A} at the i -th row and j -th column. We use \mathbf{a}_i to denote the i -th element of the vector \mathbf{a} . We use \mathbf{A}^T and \mathbf{a}^T to denote the transpose of the matrix \mathbf{A} and the vector \mathbf{a} respectively. We use $\text{tr}(\mathbf{A})$ to denote the trace of the matrix \mathbf{A} . The function $\text{sign}(\cdot)$ is defined as,

$$\text{sign}(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ -1, & \text{otherwise} \end{cases} \quad (2)$$

II. SYSTEM MODEL

We assume that the message is transmitted in a block by block fashion. The transmitted signal in one block is

$$s(t) = \sum_{n=0}^{N_b-1} \sum_{i=0}^{N_p-1} a_i[n] \bar{w}(t - t_i[n]) \quad (3)$$

where $\bar{w}(t)$ is the transmitted pulse, $a_i[n]$ is the pulse polarity for the i -th pulse of the n -th symbol, $t_i[n]$ is the pulse time for the i -th pulse of the n -th symbol. Each block has N_b symbols, and each symbol corresponds to N_p pulses.

Denote the data symbol by $d[n] \in \{-1, +1\}$. The data symbols are differentially encoded as,

$$a_i[n] = \begin{cases} a_{N_p-1}[n-1]d[n-1]b_{N_p-1}, & \text{if } i = 0 \\ a_{i-1}[n]d[n]b_{i-1}, & \text{otherwise} \end{cases} \quad (4)$$

where, $b_0, b_1, \dots, b_{N_p-1}$ is the pseudo-random amplitude code sequence, $b_i \in \{-1, +1\}$. The pulse time

$$t_i[n] = nT_s + c_i \quad (5)$$

where T_s is the symbol duration, c_i is the relative pulse timing. The relative pulse timing c_i is related to the pseudo-random

delay hopping code $\{D_i\}$,

$$D_i = \begin{cases} T_s + c_0 - c_{N_p-1}, & \text{if } i = N_p - 1 \\ c_{i+1} - c_i, & \text{otherwise} \end{cases} \quad (6)$$

The pseudo-random amplitude code and delay hopping code are used to facilitate multiple access.

The received signal is,

$$r(t) = \sum_{n=0}^{N_b-1} \sum_{i=0}^{N_p-1} a_i[n]g(t - t_i[n]) + n(t), \quad (7)$$

where, $g(t)$ is the channel response for the pulse $\bar{w}(t)$, $n(t)$ is the noise. The receiver front end is shown in Fig. 1. Denote the decoding decision variable for the n -th symbol by $z[n]$,

$$y_i[n] = \int_{t_i[n]}^{t_i[n]+T_I} r(t)r(t+D_i)dt, \quad (8)$$

$$z[n] = \sum_{i=0}^{N_p-1} y_i[n]b_i, \quad (9)$$

where, T_I is the integral time.

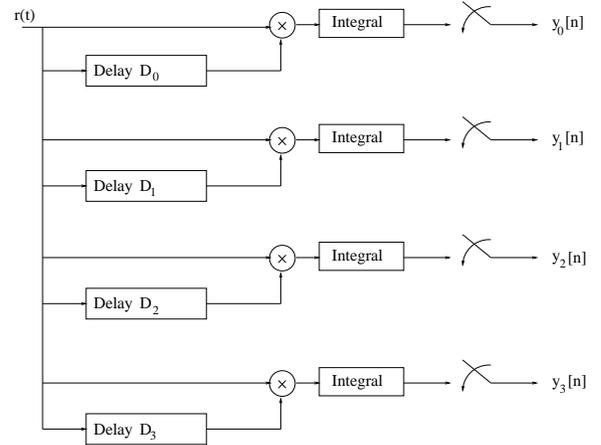


Fig. 1. Block diagram of the autocorrelator receiver

Let us define

$$I_g(t_1, t_2; \tau) = \int_{t_1}^{t_2} g(t)g(t+\tau)dt \quad (10)$$

Denote the data vector by \mathbf{d} ,

$$\mathbf{d} = [d_0, d_1, \dots, d_{N_b-1}]^T. \quad (11)$$

Define the column vector,

$$\mathbf{a} = [a_0[0], a_1[0], \dots, a_0[n], a_1[n], \dots, a_{N_p-1}[N_b-1]]^T. \quad (12)$$

Neglecting noise, we have

$$y_i[n] = \mathbf{a}^T \mathbf{A}_i[n] \mathbf{a}. \quad (13)$$

In the above equation, $\mathbf{A}_i[n]$ is a matrix, such that the $(n'N_p + i' + 1, n''N_p + i'' + 1)$ element is

$$I_g(t_i[n] - t_{i'}[n'], t_i[n] - t_{i'}[n'] + T_I; t_{i'}[n'] - t_{i''}[n''] + D_i). \quad (14)$$

Finally, the decoding decision variables can be written as,

$$z[n] = \sum_{i=0}^{N_p-1} b_i \mathbf{a}^T \mathbf{A}_i[n] \mathbf{a} = \mathbf{a}^T \mathbf{B}[n] \mathbf{a}, \quad (15)$$

where, $\mathbf{B}[n] = \sum_{i=0}^{N_p-1} b_i \mathbf{A}_i[n]$.

The vector \mathbf{a} can be written as,

$$\mathbf{a} = \mathbf{Q}(\mathbf{r} + \mathbf{P}\mathbf{d}). \quad (16)$$

In the above equation, \mathbf{Q} is a diagonal matrix,

$$\mathbf{Q} = \text{diag}[1, b_0, b_0 b_1, \dots], \quad (17)$$

$$[\mathbf{Q}]_{k,k} = \prod_{j=0}^{k-2} b_{j \bmod N_p}. \quad (18)$$

The matrix \mathbf{P} ,

$$\mathbf{P} = \mathbf{I}_{N_b} \otimes \mathbf{s}, \quad (19)$$

where, \mathbf{I}_{N_b} is an identical matrix, and \mathbf{s} is a vector with length N_p of alternating 0, 1,

$$\mathbf{s} = [0, 1, 0, 1, \dots, 1]^T. \quad (20)$$

The vector \mathbf{r} is,

$$\mathbf{r} = \mathbf{i}_{N_b} \otimes (\mathbf{i}_{N_p} - \mathbf{s}), \quad (21)$$

where \mathbf{i}_{N_b} and \mathbf{i}_{N_p} are the all one column vectors with length N_b and N_p respectively.

With the above notation, the second-order Volterra model of the system is,

$$z[n] = (\mathbf{r} + \mathbf{P}\mathbf{d})^T \mathbf{Q}^T \mathbf{B}[n] \mathbf{Q}(\mathbf{r} + \mathbf{P}\mathbf{d}) + \text{noise terms}, \quad (22)$$

where the noise terms are data dependent as shown in [7].

III. JOINT DEMODULATION AND EQUALIZATION ALGORITHM

In this section, we present the proposed demodulation and equalization algorithm. The algorithm is obtained by formulating the demodulation problem as a nearest neighborhood decoding problem, reformulating into mixed quadratic programming, and using SDP relaxation.

In the first step, we formulate the demodulation problem as a nearest neighborhood decoding problem as follows.

$$\min \sum_{n=0}^{N_b-1} \{z[n] - (\mathbf{r} + \mathbf{P}\mathbf{d})^T \mathbf{Q}^T \mathbf{B}[n] \mathbf{Q}(\mathbf{r} + \mathbf{P}\mathbf{d})\}^2 \quad (23)$$

$$\text{subject to } d_n \in \{-1, 1\}. \quad (24)$$

Note that the nearest neighborhood decoding is not the maximal likelihood decoding in the considered scenario, because noise is signal dependent.

The above nearest neighborhood decoding problem can be reformulated as a mixed quadratic programming by introducing auxiliary variables s_n , $n = 0, \dots, N_b - 1$.

$$\min \sum_{n=0}^{N_b-1} (s_n)^2 \quad (25)$$

$$\text{subject to} \quad (26)$$

$$s_n = z[n] - (\mathbf{r} + \mathbf{P}\mathbf{d})^T \mathbf{Q}^T \mathbf{B}[n] \mathbf{Q}(\mathbf{r} + \mathbf{P}\mathbf{d}), \quad (27)$$

$$d_n \in \{-1, 1\}. \quad (28)$$

Now, we claim that the above mixed quadratic programming is equivalent to the following matrix optimization problem.

$$\min \sum_{n=2}^{N_b+1} \mathbf{U}_{n,n} \quad (29)$$

$$\text{subject to} \quad (30)$$

$$\begin{aligned} \mathbf{U}_{1,n} &= z[n-2] - \mathbf{r}^T \mathbf{Q}^T \mathbf{B}[n-2] \mathbf{Q} \mathbf{r} \\ &\quad - \mathbf{r}^T \mathbf{Q}^T \mathbf{B}[n-2] \mathbf{Q} \mathbf{P} \mathbf{d}' \\ &\quad - \mathbf{r}^T \mathbf{Q}^T \mathbf{B}[n-2] \mathbf{Q} \mathbf{P} \mathbf{d}' \\ &\quad - \text{tr} \{ \mathbf{D}' \mathbf{P}^T \mathbf{Q}^T \mathbf{B}[n-2] \mathbf{Q} \mathbf{P} \}, \end{aligned} \quad (31)$$

$$\text{for } n = 2, \dots, N_b + 1$$

$$\mathbf{U}_{1,1} = 1, \quad (32)$$

$$\mathbf{U}_{n,n} = 1, \text{ for } n = N_b + 2, \dots, 2N_b + 1, \quad (33)$$

$$\mathbf{d}' = [\mathbf{U}_{1,N_b+2}, \dots, \mathbf{U}_{1,2N_b+1}]^T, \quad (34)$$

$$\mathbf{U} \in \mathcal{S}, \quad (35)$$

$$\mathbf{U} \succeq 0, \quad (36)$$

$$\mathbf{U} \text{ has rank one}, \quad (37)$$

where, \mathbf{U} denotes a matrix of size $2N_b + 1$ by $2N_b + 1$, \mathbf{D}' denote the sub-matrix of \mathbf{U} formed by selecting the last N_b rows and columns, and the optimization variables are the elements of the matrix \mathbf{U} . Because the matrix \mathbf{U} has rank one, is symmetric and positive semi-definite, it is well known [20] that there exists a vector \mathbf{u} , such that

$$\mathbf{U} = \mathbf{u}\mathbf{u}^T. \quad (38)$$

If we further assume that $u_1 = 1$, then the vector \mathbf{u} is unique. In addition, there is an one-to-one correspondence between the solution of the mixed quadratic programming and the solution of the matrix optimization problem,

$$\mathbf{u} = [1, s_0, s_2, \dots, s_{N_b-1}, d_0, d_1, \dots, d_{N_b-1}]^T. \quad (39)$$

Therefore, the mixed quadratic programming is equivalent to the matrix optimization problem.

In the matrix optimization problem, all objective function and constraints are convex except the rank one constraint in Eq. 37. If the rank one constraint is relaxed, then we obtain a convex relaxation (SDP relaxation). The convex optimization problem can then be efficiently solved by polynomial-time algorithms and softwares, for example, by using the SeDuMi package [21]. Previous research has shown that such SDP relaxations are tight approximations to the original problems [14]. Near-optimal solutions of the original problems can be obtained from SDP relaxations by random rounding.

Finally, the proposed joint demodulation and equalization algorithm consists of two steps. In the first step, the following SDP relaxation problem is solved.

$$\min \sum_{n=2}^{N_b+1} \mathbf{U}_{n,n} \quad (40)$$

subject to (41)

$$\begin{aligned} \mathbf{U}_{1,n} = & z[n-2] - \mathbf{r}^T \mathbf{Q}^T \mathbf{B}[n-2] \mathbf{Q} \mathbf{r} \\ & - \mathbf{r}^T \mathbf{Q}^T \mathbf{B}[n-2] \mathbf{Q} \mathbf{P} \mathbf{d}' \\ & - \mathbf{r}^T \mathbf{Q}^T \mathbf{B}[n-2]^T \mathbf{Q} \mathbf{P} \mathbf{d}' \end{aligned} \quad (42)$$

$$- \text{tr} \{ \mathbf{D}' \mathbf{P}^T \mathbf{Q}^T \mathbf{B}[n-2] \mathbf{Q} \mathbf{P} \},$$

$$\text{for } n = 2, \dots, N_b + 1$$

$$\mathbf{U}_{1,1} = 1, \quad (43)$$

$$\mathbf{U}_{n,n} = 1, \text{ for } n = N_b + 2, \dots, 2N_b + 1, \quad (44)$$

$$\mathbf{d}' = [\mathbf{U}_{1,N_b+2}, \dots, \mathbf{U}_{1,2N_b+1}]^T, \quad (45)$$

$$\mathbf{U} \in \mathcal{S}, \quad (46)$$

$$\mathbf{U} \succeq 0, \quad (47)$$

\mathbf{D}' is the submatrix of \mathbf{U} formed by selecting the last N_b rows and columns. (48)

In the second step, the demodulation decision is made by thresholding,

$$d_n = \text{sign}(\mathbf{U}_{1,n+N_b+2}). \quad (49)$$

IV. NUMERICAL RESULTS

In this section, we present simulation results for the proposed demodulation and equalization scheme. We assume that the channel can be modeled by the S-V model [22]. The received signal for each transmitted pulse $\bar{w}(t)$ is,

$$g(t) = \sum_{j=1}^{N_m} \alpha_j w(t - \delta_j). \quad (50)$$

In the above equation, $w(t)$ is the second derivative Gaussian monocycle,

$$w(t) = \left[1 - 4\pi (t/\tau_m)^2 \right] \exp \left\{ -2\pi (t/\tau_m)^2 \right\} \quad (51)$$

where $\tau_m = 0.2877$ nanosecond. N_m is the total number of multiple paths. α_j and δ_j are amplitude and delay of the j -th path.

We assume that the delays of the paths follow the Poisson process with the expected interval between two consecutive paths being 10 nanoseconds. The amplitude is Raleigh distributed, such that the expectation of the amplitude α_j is $\exp(-\delta_j/T_e)$, where $T_e = 20$ nanoseconds. We assume that the amplitudes and delays α_j , δ_j vary slowly, so that the matrices $\mathbf{B}[n]$ can be accurately estimated (for example, by using pilot signals), and considered perfectly known at the demodulator.

For the transmitted signal, we assume that each block has $N_b = 10$ symbols and each symbol corresponds to $N_p = 4$ pulses. The pseudo-random delay hopping code is [1.7, 1.9, 2.1, 2.3]. The symbol duration $T_s = 8$ nanoseconds. The integral time $T_I = T_s = 8$ nanoseconds.

We illustrate the bit error probability of the proposed demodulation and equalization algorithm in three different cases. In the first case, we assume that the delay spread extends over a range of 200 nanoseconds. Therefore, there exists severe non-linearity in the system. The bit error probability of the proposed scheme for this case is shown in Fig. 2. The bit error probability of the maximal likelihood detection algorithm is also plotted.

In the second case, we assume that the delay spread extends over a range of 30 nanoseconds. The non-linearity is mild in this case. The bit error probabilities of the proposed scheme and the maximal likelihood decoding are shown in Fig. 3. In the third case, we assume that there is no ISI. And the bit error probabilities are shown in Fig. 4. From all these results, we conclude that even though the proposed scheme is sub-optimal, the performance loss is negligible.

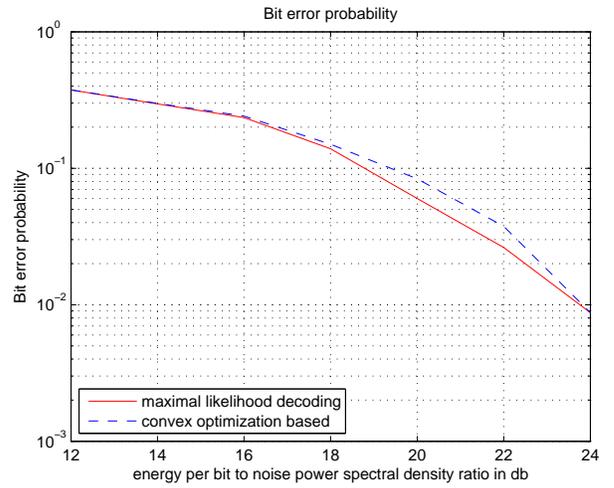


Fig. 2. Bit error probabilities in the case of severe non-linearity. The length of delay spread is 200 nanoseconds. The solid curve represents the bit error probabilities of the maximal likelihood detection algorithm. The dashed curve represents the bit error probabilities of the proposed reduced complexity detection algorithm. The X-axis shows energy per bit to noise power spectral density ratio E_b/N_0 in dB.

V. CONCLUSION

In this paper, we propose a convex optimization based demodulation and equalization algorithm with low complexity for the differential IR UWB systems. The complexity of the proposed algorithm grows polynomially with respect to the blocklengths, and is independent of the length of delay spread. Even though the proposed algorithm is sub-optimal, we show by simulation results that the performance loss is negligible. The proposed demodulation and equalization algorithm is a near-optimal algorithm with significantly reduced computational complexity.

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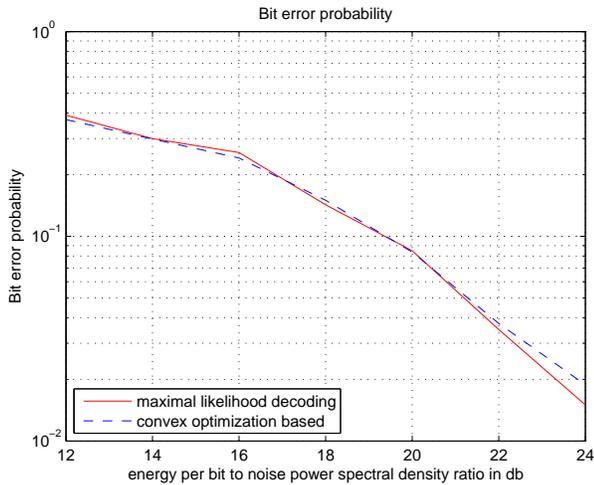


Fig. 3. Bit error probabilities in the case of mild non-linearity. The length of delay spread is 30 nanoseconds. The solid curve represents the bit error probabilities of the maximal likelihood detection algorithm. The dashed curve represents the bit error probabilities of the proposed reduced complexity detection algorithm. The X-axis shows energy per bit to noise power spectral density ratio E_b/N_0 in dB.

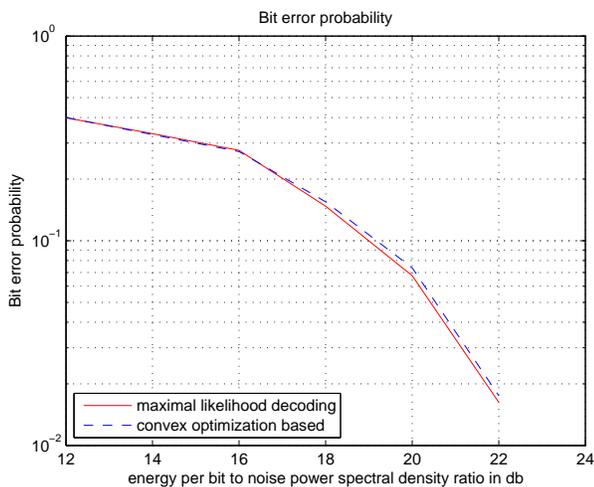


Fig. 4. Bit error probabilities in the case of linear channels. The frequency response of the channel is flat. The solid curve represents the bit error probabilities of the maximal likelihood detection algorithm. The dashed curve represents the bit error probabilities of the proposed reduced complexity detection algorithm. The X-axis shows energy per bit to noise power spectral density ratio E_b/N_0 in dB.

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