A Simple FFSK Modulator and its Coherent Demodulator

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Summary
In this letter, a simple binary Fast Frequency Shift Keying (FFSK) modulator and its coherent demodulator is proposed. The performance of the proposed modem is in between a coherently detected and a non-coherently detected binary FSK, but its bandwidth requirement is the same as for the Minimum Shift Keying (MSK) modulation.

Key words:
CPFSK, MSK, FFSK.

1. The proposed FFSK modem
A continuous-phase, frequency-shift keying (CPFSK) signal can be described as a phase-modulated signal, as shown by:

\[
s(t) = \sqrt{2E_b} \cos \left[ 2\pi f_r t + \Theta(t) \right]
\]

(1)

where \(E_b\) is the average energy per bit and \(T_b\) is the bit duration. In a given symbol interval \(\Theta(t)\) increases or decreases linearly, depending on the desired transmitted tone, \(f_1\) or \(f_2\), respectively, as described by:

\[
\Theta(t) = \frac{\pi h}{T_b} \int_0^t b(t) dt
\]

(2)

where \(\Theta(0)\) accounts for the accumulated phase history until instant \(t = 0\), \(h\) is a measure of the frequency deviation and, for the binary case, \(b(t) \in \{-1, 1\}\) is the waveform related to the information sequence, such that \(a\), \(-1\) represents a bit 0 and \(a\), \(+1\) represents a bit 1.

If \(h = \frac{1}{2}\) in (2), an MSK (Minimum Shift Keying) or FFSK (Fast Frequency Shift Keying) signal \([1]\) is obtained.

The realization of (1) can be accomplished via a VCO (voltage controlled oscillator) having \(h(t)\) as its input, and configured with free-running frequency \(f_r\) Hz and gain \(1/(4T_b)\) Hz/volt.

It is known that an FFSK signal has the most compact spectrum among the orthogonal CPFSK modulations. It is also known that, by exploring the phase information in the modulated signal, a coherent FSK receiver exhibits a performance 3 dB better than that obtained with a non-coherent FSK receiver \([2, pp. 387-396]\). In this letter it is proposed to detect the spectrally-efficient FFSK signal by using a modified version of a simple FSK correlator detector. In this case, as a consequence of using the frequency separation of \(1/(2T_b)\) Hz, in several correlation intervals there will be no phase coherence between the modulated signal and the base-functions with the same frequency, a behavior that would lead to detection errors. This is illustrated in Figure 1, were are plotted a binary FFSK signal and the cosine base-functions with frequencies \(f_1\) and \(f_2\) separated by \(1/(2T_b)\) Hz. It can be noticed from this figure that when no phase coherence occurs, the FFSK signal is at 180° out of phase from the corresponding base-function. Nevertheless, by comparing the magnitudes of the correlators outputs, it is still possible to make correct decisions. The proposed structure for this sub-optimal simple FFSK receiver is shown in Figure 2.

2. Performance analysis
Differently from the conventional MSK coherent receiver, the proposed receiver shown in Figure 2 is not exploring
any phase information. Then, it is expected a worse performance as compared to the one provided by a conventional MSK receiver on an additive white Gaussian noise (AWGN) channel. A numerical calculation of the bit error probability $P_e$ for the proposed system is presented in what follows.

Consider a reference Gaussian density function with variance $\sigma^2 = N_0/2$ and mean $\mu_x$ be written as $p_x(x, \mu_x)$. Also, consider $x_1$ and $x_2$ as the samples of the random variables $X_1$ and $X_2$ at the output of the upper and lower correlators shown in Figure 2, respectively. Referring to this figure and considering equally likely symbols, it can be written the following set of conditional densities, which are plotted in Figure 3.

$$p_{X_1}(x_1 | 1) = \frac{1}{2} p_x(x_1, \sqrt{E_b}) + \frac{1}{2} p_x(x_1, -\sqrt{E_b})$$ (3)

$$p_{X_1}(x_1 | 0) = p_x(x_1, 0)$$ (4)

$$p_{X_1}(x_1 | 1) = p_x(x_1, 0)$$ (5)

$$p_{X_2}(x_2 | 0) = \frac{1}{2} p_x(x_2, \sqrt{E_b}) + \frac{1}{2} p_x(x_2, -\sqrt{E_b})$$ (6)

Due to the quadrature carriers, the random variables $|X_1|$ and $-|X_2|$ are independent and the density function of the decision variable $Y = |X_1| - |X_2|$ is given by the convolution between the density functions of $|X_1|$ and $-|X_2|$:

$$p_{Y} (y | 1) = p_{P_{Y|1}}(y | 1) * p_{P_{Y|1}}(-y | 1)$$ (12)

These conditional densities are plotted in Figure 4 for $E_b = 1$, and for $E_b/N_0 = 2$ dB and 8 dB. It can be noticed that they are not Gaussian, although, for high $E_b/N_0$ they tend to become (visually) more similar to Gaussian densities.

The conditional densities related to the outputs of the absolute-value operations shown in Figure 2 are:

$$p_{P_{Y|1}}(|X_1| | 1) = [p_{X_1}(x_1 | 1) + p_{X_1}(-x_1 | 1)] u(x_1)$$ (7)

$$p_{P_{Y|1}}(|X_1| | 0) = [p_{X_1}(x_1 | 0) + p_{X_1}(-x_1 | 0)] u(x_1)$$ (8)

$$p_{P_{Y|1}}(|X_2| | 1) = [p_{X_2}(x_2 | 1) + p_{X_2}(-x_2 | 1)] u(x_2)$$ (9)

$$p_{P_{Y|1}}(|X_2| | 0) = [p_{X_2}(x_2 | 0) + p_{X_2}(-x_2 | 0)] u(x_2)$$ (10)

where $u(x)$ is the unit-step function.

A simulation of the proposed system was carried out. Numerical and simulation results agreed and showed that the performance of this system lies in between a coherently detected and a non-coherently detected binary FSK, as shown in Figure 5, and is approximately 3.05 dB worse than the $P_e$ obtained with a conventional MSK receiver.

This is an attractive result, since the $P_e$ curves for the coherent and the non-coherent FSK differs asymptotically in about only 1 dB, and an MSK transmitted signal that has the most compact spectrum among the orthogonal CPFSK modulations is being used.
Fig. 5. Performance results for MSK, coherent and non-coherent BFSK and for the receiver depicted in Figure 2. The channel is AWGN.

In Figure 5 it is also plotted the theoretical $P_e$ obtained by considering that the noise in the decision variable is Gaussian, with mean and standard deviation given according to the densities (11) and (12). As can be noticed from Figure 5, this approximation tends to become poorer as $E_b/N_0$ increases, despite the visual similarity with Gaussian densities for high values of $E_b/N_0$, mentioned earlier in this letter and shown in Figure 4.

References


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