LETTER

A Simple FFSK Modulator and its Coherent Demodulator

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Summary
In this letter, a simple binary Fast Frequency Shift Keying (FFSK) modulator and its coherent demodulator is proposed. The performance of the proposed modem is in between a coherently detected and a non-coherently detected binary FSK, but its bandwidth requirement is the same as for the Minimum Shift Keying (MSK) modulation.
Key words:
CPFSK, MSK, FFSK.

1. The proposed FFSK modem

A continuous-phase, frequency-shift keying (CPFSK) signal can be described as a phase-modulated signal, as shown by:

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left[2\pi f_c t + \theta(t)\right] \qquad (1)$$

where E_b is the average energy per bit and T_b is the bit duration. In a given symbol interval $\theta(t)$ increases or decreases linearly, depending on the desired transmitted tone, f_1 or f_2, respectively, as described by:

$$\theta(t) = \theta(0) + \frac{\pi h}{T_b} \int_0^t b(t) dt \qquad (2)$$

where $\theta(0)$ accounts for the accumulated phase history until instant $t = 0$, h is a measure of the frequency deviation and, for the binary case, $b(t) \in \{\pm 1\}$ is the waveform related to the information sequence, such that a −1 represents a bit 0 and a +1 represents a bit 1.

If $h = ½$ in (2), an MSK (Minimum Shift Keying) or FFSK (Fast Frequency Shift Keying) signal [1] is obtained.

The realization of (1) can be accomplished via a VCO (voltage controlled oscillator) having $b(t)$ as its input, and configured with free-running frequency f_c Hz and gain $1/(4T_b)$ Hz/volt.

It is known that an FFSK signal has the most compact spectrum among the orthogonal CPFSK modulations. It is also known that, by exploring the phase information in the modulated signal, a coherent FFSK receiver exhibits a performance 3 dB better than that obtained with a non-coherent FSK receiver [2, pp. 387-396]. In this letter it is proposed to detect the spectrally-efficient FFSK signal by using a modified version of a simple FSK correlator detector. In this case, as a consequence of using the frequency separation of $1/(2T_b)$ Hz, in several correlation intervals there will be no phase coherence between the modulated signal and the base-functions with the same frequency, a behavior that would lead to detection errors. This is illustrated in Figure 1, were are plotted a binary FFSK signal and the cosine base-functions with frequencies f_1 and f_2 separated by $1/(2T_b)$ Hz. It can be noticed from this figure that when no phase coherence occurs, the FFSK signal is at 180º out of phase from the corresponding base-function. Nevertheless, by comparing the magnitudes of the correlators outputs, it is still possible to make correct decisions. The proposed structure for this sub-optimal simple FFSK receiver is shown in Figure 2.

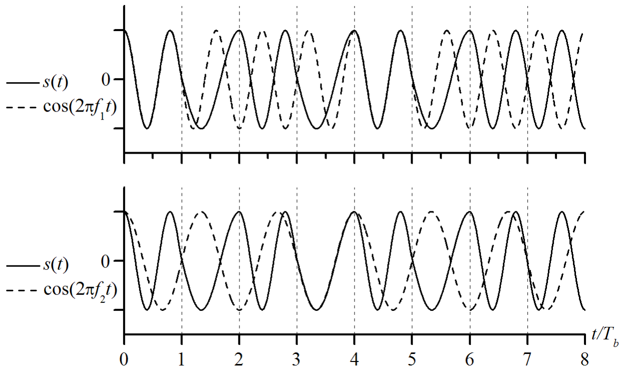

Fig. 1. FFSK signal $s(t)$ and base-functions.

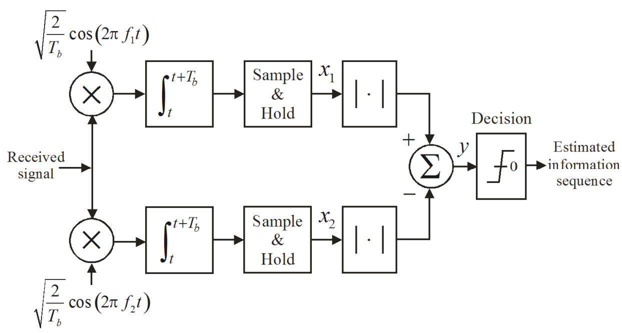

Fig. 2. Proposed coherent binary FFSK receiver.

2. Performance analysis

Differently from the conventional MSK coherent receiver, the proposed receiver shown in Figure 2 is not exploring

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any phase information. Then, it is expected a worse performance as compared to the one provided by a conventional MSK receiver on an additive white Gaussian noise (AWGN) channel. A numerical calculation of the bit error probability P_e for the proposed system is presented in what follows.

Consider a reference Gaussian density function with variance $\sigma^2 = N_0/2$ and mean μ_X be written as $p_X(x, \mu_X)$. Also, consider x_1 and x_2 as the samples of the random variables X_1 and X_2 at the output of the upper and lower correlators shown in Figure 2, respectively. Referring to this figure and considering equally likely symbols, it can be written the following set of conditional densities, which are plotted in Figure 3.

$$p_{X_1}(x_1 \mid 1) = \frac{1}{2} p_X\left(x, \sqrt{E_b}\right) + \frac{1}{2} p_X\left(x, -\sqrt{E_b}\right) \quad (3)$$

$$p_{X_1}(x_1 \mid 0) = p_X(x, 0) \quad (4)$$

$$p_{X_2}(x_2 \mid 1) = p_X(x, 0) \quad (5)$$

$$p_{X_2}(x_2 \mid 0) = \frac{1}{2} p_X\left(x, \sqrt{E_b}\right) + \frac{1}{2} p_X\left(x, -\sqrt{E_b}\right) \quad (6)$$

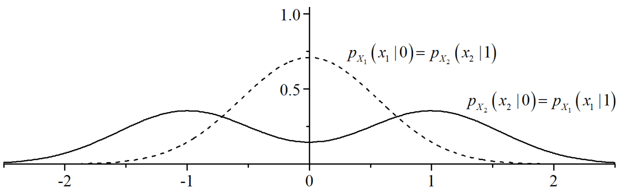

Fig. 3. Conditional densities for x_1 and x_2. $E_b = 1$ and $E_b/N_0 = 2$ dB.

The conditional densities related to the outputs of the absolute-value operations shown in Figure 2 are:

$$p_{|X_1|}(|x_1| \mid 1) = \left[p_{X_1}(x_1 \mid 1) + p_{X_1}(-x_1 \mid 1)\right] u(x_1) \quad (7)$$

$$p_{|X_1|}(|x_1| \mid 0) = \left[p_{X_1}(x_1 \mid 0) + p_{X_1}(-x_1 \mid 0)\right] u(x_1) \quad (8)$$

$$p_{|X_2|}(|x_2| \mid 1) = \left[p_{X_2}(x_2 \mid 1) + p_{X_2}(-x_2 \mid 1)\right] u(x_2) \quad (9)$$

$$p_{|X_2|}(|x_2| \mid 0) = \left[p_{X_2}(x_2 \mid 0) + p_{X_2}(-x_2 \mid 0)\right] u(x_2) \quad (10)$$

where $u(x)$ is the unit-step function.

Due to the quadrature carriers, the random variables $|X_1|$ and $-|X_2|$ are independent and the density function of the decision variable $Y = |X_1| - |X_2|$ is given by the convolution between the density functions of $|X_1|$ and $-|X_2|$:

$$p_Y(y \mid 1) = p_{|X_1|}(|x_1| \mid 1) * p_{|X_2|}(-|x_2| \mid 1) \quad (11)$$

$$p_Y(y \mid 0) = p_{|X_1|}(|x_1| \mid 0) * p_{|X_2|}(-|x_2| \mid 0) \quad (12)$$

These conditional densities are plotted in Figure 4 for $E_b = 1$, and for $E_b/N_0 = 2$ dB and 8 dB. It can be noticed that they are not Gaussian, although, for high E_b/N_0 they tend to become (visually) more similar to Gaussian densities.

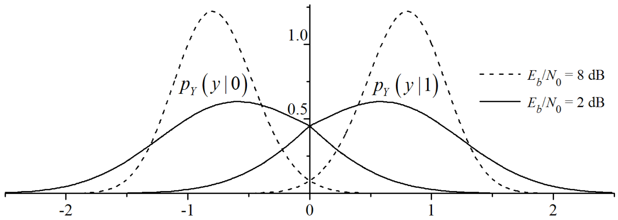

Fig. 4. Conditional densities for the decision variable y. $E_b = 1$, and $E_b/N_0 = 2$ dB or 8 dB.

Then, for equally likely symbols the probability of symbol error P_e for the receiver shown in Figure 2 can be numerically calculated through:

$$P_e = \int_0^\infty p_Y(y \mid 0)\, dy \quad (13)$$

A simulation of the proposed system was carried out. Numerical and simulation results agreed and showed that the performance of this system lies in between a coherently detected and a non-coherently detected binary FSK, as shown in Figure 5, and is approximately 3.05 dB worse than the P_e obtained with a conventional MSK receiver. This is an attractive result, since the P_e curves for the coherent and the non-coherent FSK differs asymptotically in about only 1 dB, and an MSK transmitted signal that has the most compact spectrum among the orthogonal CPFSK modulations is being used.

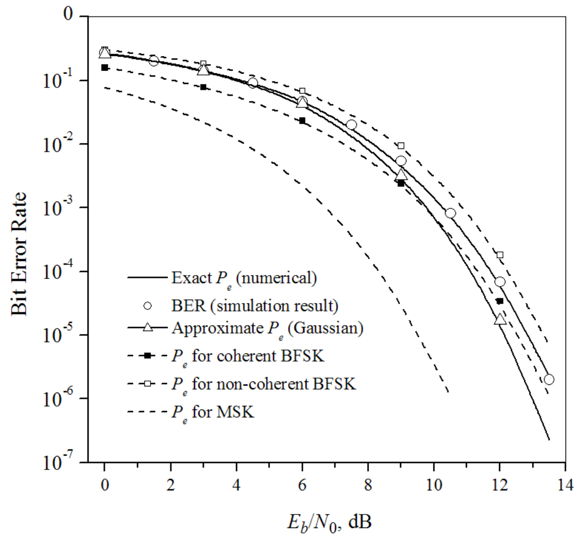

Fig. 5. Performance results for MSK, coherent and non-coherent BFSK and for the receiver depicted in Figure 2. The channel is AWGN.

In Figure 5 it is also plotted the theoretical P_e obtained by considering that the noise in the decision variable is Gaussian, with mean and standard deviation given according to the densities (11) and (12). As can be noticed from Figure 5, this approximation tends to become poorer as E_b/N_0 increases, despite the visual similarity with Gaussian densities for high values of E_b/N_0, mentioned earlier in this letter and shown in Figure 4.

References

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[2] S. Haykin, *Communication Systems*, 4th Edition - John Wiley & Sons, Inc.: New York, USA, 2001.

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