The Expectation Maximization (EM) Algorithm Applied to the MI-SBTVD Channel Estimation

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***Abstract*— In this paper we present results concerning the use of the EM algorithm as an improvement of the original MI-SBTVD channel estimation process. This original channel estimation is employed to initialize the EM refinement algorithm, which iterates with the space-time decoding to get an estimate of the channel frequency response. Simulation results show that the proposed algorithm can significantly improve the performance of the channel estimation process and of the overall system. It also reveals that it is possible to increase the system throughput by reducing the usage of redundant OFDM pilot sub-carriers.**

***Index Terms*—Expectation Maximization (EM) algorithm, channel estimation, MI-SBTVD, STC-OFDM, Alamouti.**

I. Introduction

In 2005, the Brazilian government financed projects for the developing of the SBTVD (acronym for the Brazilian Digital TV System). One of the components of this system is the MI-SBTVD, a novel modulation and coding subsystem [3]. The MI-SBTVD project was conducted by a consortium formed by the institutions: *Instituto Nacional de Telecomunicações - Inatel*, *Universidade Estadual de Campinas - Unicamp*, *Universidade Federal de Santa Catarina - UFSC*, *Centro Federal de Educação Tecnológica do Paraná - CEFET/PR* and the industry *Linear Equipamentos Eletrônicos S.A*. The MI-SBTVD employs OFDM (Orthogonal Frequency Division Multiplexing) signaling [1] and STBC (Space-Time Block Coding) coding [2] as a transmit diversity solution employing two transmit antennas and one receiving antenna.

For the transmit diversity to be effective, the magnitude and phase response of the channel for each OFDM carrier must be estimated at the receiver. The original MI-SBTVD channel estimation uses a linear interpolation of the estimates made through pilot carriers. In this work we apply the algorithm proposed by Xiaoqiang Ma [5], which is based on the EM (Expectation Maximization) algorithm, as a refinement on the MI-SBTVD channel estimation process. The original MI-SBTVD channel estimation is employed to initialize the EM refinement algorithm, which iterates with the space-time decoding to get a Maximum Likelihood estimate of the channel frequency response.

The next sections are organized as follows: in Section III the original channel estimation of the MI-SBTVD is described. Section III discusses the EM theory and its application in the MI-SBTVD. Simulation results are presented in Section IV, and Section V concludes the paper.

II. MI-SBTVD Original Channel Estimation

Following [3] and [4], Figure 1 depicts the simplified MI-SBTVD block diagram. It was assumed perfect synchronism between the transmitter and the receiver in this diagram and throughout this paper. The notation used in Figure 1 means: $a[b]_t$ is the sequence $a[b]$ of length b operated or generated in the discrete-time instant t and $c(k)$ is a discrete-time signal whose samples are indexed by the integer k.

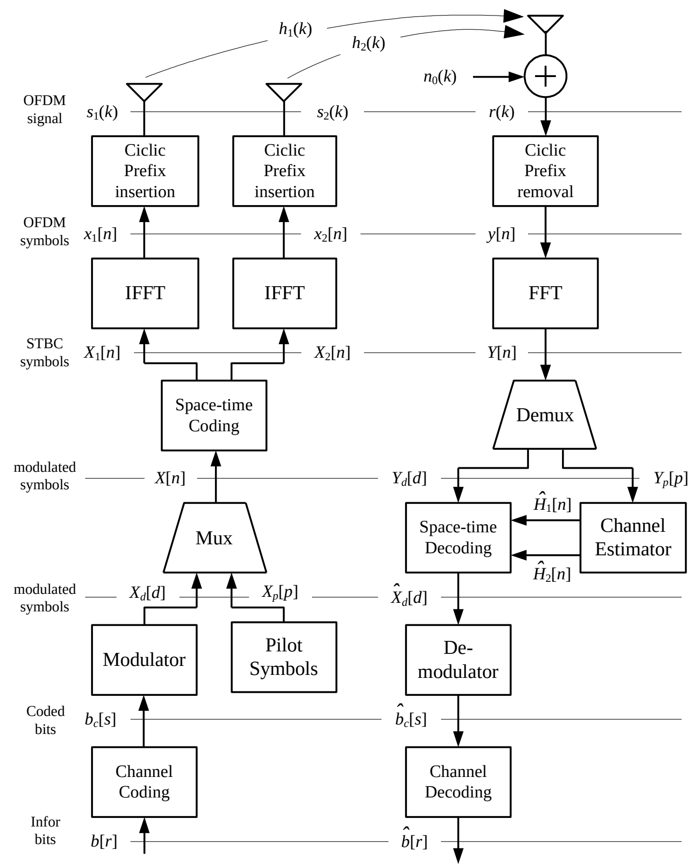

Figure 1. MI-SBTVD simplified block diagram.

The information sequence $b[r]$ is applied to the channel coding block, which adds redundancy due to FEC (Forward Error Correction), resulting in the sequence $b_c[s]$, $s > r$. Groups of M bits, $M = 2$, 4 or 6, of the sequence $b_c[s]$ are then mapped onto the 2^M symbols of a QPSK (Quadrature Phase Shift Keying), 16QAM (16-level Quadrature Amplitude Modulation) or 64QAM modulation. The sequence of complex symbols $X_d[d]$ and the sequence of Binary PSK pilot symbols $X_p[p]$ are multiplexed in a way that pilot and data symbols are, later, correctly distributed in the OFDM spectrum. The sequence of multiplex symbols is denoted by $X[n]$ and represents a sequence of symbols in the frequency domain.

The space-time block coding (STBC) uses the Alamouti technique [2]. It stores two consecutive sequences of multiplexed data and pilot symbols, $X[n]_{t0}$ and $X[n]_{t1}$, and produces the outputs $X_1[n]$ and $X_2[n]$ according to Table I, where the symbol $*$ means complex conjugate.

TABLE I
ALAMOUTI MAPPING USED BY THE MI-SBTVD.

	$X_1[n]_t$	$X_2[n]_t$
$t = t_2$	$X[n]_{t0}$	$X[n]_{t1}$
$t = t_3$	$-X^*[n]_{t1}$	$X^*[n]_{t0}$

The STBC symbols $X_1[n]$ and $X_2[n]$ are then processed via IFFT (Inverse Fast Fourier Transform) operations and the result are the OFDM symbols $x_1[n]$ and $x_2[n]$ in the time domain. After cyclic prefix insertion, the OFDM symbols become the discrete-time signals $s_1(k)$ and $s_2(k)$.

It was considered that the channels between the transmit antennas and the receiver antenna, with impulse response given by $h_1(k)$ and $h_2(k)$, are frequency-selective. It was also considered that the fading is slow enough to be considered approximately constant during two consecutive OFDM symbol intervals. In Figure 1 $n_0(k)$ represents the additive white Gaussian noise (AWGN). The, the received signal can be written as follows:

$$r(k) = s_1(k) * h_1(k) + s_2(k) * h_2(k) + n_0(k) \tag{1}$$

Let $H_1[n]$, $H_2[n]$ and $N_0[n]$ be the discrete Fourier transform (DFT) of $h_1(k)$, $h_2(k)$ and $n_0(k)$, respectively, realized by an FFT operation. After de-multiplexing, the data symbols $Y_d[d]$ and the pilot symbols $X_p[d]$ are separated. Then, using the mapping given in Table I, we can write:

$$Y_p[p]_{t2} = X_p[p]_{t0}\, H_1[p] + X_p[p]_{t1}\, H_2[p] + N_0[p]_{t2} \tag{2}$$

$$Y_p[p]_{t3} = -X_p^*[p]_{t1} H_1[p] + X_p^*[p]_{t0}\, H_2[p] + N_0[p]_{t3} \tag{3}$$

Under the condition of high signal-to-noise ratio, the noise terms in (2) and (3) can be ignored. In this case we have the following system of equations for the channel estimates:

$$\hat{H}_2[p] = \frac{Y_p[p]_{t3}\, X_p[p]_{t0} + Y_p[p]_{t2}\, X_p^*[p]_{t1}}{\left|X_p[p]_{t0}\right|^2 + \left|X_p[p]_{t1}\right|^2} \tag{4}$$

$$\hat{H}_1[p] = \frac{X_p[p]_{t1}\, Y_p[p]_{t3}}{\left|X_p[p]_{t0}\right|^2 + \left|X_p[p]_{t1}\right|^2} + \frac{Y_p[p]_{t2}}{X_p[p]_{t0}} \left(1 - \frac{\left|X_p[p]_{t1}\right|^2}{\left|X_p[p]_{t0}\right|^2 + \left|X_p[p]_{t1}\right|^2}\right) \tag{5}$$

Since the receiver has knowledge about the transmitted pilot symbols $X_p[p]_{t0}$ and $X_p[p]_{t1}$, using them and the received pilot symbols $Y_p[p]_{t2}$ and $Y_p[p]_{t3}$ in (4) and (5) it is possible to estimate the frequency responses of the channels associated to the positions of the pilot carriers. The MI-SBTVD generates the channel estimates for all carriers, $\hat{H}_1[n]$ and $\hat{H}_2[n]$, through linear interpolation of the above estimates.

III. APPLICATION OF THE EM ALGORITHM TO THE MI-SBTVD CHANNEL ESTIMATION

The Expectation Maximization (EM) algorithm is an iterative method for obtaining a Maximum Likelihood (ML) estimate of the parameter θ from a random process with probability density function $f(x,\theta)$.

Equations (6), (7) and (8) below synthesize one of the possible variations of the EM algorithm, adapted to channel estimation in OFDM systems with space-time coding [5]. The algorithm comprises two steps: the E-step, given by equations (7) and (8), and the M-step, given by equation (6).

$$h_i^{(e+1)} = \mathbf{W}_L^H \mathbf{X}_i^{-1} \hat{Z}_i^{(e)}, \;\; i = 1,\ 2, \tag{6}$$

where

$$\hat{Z}_i^{(e)} = Z_i^{(e)} + \beta_i \left(Y - \sum_{j=1}^{2} Z_j^{(e)} \right) \tag{7}$$

$$Z_i^{(e)} = \mathbf{X}_i \mathbf{W}_L h_i^{(e)} \tag{8}$$

The index i, $i = 1$, 2 identify the links between the transmitter antennas and the receive antenna. $\boldsymbol{X}_i$ represent the transmit data; $h_i^{(e)}$ is the channel impulse response on iteration e; Y is the received data; and β_i are semi-empirical coefficients, with $\beta_1 = \beta_2 = 0.5$ as their typical values [5]; $\boldsymbol{W}_L$ is the sub matrix of the FFT matrix with its first L columns, L being determined by the length of the channel impulse response. In the context of the EM algorithm, Y corresponds to the incomplete observed data and Z_i is the complete, non-observed data, obtained by decomposing the observed data in its two components, Z_1 and Z_2. Then, Z_i can be viewed as the component of Y which was transmitted by the antenna i.

This algorithm was applied to the MI-SBTVD according to the block diagram shown in Figure 2. The linearly-interpolated channel estimates made by the original MI-SBTVD procedure are used as initial estimates, $\hat{H}_1[n]^{(0)}$ and $\hat{H}_2[n]^{(0)}$, for the EM algorithm, the superscripts identifying the iteration number. The EM algorithm then starts iterating with the space-time decoding process, improving the channel estimates and, consequently, improving the reliability of the estimated data.

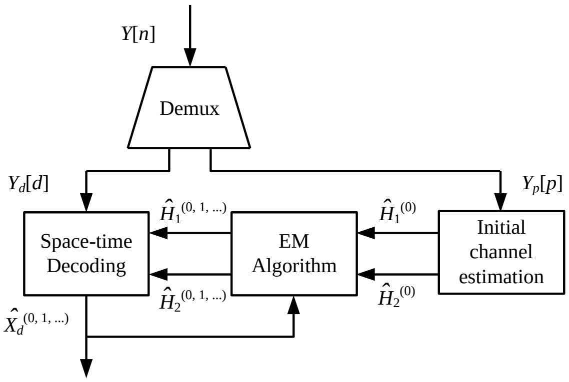

Figure 2. Proposed adaptation of the EM algorithm to the MI-SBTVD.

The estimates of the transmitted symbols $\hat{X}[n]$ comprises the estimated data symbols $\hat{X}_d[d]$ available at the output of the space-time decoder, plus the pilot symbols $X_p[p]$ known by the receiver.

Then, the estimates of Z_1 and Z_1 on iteration e and at time instant $t = t_2$ can be determined as follows:

$$Z_1^{(e)} = \hat{H}_1[n]^{(e)}\,\hat{X}[n]_{t0}^{(e)} \tag{9}$$

$$Z_2^{(e)} = \hat{H}_2[n]^{(e)}\,\hat{X}[n]_{t1}^{(e)} \tag{10}$$

Taking into account the received signal at $t = t_2$, $Y[n]_{t2}$, making use of (7) we obtain:

$$\hat{Z}_1^{(e)} = Z_1^{(e)} + \beta_1\left\{Y[n]_{t2} - \left(Z_1^{(e)} + Z_2^{(e)}\right)\right\} \tag{11}$$

$$\hat{Z}_2^{(e)} = Z_2^{(e)} + \beta_2\left\{Y[n]_{t2} - \left(Z_1^{(e)} + Z_2^{(e)}\right)\right\} \tag{12}$$

Through the knowledge of $\hat{Z}_1^{(e)}$, $\hat{Z}_2^{(e)}$, $\hat{X}[n]_{t0}^{(e)}$, $\hat{X}[n]_{t1}^{(e)}$ and $Y[n]_{t2}$ it is already possible to obtain an estimate of the channel frequency response, but $Y[n]_{t2}$ is fed back to the EM algorithm in order to get new estimates also based on the received signal at $t = t_3$, $Y[n]_{t3}$, as shown by:

$$Z_1^{(e)} = -\frac{\hat{Z}_1^{(e)}}{\hat{X}[n]_{t0}^{(e)}}\,\hat{X}^*[n]_{t1}^{(e)} \tag{13}$$

$$Z_2^{(e)} = \frac{\hat{Z}_2^{(e)}}{\hat{X}[n]_{t1}^{(e)}}\,\hat{X}^*[n]_{t0}^{(e)} \tag{14}$$

Applying (7), now as a function of $Y[n]_{t3}$ we get:

$$\hat{Z}_1^{(e)} = Z_1^{(e)} + \beta_1\left\{Y[n]_{t3} - \left(Z_1^{(e)} + Z_2^{(e)}\right)\right\} \tag{15}$$

$$\hat{Z}_2^{(e)} = Z_2^{(e)} + \beta_2\left\{Y[n]_{t3} - \left(Z_1^{(e)} + Z_2^{(e)}\right)\right\} \tag{16}$$

Finally, using the results given by (15) and (16), the channel estimates $\hat{H}_1[n]^{(e+1)}$ and $\hat{H}_2[n]^{(e+1)}$ can be obtained through the expressions:

$$\hat{H}_1[n]^{(e+1)} = -\frac{\hat{Z}_1^{(e)}}{\hat{X}^*[n]_{t1}^{(e)}} \tag{17}$$

$$\hat{H}_2[n]^{(e+1)} = \frac{\hat{Z}_2^{(e)}}{\hat{X}^*[n]_{t0}^{(e)}} \tag{18}$$

The estimates $\hat{H}_1[n]^{(e+1)}$ and $\hat{H}_2[n]^{(e+1)}$ are used by the space-time decoder that, by operating with $Y[n]_{t2}$ and $Y[n]_{t3}$, gives the data estimates $\hat{X}_d[d]_{t0}^{(e+1)}$ and $\hat{X}_d[d]_{t1}^{(e+1)}$. These estimates, combined with the pilot symbols $X_p[p]$, form $\hat{X}[n]_{t0}^{(e+1)}$ and $\hat{X}[n]_{t1}^{(e+1)}$. Using these sequences, the EM algorithm calculates $\hat{H}_1[n]^{(e+2)}$ and $\hat{H}_2[n]^{(e+2)}$, and so on, improving successively the channel frequency response estimations.

IV. Simulation results

In this Section we present simulation results of the EM algorithm applied to the MI-SBTVD channel estimation process. The results consider the *Typical Urban GSM* channel model [6] with static paths, but with different values for each channel between the transmit antennas do the receive antenna.

Figure 3 shows the superposition of three frequency response magnitudes: for one of the channels between the transmit antennas to the receive antenna, named channel #1, for the original MI-SBTVD estimation result and for the estimate with the proposed EM algorithm. The parameters used by the EM estimation process were: E_b/N_0 = 10 dB, 21 iterations, $\beta_1 = \beta_2 = 0.0005$ and 200 OFDM symbols.

Table II shows the *mean square errors* (MSE) between the real (*Re*) and imaginary (*Im*) parts of the frequency response for channel #1 and the real and imaginary parts of the corresponding channel estimations, calculated using 2,048-element vectors. It can be seen from Table II and Figure 3 that the EM estimate has a good convergence to the actual channel frequency response, a result that is clearly far superior to the one obtained with the original channel estimation process. The same results apply for the channel #2 estimation.

Table II
MSE between the channel #1 response and the estimations.

	MI-SBTVD		MI-SBTVD+EM	
	Re	*Im*	*Re*	*Im*
MSE	0.013	0.014	0.001	0.001

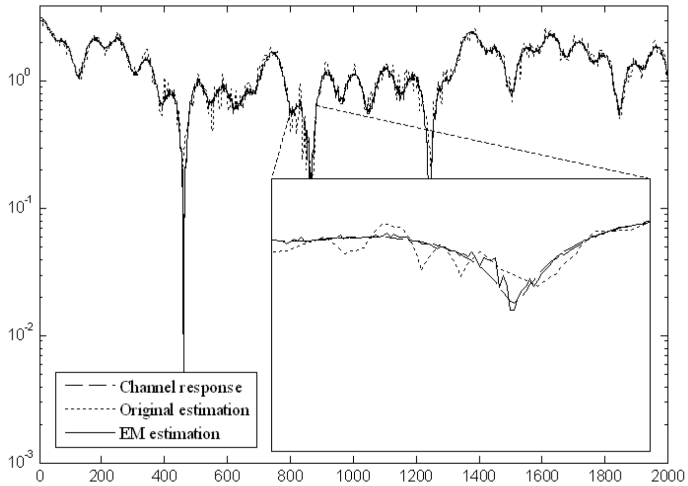

Figure 3. Some channel estimation results.

V. Conclusions and Final Remarks

This paper presented results concerning the use of the Expectation Maximization (EM) algorithm as an improvement of the original MI-SBTVD channel estimation technique. Simulation results show that the proposed algorithm can significantly improve the performance of the channel estimation process, bringing the possibility of improvements in the performance of the overall system.

The proposed channel estimation scheme makes use of the OFDM pilot carriers only to initialize the EM algorithm. In a time-varying channel this might be done more frequently, but in any case the system throughput can be increased by the reduction in the pilot carriers usage.

We are still engaged with the studies concerning the proposed application, and the results presented here, though attractive, may be considered as preliminary, with great chances of future improvements. Our intention is to explore more deeply the influence of the EM parameters on the convergence properties of the estimation process, with special attention to the number of iterations, the parameter β and the number of OFDM symbols necessary for convergence. These parameters must also be adjusted to permit good estimations in cases when the channel is time varying.

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