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TURBO PRODUCT CODES FOR AN ORTHOGONAL MULTICARRIER DS-CDMA SYSTEM

Abstract. In this article it is suggested a class of product codes and its application to a multicarrier DS-CDMA system. The iterative (turbo) decoding of this class is based on a very simple minimum distance decoding of the component codes. Performance simulation results for various coded systems are compared with capacity calculations and reveal a good trade-off between complexity and performance.

1. INTRODUCTION

It is somewhat a consensus that multicarrier systems, especially those combined with the code-division multiple access technique, are potential candidates to be used in fourth-generation wireless communication systems. Therefore, it is of interest to develop supporting techniques for them that will actually turn this potential into reality. Among these techniques, efficient and simple channel coding/decoding strategies represent one challenge.

In this context, this article describes a class of low rate multidimensional product codes and its iterative (turbo) decoding applied to the orthogonal multicarrier DS-CDMA system suggested in [1] for multi-path Rayleigh fading channels.

The article is organized as follows: in Section 2 the multicarrier system is described and in Section 3 some results concerning the channel capacity analysis for this system are presented. Section 4 describes the proposed class of product codes whereas Section 5 presents the iterative (turbo) decoding process. In Section 6, the performance of the turbo product codes when applied to the multicarrier CDMA system is investigated. Finally, Section 7 is devoted to the concluding remarks.

2. DESCRIPTION OF THE MC-DS-CDMA SYSTEM

In the system suggested in [1], transmitted data bits are serial-to-parallel converted to M parallel branches. On each branch, each bit is repeated S times and the replicas feed different block interleavers. Then, these identical bits are direct sequence spread spectrum BPSK modulated and transmitted using orthogonal carries. Hence, there are a total number of MS carriers and time-frequency diversity is achieved. At the receiver, the matched filter outputs of the S identical-bit carriers are combined prior to decoding. The system of [1] can be viewed as a combination or generalization of the *copy-type* and *S/P-type* configurations described in [2]. The main attributes of this system are: 1) the possibility of overcoming the performance of the conventional single-carrier CDMA system, and 2) the reduction of complexity through the use of one matched filter per carrier, instead of a RAKE receiver, situation that is achieved if the number of carriers satisfies [1]

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$$MS \geq 2L_1 - 2 \tag{1}$$

where L_1 is the number of resolvable propagation paths for a single-carrier CDMA system with the same total bandwidth as that of the MC-DS-CDMA system. In [1] it is further assumed a 50% of spectrum overlap of adjacent modulated and orthogonal carriers.

3. CHANNEL CAPACITY ANALYSIS

In this article it is assumed that the receiver has perfect knowledge of the channel state information and that there is no transmit power adaptation scheme. It is further assumed that the compatibility constraint of [5] is satisfied, that is, the channel gains are independent and identically distributed (i.i.d.) random variables, and the input distribution that maximizes mutual information is the same, regardless of the channel state.

Let $g[i]$ represent the channel state information at the discrete-time instant i, and assume that it is possible to generate by computer a sufficient large number X of values for g, based on a known probability distribution. Then, the Shannon capacity of the fading channel with side information at the *receiver only* given in [5], can be estimated as follows

$$C = \frac{1}{X}\sum_{i=1}^{X} B\log_2\left(1+\gamma g^2[i]\right) \tag{2}$$

where B is the channel bandwidth and γ is the average received signal-to-noise ratio (SNR). The value of X is the one enough for convergence in (2).

For BPSK signalling on a fading channel, the capacity can be estimated through

$$C_{\text{BPSK}} = \frac{1}{X}\sum_{i=1}^{X}\int_{-\infty}^{\infty} p(y\,|\,\psi[i])\log_2\frac{p(y\,|\,\psi[i])}{p(y)}dy \tag{3}$$

where

$$\psi[i] = g[i]\sqrt{E_s} \tag{4}$$

$$p(y\,|\,\psi[i]) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[\frac{-(y-\psi[i])^2}{2\sigma^2}\right] \quad \text{and} \tag{5}$$

$$p(y) = \frac{\exp\left[\frac{-(y-\psi[i])^2}{2\sigma^2}\right]+\exp\left[\frac{-(y+\psi[i])^2}{2\sigma^2}\right]}{\sqrt{8\pi\sigma^2}} \tag{6}$$

and where E_s is the BPSK symbol energy and σ^2 accounts for the variances of the interference plus noise. For an AWGN channel $g[i]$ = 1. The results obtained through (2) and (3) demonstrate perfect agreement [6] with those obtained through their analytical counterparts, showing the applicability of the method for both unconstrained and constrained input alphabets.

The reverse link of the MC-DS-CDMA system for a user of reference can be interpreted, in one hand, as a set of *M* parallel channels with BPSK signalling. These channels are defined from each of the *M* serial-to-parallel converter outputs at the transmitter to each of the *M* combiner's outputs at the receiver. On the other hand, this link can be interpreted as a set of *MS* parallel channels defined from each of the *MS* modulator inputs at the transmitter to each of the *MS* matched filters outputs at the receiver. Then the channel capacity for the system can be estimated as the sum of *M* or *MS* individual capacities [6], depending on the case under consideration. This sum is possible if it is presumed independence among the *M* or *MS* channels, a reasonable assumption when the bandwidth occupied by each modulated carrier is smaller than the coherence bandwidth of the channel.

In this article we consider three situations for the MC-DS-CDMA system configuration: Equal Gain Combining (EGC), Maximal Ratio Combining (MRC) and no combining. The first two situations are interpreted as *M* parallel channels. The last one is interpreted as *MS* parallel channels. It is worth noting that, even when EGC combining is considered for capacity estimation, the receiver has knowledge of the channel state information.

If the sum of the interference at the receiver input is modelled as Gaussian, the capacity for each of the *M* or *MS* channels of the MC-DS-CDMA system can be approximately estimated using (3). However, the value of $\sqrt{E_s}$ in this expression should be substituted by [1] $T\sqrt{P/2}$, where *P* is the average transmitted power per carrier and *T* is the BPSK symbol duration. Furthermore, Table I shows the values for $g[i]$ and for the variances of the interference plus noise, σ^2, to be operated in (3), according to each case taken into consideration here. The values of the average signal-to-noise ratio, γ, in each situation are also given in Table I. In this table, the value of $\beta[i]_\nu$, $i = 1, 2, ... X$, corresponds to the *i*-th value of the computer generated Rayleigh random variable, i.i.d. for all *i* and ν, and J_ν accounts for the interference variances at the output of each matched filter, at the receiver side. In fact, the values of J_ν are different for different combiner's outputs [1], but if this difference is not taken into account, the channel capacity results are not significantly affected [6].

Figure 1 shows capacity results, in terms of spectral efficiency versus the minimum average SNR per information bit for error-free transmission on the Rayleigh channel, for $MS = 6$ and variable *M* and *S*. For *M* channels with diversity, the results shown are for MRC combining. The systems with EGC combining have average capacities identical to those with MRC combining, although, for a given SNR, the necessary transmitted power per carrier with EGC is grater than that necessary for MRC combining. The length N_1 of the spreading code for a single-carrier CDMA system taken for reference was made equal to 60, and the number of resolvable paths L_1 for this single-carrier case was made equal to 4. In this case (1) is satisfied and the number *L* of resolvable paths per carrier reduces to 1. The multi-path intensity profile was considered uniformly distributed and the number of active users, K, was made equal to 10 (the capacity reduces, as the number of users increases, as expected).

Table I. Values operated in (3), (4), (5) *and* (6).

	γ	$g[i]$	σ^2
M channels EGC combiner	$\frac{PT}{N_0 S}\mathrm{E}\left[\left(\sum_{\nu=1}^{S}\beta_\nu\right)^2\right]$	$\sum_{\nu=1}^{S}\beta[i]_\nu$	$\sum_{\nu=1}^{S}J_\nu+\frac{N_0 TS}{4}$
M channels MRC combiner	$\frac{PT}{N_0}\mathrm{E}\left(\sum_{\nu=1}^{S}\beta_\nu^2\right)$	$\sum_{\nu=1}^{S}\beta[i]_\nu^2$	$g[i]\left(\frac{1}{S}\sum_{\nu=1}^{S}J_\nu+\frac{N_0 T}{4}\right)$
MS channels	$\frac{PT}{N_0}\mathrm{E}\left(\beta^2\right)$	$\beta[i]$	$\frac{1}{S}\sum_{\nu=1}^{S}J_\nu+\frac{N_0 T}{4}$

where [1]: $J_\nu=\frac{PT^2}{6N^3}r+\frac{1}{M}\sum_{p=1}^{M}\frac{PT^2}{2\pi^2N^3}\mu Q_{p,\nu}$ and where: $N=\frac{2M}{MS+1}N_1$,

$r=2(K-1)N^2$, $\mu=(K-1)N^2$ and $Q_{p,\nu}=\sum_{\substack{m=1\\ \neq p+(\nu-1)M}}^{MS}\frac{1}{[m-p-(\nu-1)M]^2}$

$N_0/2$ is the received noise p.s.d. and $\mathrm{E}(x)$ is the expectation operator.

It can be observed from Figure 1 that it is more advantageous to explore the maximum order of diversity (M = 1 and S = 6) instead of exploring the maximum parallelism of the data stream and that, as the order of diversity, S, increases, the channel capacity of the MC-DS-CDMA channel approximates the AWGN one. The observation of MS channels, without diversity, significantly reduces the capacity, especially for high information rates.

The results shown in Figure 1 indicate that it should be preferable to use a low-rate outer code concatenated with the inner repetition of the MC-DS-CDMA system in order to aim the best performance, since the capacity is increased with increasing S and it is changed less than 1 dB for code rates below 0.2. At the receiver, the outputs of the combiners can be viewed as soft inputs for the outer decoder. Furthermore, the length of the spreading code per carrier, N, can be adjusted [6] to compensate for the reduced coded symbols duration R_cT due to coding of rate R_c, keeping unchanged the transmission bandwidth and the information data rate relative to the uncoded system. In this case, however, the channel capacity is reduced, since the total variance of interference in the decision variable is increased.

4. DESCRIPTION OF THE CLASS OF PRODUCT CODES

Let $\boldsymbol{c}_1$ be a codeword of the binary repetition code $\boldsymbol{C}_1$ = (n/2, 1, n/2) and $\boldsymbol{c}_2$ be a codeword of the binary single parity-check code $\boldsymbol{C}_2$ = (n/2, n/2–1, 2). A codeword $\boldsymbol{c}$ of the non-systematic code $\boldsymbol{C}$ = (n, k, $d_{\min}$) = (n, n/2, 4) can be expressed as [7]

$$\boldsymbol{c} = [01]\boldsymbol{c}_1 \oplus [11]\boldsymbol{c}_2 \tag{7}$$

where the sum $\oplus$ is over GF(2) and the product $[01]\boldsymbol{c}_1$ is calculated by substituting a 0 in $\boldsymbol{c}_1$ by 00 and a 1 by 01. The same is done for $[11]\boldsymbol{c}_2$, where now a 1 in $\boldsymbol{c}_2$ becomes 11. By using the same non-systematic code $\boldsymbol{C}$ as the component code in each of the D dimensions, a product code of length n^D, rate $(½)^D$ and minimum distance 4^D is obtained.

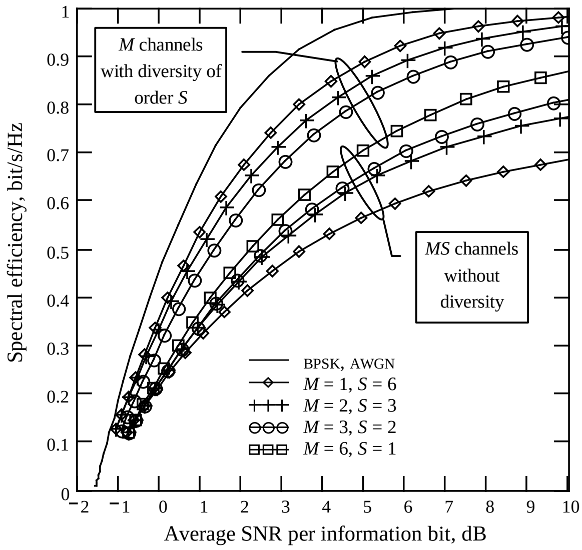

Figure 1. Spectral efficiency for the MC-DS-CDMA system, MS = 6, variable M and S.

A D-dimensional product code can be interpreted as a serial concatenation of D codes separated by D–1 row-column type block interleavers in which the number of columns is $N_c = n$ and the number of rows of the interleaver between the code d and d+1 is $N_r = n^{d-1}k^{D-d}$ [6]. If this rule is accomplished, it is possible to verify [6] that all the n^{D-1} n-element vectors oriented in the "direction" of each dimension of the D-dimensional hypercube of n^D coded bits are codewords of $\boldsymbol{C}$. The key feature of this component code $\boldsymbol{C}$ is that it is possible to derive for it a very simple minimum distance decoding algorithm: set $\boldsymbol{c}_1 = \mathbf{0}$ (the all-zero codeword) and apply Wagner decoding [3] for a single parity-check code of length n/2 over the binary alphabet {00, 11}. The decision is $\hat{\boldsymbol{c}}$. Then set $\boldsymbol{c}_1 = \mathbf{1}$ (the all-one codeword) and again apply Wagner decoding for a parity-check code over the alphabet {01, 10}. The decision is $\hat{\boldsymbol{c}}$'. Compare the Euclidean distances from $\hat{\boldsymbol{c}}$ and $\hat{\boldsymbol{c}}$' to the received codeword $\boldsymbol{r}$ and choose as the final decision the shortest one.

5. TURBO DECODING ALGORITHM

In the initialisation phase of the iterative decoding algorithm, the channel likelihood ratios for all received noisy symbols at the discrete-time instant i are estimated using $\Lambda(c \mid r, g[i]) = g[i]r$, where $g[i]$ is the fading amplitude estimated according to Table I, c is the transmitted codeword symbol and r is the received channel value. For notational simplicity, hereafter the index i in $g[i]$ will be dropped.

The turbo-decoding algorithm is essentially the Pyndiah's one [4]. However, instead of using Chase algorithm to decode the codewords in each dimension, the Wagner algorithm is applied here. Figure 2 shows a block diagram representing operations for the j-th decoding step of the Pyndiah's SISO algorithm, where the maximum value of j, say j_{max}, is the total number of iterations multiplied by D. The vector $\boldsymbol{R}$ represents all n^D received noisy symbols and $\overrightarrow{\boldsymbol{gR}}$ represents the symbol-by-symbol multiplication by the respective fading amplitudes. The expression "decoding in one dimension" in this figure means decoding n^{D-2} $n \times n$ arrays of the soft input $\boldsymbol{E}(j)$ in the "direction" of one dimension. Decoding an array consists of decoding n rows (or n columns). Hence, "decoding in one dimension" implies applying the SISO decoding algorithm n^{D-1} times.

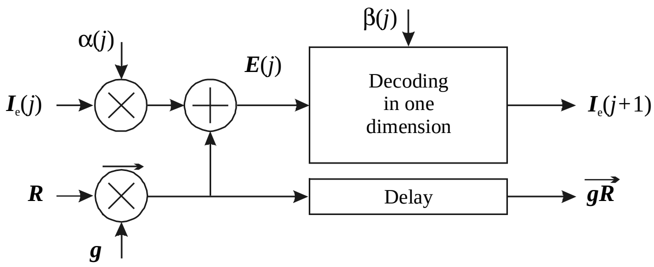

Figure 2. Turbo decoding structure for the j-th decoding step.

The variations of the parameter $\beta(j)$ were chosen to follow a linear rule, and the variations of the parameter $\alpha(j)$ used to weight the extrinsic information, $\boldsymbol{I}_e(j)$, were chosen to follow a logarithmic rule [7]. The reliabilities of decisions, r_d, were always obtained through $r_d = \beta \hat{c}_d$, where $\hat{c}_d$ represents a symbol of the final decision $\hat{\boldsymbol{c}}_d$.

6. SIMULATION RESULTS

Figure 3 presents some simulation results for the uncoded and coded MC-DS-CDMA system for $MS = 6$, variable M and S, $N_1 = 60$, $L_1 = 4$, $K = 10$, uniform multi-path power delay profile and uncorrelated channel gains in time and frequency. The number of iterations in the turbo decoding process was made equal to 10. The performance of the uncoded single-carrier system with a four-tap RAKE receiver is also presented. It can be seen that for $M = 1$, $S = 6$ and $M = 2$, $S = 3$ the performance of the multicarrier system overcomes the performance of the single-carrier one. It also can be seen that the performance of the uncoded system with MRC combining overcomes the performance with the EGC combining rule. This is shown for $M = 1$ and $S = 6$. However, the use of MRC combining did not bring performance gains in the case of the coded system, as compared to the use of EGC combining. It can be further observed in Figure 3 that, for bit error rates below 2×10^{-4}, infinite coding gains can be obtained for all cases considered in this figure. The best performance result is 5.8 dB away from capacity (approximately −0.8 dB for code rate 1/8, $M = 1$ and $S = 6$, according to Figure 1), for a bit error rate of 10^{-5}.

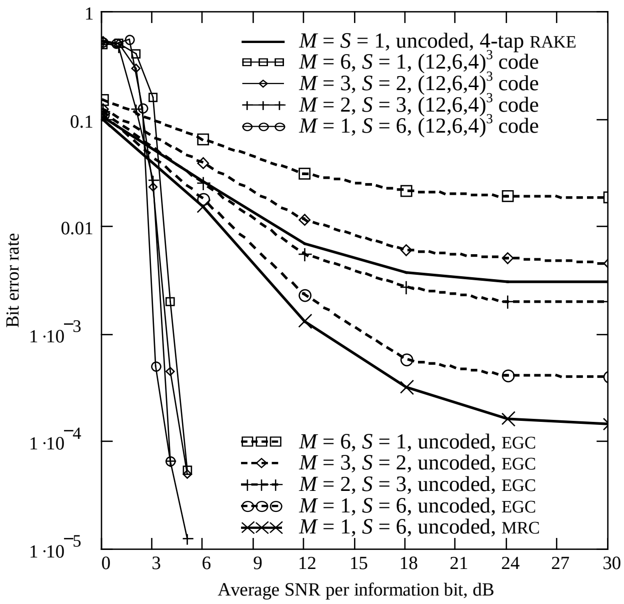

Figure 3. Uncoded and coded MC-DS-CDMA system, MS = 6 and variable M and S.

For the coded system, it is possible to modify the length of the spreading code for each carrier in such way that the information rate and the total occupied bandwidth are kept unchanged, as compared to the uncoded system. This situation was investigated in [6] and demonstrated a decrease in performance less than 0.5 dB for all the cases considered in Figure 3. Although this is an attractive solution, the channel capacity is reduced, since the variance of the total interference is increased. However, in this case, the gap between performance and capacity decreases.

Another situation investigated in [6] was the modification of the channel parameters in such a way that the performance of the uncoded system becomes better than those showed in Figure 3. It was verified that, in this situation, the performance of the coded system is roughly the same as in the cases reported in Figure 3. This verification indicates that greater coding gains can be achieved if the channel becomes worse.

7. CONCLUDING REMARKS

This work described a class of low rate multidimensional product codes and its iterative (turbo) decoding applied to the orthogonal multicarrier DS-CDMA system suggested in [1] for multi-path fading channels. The key feature of this class is that the component code can be decoded through a very simple minimum-distance algorithm based on applying the Wagner rule [3]. The turbo decoding algorithm used a simplified form of Pyndiah's SISO decoding algorithm [4]. Some performance results for this class of codes on the MC-DS-CDMA system of [1] were reported here, unveiling a good trade-off between performance and complexity. It was verified that the best choice for the system parameters corresponds to the use of the maximum allowable frequency diversity, like the one attained by the *copy-type* configuration [2], instead of the maximum parallelism of the data stream, as is the case for the *S/P-type* configuration (OFDM-CDMA) [2]. It was also pointed out that, for the coded system, a very low decrease in performance is observed if the length of the spreading code is changed in order to keep unchanged the information rate and the occupied bandwidth, as compared to the uncoded system.

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