# GLRT Based Spectrum Sensing Techniques for Pulse Radar Signals

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***Abstract*—Radar frequency bands are known to be relatively wide and underutilized, potentially allowing for higher spectral usage by means of the cognitive radio technology. One of the most crucial functionalities in cognitive radio is spectrum sensing, which aims at detecting the presence of a primary user signal in the band of interest. Based on the fact that pulse radar signals can be modeled as a train of rectangular pulses with two amplitude levels, we develop a generalized likelihood ratio test spectrum sensing scheme, as well as its less complex sub-optimal variations, to detect the presence of such signals. The performances of the developed schemes are compared in terms of the probabilities of detection and false alarm via computer simulation.**

***Index Terms*—Cognitive radio, generalized likelihood ratio test, pulse radar, spectrum sensing.**

## I. Introduction

LATELY, many research efforts have been directed at enhancing the radio-frequency (RF) spectrum utilization. A promising alternative is the cognitive radio (CR) technology [1], which enables opportunistic dynamic spectrum access by allowing a secondary user to use a frequency band temporarily, whenever it is found to be vacant.

Radar bands have recently emerged as potential candidates for CR, since they are relatively wide and turn out to be underutilized [2]. Therefore, several initiatives for introducing spectrum sharing in radar bands have been reported in the literature. These include evaluating the interference between radio and radar systems [3], realizing spectrum sharing of radar bands by means of spectrum sensing [4], and minimizing the effects of radar systems on secondary networks [5]. A popular realization of spectrum sharing in radar bands allows the access of the 5 GHz band by a wireless local area network (WLAN). The dynamic frequency selection (DFS) [6] strategy allows the WLAN terminals detect the presence of the radar signal and access vacant bands without interfering the radar system operation.

A crucial functionality of a CR is the capability of spectrum sensing to detect the primary user (PU) signal. Spectrum sensing techniques can be categorized as a matched filter, energy detection, feature based, and eigenvalue based schemes [7]. A matched filter provides optimal sensing performance, but requires the prior knowledge of the exact form of the received signal, which renders it impractical. Energy detection decides on the presence of the PU signal by comparing the received signal energy with a threshold, yielding optimal performance for independent and identically distributed signals. However, its performance severely degrades in the presence of noise power uncertainty. Feature based detection exploits intrinsic characteristics of the PU signal such as preamble, pilot, and cyclostationarity. Eigenvalue based detection applies a test statistic made from the eigenvalues of the received signal sample covariance matrix, and is known to perform well for time-correlated signals.

A pulse radar system, which is the PU in radar bands, usually emits a train of pulses periodically and extracts information about targets by examining the characteristics of the signal reflecting off them. Here we consider the problem of detecting radar signals for the purpose of spectrum sharing with secondary networks. In this case, a simple detection strategy compares the received power with a predetermined threshold, which can be categorized as an energy detection. Multiple pulse energy detection [8]-[10] can also be applied for improved performance. Since these strategies are based on the energy of the received signal, they are susceptible to noise power uncertainty, which is unavoidable in practice.

In order to solve the noise power uncertainty problem, the inherent characteristics of the PU signal can be explored. This strategy has been adopted in [11], where a generalized likelihood ratio test (GLRT) detection scheme has been devised for the case of a single radar pulse partially or fully contained in an observation window. However, this single pulse constraint is too restrictive to apply in practice. In this letter we generalize the GLRT to the case of multiple radar pulses in an observation window. Given the high computational complexity of this optimal GLRT, we also devise sub-optimal, lower complexity detectors. Performance comparisons among these detector and other competing ones are also provided.

In the following, Section II describes the system model. Section III introduces the GLRT based spectrum sensing and its sub-optimal variations. Section IV gives simulation results and discussions, and Section V concludes the work.

## II. System Model

The pulse radar signal without pulse compression can be described by periodic rectangular pulses. The received signal can be seen as a faded pulse train and approximated by a two-level signal under the assumption that the channel fading is static over the observation window. Let the hypotheses $H_1$

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and $H_0$ denote the presence and absence of the radar signal, respectively. Under $H_0$, the received baseband signal sample $r(n)$ in the discrete-time instant $n$ is given by

$$r(n) = w(n), \quad n = 1, \cdots, N, \tag{1}$$

where $w(n)$ is the $n$-th zero-mean and circularly symmetric complex additive white Gaussian noise sample with unknown variance $\sigma^2$, and $N$ is the length of the observation window, in samples.

Under $H_1$, assuming $P$ pulses contained in the observation window, the received signal sample $r(n)$ can be written as

$$r(n) = \begin{cases} A_0 + w(n), & n = 1, \cdots, n_0 \\ A_1 + w(n), & n = n_0 + 1, \cdots, n_1 \\ A_0 + w(n), & n = n_1 + 1, \cdots, n_2 \\ A_1 + w(n), & n = n_2 + 1, \cdots, n_3 \\ A_0 + w(n), & n = n_3 + 1, \cdots, n_4 \\ \cdots & \\ A_0 + w(n), & n = n_{2P-1} + 1, \cdots, N \end{cases}, \tag{2}$$

where $A_0$ and $A_1$ are unknown pulse levels that include the fading effect; the instants $n_i$, $i = 0, 1, \cdots, 2P-1$ are also unknown, with $1 \leq n_0 \leq n_1 \leq \cdots \leq n_{2P-1} \leq N$. For example, Fig. 1 illustrates the received signal during an observation window under $H_1$, with $P = 1$ and $w(n) = 0$, assuming a partial received pulse in the upper two sub-figures, and an entire received pulse in the bottom.

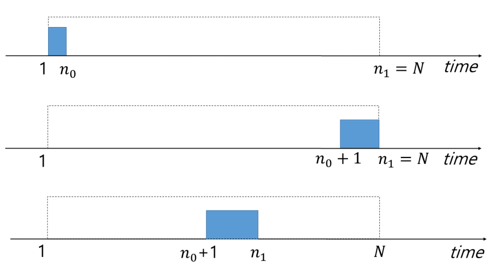


Fig. 1. Three possible cases for a pulse radar signal contained in an observation window for $P = 1$.

## III. GLRT Based Detection of Pulse Radar Signals

For convenience in notation, define $\mathbf{n} = [n_0, n_1 \ldots, n_{2P-1}]$ and $\mathbf{r} = [r(1), r(2), \cdots, r(N)]$. When $\mathbf{n}$ is assumed to be known, the GLRT based test statistic $L(\mathbf{r})$ for estimated $A_0$, $A_1$, and $\sigma^2$ can be expressed as [12]

$$L(\mathbf{r}|\mathbf{n}) = \frac{f(\mathbf{r}|\mathbf{n}, H_1)}{f(\mathbf{r}|\mathbf{n}, H_0)}, \tag{3}$$

where $f(\mathbf{r}|\mathbf{n}, H_1)$ and $f(\mathbf{r}|\mathbf{n}, H_0)$ are the joint probability density functions (PDFs) of $\mathbf{r}$ under $H_1$ and $H_0$, respectively, with $f(\mathbf{r}|\mathbf{n}, H_1)$ determined according to [12, Ch. 6] as

$$f(\mathbf{r}|\mathbf{n}, H_1) = \left(\frac{1}{\sqrt{\pi\hat{\sigma}_1^2}}\right)^{2N} \left\{\prod_{i=0}^{P} \prod_{n=n_{2i-1}+1}^{n_{2i}} \exp\left(-\frac{\left|r(n) - \hat{A}_0\right|^2}{\hat{\sigma}_1^2}\right)\right\} \times \left\{\prod_{i=0}^{P-1} \prod_{n=n_{2i}+1}^{n_{2i+1}} \exp\left(-\frac{\left|r(n) - \hat{A}_1\right|^2}{\hat{\sigma}_1^2}\right)\right\}, \tag{4}$$

where $n_{-1} = 0$, $n_{2P} = N$, and $\hat{A}_0$, $\hat{A}_1$ and $\hat{\sigma}_1^2$ are the maximum likelihood (ML) estimates of $A_0$, $A_1$, and $\sigma^2$, respectively, which in light of [12, Ch. 6] are given by

$$\hat{A}_0 = \sum_{i=0}^{P} \frac{1}{n_{2i} - n_{2i-1}} \sum_{n=n_{2i-1}+1}^{n_{2i}} r(n), \tag{5}$$

$$\hat{A}_1 = \sum_{i=0}^{P-1} \frac{1}{n_{2i+1} - n_{2i}} \sum_{n=n_{2i}+1}^{n_{2i+1}} r(n), \tag{6}$$

$$\hat{\sigma}_1^2 = \frac{1}{N}\left\{\sum_{i=0}^{P} \sum_{n=n_{2i-1}+1}^{n_{2i}} |r(n) - \hat{A}_0|^2 + \sum_{i=0}^{P-1} \sum_{n=n_{2i}+1}^{n_{2i+1}} |r(n) - \hat{A}_1|^2\right\}. \tag{7}$$

Similarly, also based on [12, Ch. 6], $f(\mathbf{r}|\mathbf{n}, H_0)$ is given by

$$f(\mathbf{r}|\mathbf{n}, H_0) = \left(\frac{1}{\sqrt{\pi\hat{\sigma}_0^2}}\right)^{2N} \prod_{n=1}^{N} \exp\left(-\frac{|r(n)|^2}{\hat{\sigma}_0^2}\right), \tag{8}$$

where $\hat{\sigma}_0^2$ is the ML estimate of $\sigma^2$, which is

$$\hat{\sigma}_0^2 = \frac{1}{N} \sum_{n=1}^{N} |r(n)|^2. \tag{9}$$

Inserting (4) and (8) into (3) yields the test statistic

$$L(\mathbf{r}|\mathbf{n}) = \left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}_1^2}\right)^N. \tag{10}$$

Moreover, if we take into account the fact that $\mathbf{n}$ and $P$ are unknown, the GLRT based test statistic can be expressed as

$$L(\mathbf{r}) = \max_{n_0, n_1, \cdots, n_{2P-1}, P} \left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}_1^2}\right)^N. \tag{11}$$

The computational complexity of (11) grows exponentially with the observation window length $N$, since $P$ is likely to increase with $N$. Moreover, the number of possible combinations of $\{n_0, n_1, \cdots, n_{2P-1}\}$ increases exponentially with $P$. Thus, the above GLRT based approach may not be a feasible solution in practice. In the following we suggest two approximate, less complex versions of the GLRT.

### A. Simplifications of the GLRT by Ordering

Equation (7) has two terms: one is related to the set of received samples with amplitude $A_0$ and the other refers to the set of samples with amplitude $A_1$. The high computational complexity of the GLRT (11) is due to the process of classifying the received samples according to each of these two sets [13]. Motivated by the fact that $A_0$ is different from $A_1$ in magnitude under $H_1$, we can simplify the categorization by arranging the received samples in ascending order of magnitude and finding a level changing instant to classify them into the above sets. The exact joint PDF of the ordered noise samples may not be Gaussian, being very complicated in some cases. For simplification, we approximate the ordered noise samples to follow a Gaussian distribution. Then we apply the GLRT to this simplified formulation and obtain the corresponding feasible spectrum sensing method, which is called ordering based GLRT (OGLRT).

Let the ascending ordered received samples be denoted by $\hat{\mathbf{r}} = [\hat{r}(1), \cdots, \hat{r}(N)]$. Then, using (11), the corresponding GLRT test statistic $\hat{L}$ reduces to

$$\hat{L}(\hat{\mathbf{r}}) = \max_{n_0} \left( \frac{\hat{\sigma_0^2}}{\hat{\sigma_1^2}} \right)^N . \quad (12)$$

One can see that (12) performs the maximization over a single parameter $n_0$, yielding a computational complexity significantly small compared to the original GLRT based statistic given in (11).

In order to reduce the computational complexity of (12), we devise a simple procedure for determining $n_0$ (see Fig. 1) by calculating the magnitude differences between every adjacent ordered samples and determining the discrete-time instant corresponding to the maximum difference as $n_0$. This is an intuitively satisfying approach, since it is very likely that the magnitude of the ordered sample jumps at the optimal $n_0$, especially at high SNR. The procedure to find $n_0$ is given by the pseudo-code in the **Algorithm 1**. The resultant test statistic is referred to as simplified ordering based GLRT (SOGLRT).

**Algorithm 1** Simple method to determine $n_0$

Generate $\{\hat{r}(i)\}$, $i = 1, \cdots, N$, as the samples $\{r(i)\}$ placed in ascending order of magnitude
$n_0 \leftarrow 0$
max $\leftarrow 0$
for $i = 2$ to $N$
    if $|\hat{r}(i)| - |\hat{r}(i-1)| >$ max
        max $\leftarrow |\hat{r}(i)| - |\hat{r}(i-1)|$
        $n_0 \leftarrow i - 1$
    end if
end for

### *B. Simplification of the GLRT by Observation Window Segmentation*

As already mentioned, the major cause of the high computational complexity of (11) is the fact that the possible combinations of $\{n_0, n_1, \cdots, n_{2P-1}\}$ may be prohibitive, especially for large $P$. Similarly to [14], if an observation window is divided into non overlapping smaller segments such that the length of each segment contains a single radar pulse, partially or entirely, the GLRT detection strategy can be applied to the segments individually. Then, the corresponding intermediate test statistics or decisions can be combined into a single test statistic or decision. Owing to this process, the computational burden can be reduced, yet exploiting the fact that a pulse radar signal is sparse in the time domain, which results in higher SNRs when small windows are adopted. Here we consider two combining rules: One is a soft-decision combining that sums the intermediate test statistics from each segment, and the other is a hard decision combining to fuse the intermediate decisions from the multiple segments by means of the OR rule. The former is named sliding window GLRT with soft-decision combining (SWGLRT+SC), and the latter is called sliding window GLRT with hard decision combining (SWGLRT+HC). These sliding window strategies can be also applied to the ordering based GLRT, yielding the techniques SWOGLRT+SC and SWOGLRT+HC.

## IV. Numerical Results

In order to assess the sensing performances of the proposed schemes, we assume that the radar pulse width is 1 $\mu$s, the pulse repetition frequency (PRF) is 10 kHz or 100 kHz, and the radar signal is sampled at 5 MHz [6, Annex D]. The relatively high PRFs were chosen to reduce the simulation time, without loss of generality. The transmitted radar signal is assumed to go through a Rayleigh fading channel with Doppler frequency of 10 Hz. For the SWGLRT, the smaller sensing window length, that is, the segment length is set to 5 $\mu$s. Furthermore, since the whole observation window is not synchronized with the radar pulses, the starting instant of a pulse is supposed to be random over the pulse repetition interval (PRI), following a uniform distribution.

The results shown hereafter are given in terms of receiver operating characteristic (ROC) curves, truncated to show only the most relevant region of low false alarm probabilities, with the target false alarm rates of $\{0.01, 0.05, 0.1\}$. Since the noise power uncertainty (NPU) is not avoidable in practice, it is also taken into account in the results.

Fig. 2 shows the performances of the GLRT and the OGLRT schemes, along with the performances of the well-known energy detection (ED), the matched filter (MF), and the max-min eigenvalue ratio detection (EIG) with a smoothing factor of 10. The PRF and the observation window length were respectively set to 100 kHz and 9 $\mu$s, such that the observation window contains a partial or an entire single radar pulse. This figure demonstrates that the GLRT based schemes suffer from a relatively small performance degradation in the presence of NPU, when compared with the ED. Both the GLRT and the OGLRT outperform the ED when the NPU is 2 dB. The robustness of the GLRT based schemes to the noise power uncertainty is due to the fact that they do not depend on the prior knowledge of the exact noise power level.

Fig. 3 and Fig. 4 show the performances of the proposed schemes for PRFs of 100 kHz and 10 kHz, and SNRs of $-10$ dB and 0 dB. These figures illustrate that the SOGLRT is a good approximation of the OGLRT for high SNR. Additionally, a comparison between these two figures unveils that the performance gain of the SWGLRT with soft-decision combining (SWGLRT+SC) over the SWGLRT with hard-decision combining (SWGLRT+HC) is reduced as the PRF decreases. This is owed to the fact that a lower PRF causes the number of segments with no radar pulses to increase, resulting in a higher number of noise only segments contributing to the combined test statistic value. Therefore, for the case of high duty cycle, the SWGLRT with soft-decision combining performs better than the one with hard-decision combining.

Back to Fig. 2, it can be noticed that the performance of the OGLRT is worse than the one attained by the GLRT. This is due to the sub-optimality of the OGLRT, which is owed to the ordering process. Fig. 3 and Fig. 4 show that this behavior

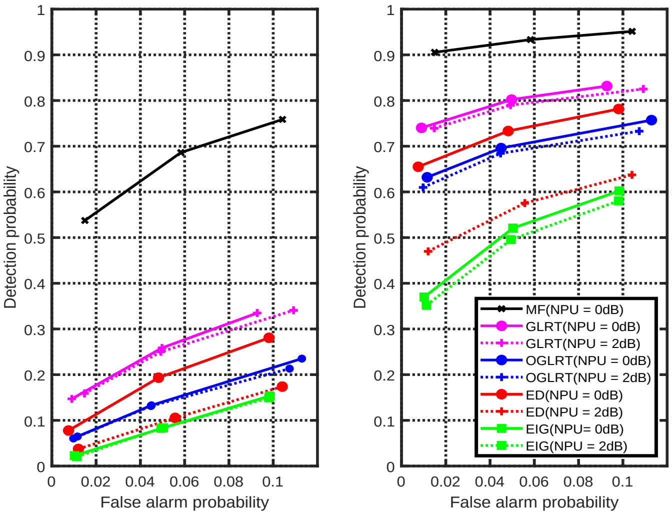


Fig. 2. Comparison of spectrum sensing performances of MF, ED, GLRT, OGLRT, and EIG, for SNR = −10 dB (left) and SNR = 0 dB (right).

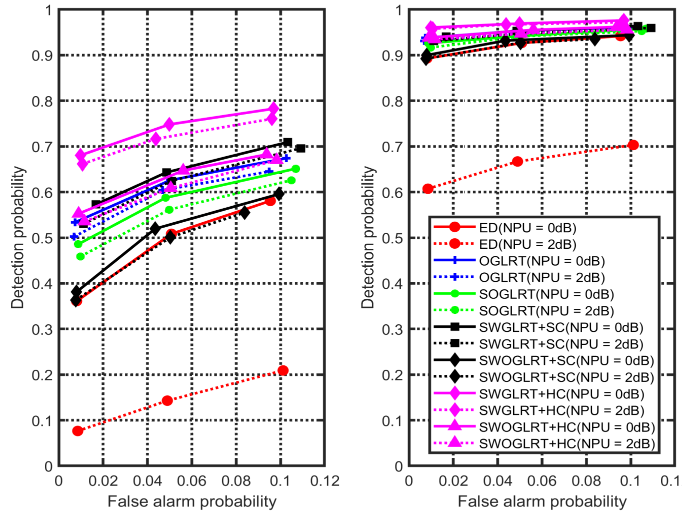


Fig. 4. Comparison of spectrum sensing performance of ED, OGLRT, SOGLRT, and variations of SWGLRT, for SNR = −10 dB (left) and SNR = 0 dB (right) when PRF = 10 kHz.

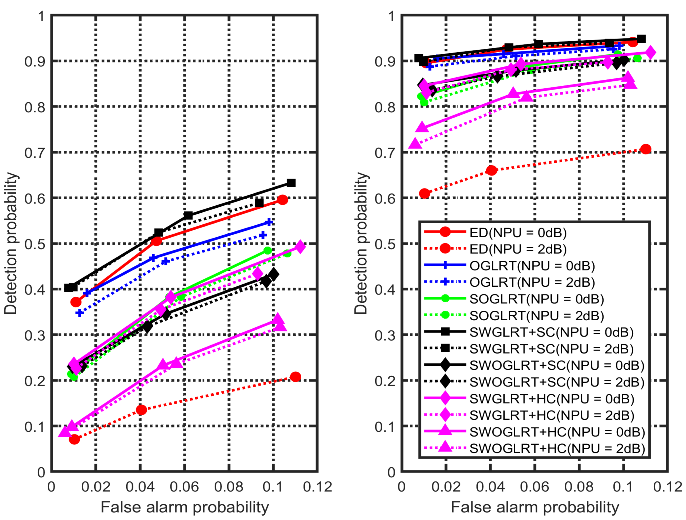


Fig. 3. Comparison of spectrum sensing performance of ED, OGLRT, SOGLRT, and variations of SWGLRT, for SNR = −10 dB (left) and SNR = 0 dB (right) when PRF = 100 kHz.

is maintained in the case of applying the observation window segmentation strategy to the ordering based GLRT.

## V. Conclusions

In this letter we have modeled the pulse radar signal as a two-level signal, derived a GLRT based detector and its sub-optimal, less complex variants, and compared their spectrum sensing performances in a variety of situations and with well-known competing detectors. The sub-optimal versions were obtained by ordering the received signal samples according to their magnitudes, and by replacing the whole observation window by multiple small sliding windows. Since the proposed schemes do not depend on the knowledge of the exact noise power, it turns out that their performances are robust against noise power uncertainty, unlike the energy detection. Simulation results showed that some sub-optimal detectors can achieve better sensing performances than the energy detector, regardless of the noise power uncertainty.

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