

Gini Index Inspired Robust Detector for Spectrum Sensing over Ricean Channels

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The Gini index is a statistical dispersion metric widely used in economic and social sciences to measure inequalities. In this letter, it is used to build the novel Gini index detector (GID) for cooperative spectrum sensing. It is shown that the GID: i) is robust against unequal and dynamical noise and signal powers; ii) is suited to line-of-sight channels having mild dominant component; iii) can outperform most of the state-of-the-art robust detectors; iv) has the constant false alarm rate property; v) is one of the simplest detectors available so far.

Introduction: The cognitive radio (CR) concept promises to alleviate the problems of congestion and scarcity of the radio-frequency spectrum inherited by the fixed allocation policy currently adopted in wireless communication networks [1]. The spectrum sensing [2] is a fundamental task within the CR framework, allowing for dynamic spectrum access of CR-enabled secondary users (SUs) to vacant bands in the primary user (PU) network. Cooperative spectrum sensing (CSS) prevails among other approaches, due to its ability of mitigating channel fading, shadowing and the hidden node problems [2]. In centralized data fusion CSS, the PU signal samples collected by the SUs are sent to a fusion center (FC), where they are processed to allow for the global decision upon the spectrum occupancy. Thanks to the cooperation, more reliable global decisions are attained when compared to local CR decisions.

Among the detection strategies for spectrum sensing, the energy detection (ED), the matched filter detection (MFD), the cyclostationary feature detection (CFD) and the eigenvalue-based detection (EVD) [2, 3] are the most widely known. The MFD and CFD are difficult to implement in practice, since they depend on PU signal or channel information that cannot be easily available to the SUs. The ED and the EVD do not need such information, being classified as blind techniques. However, the ED and some EVD rely on the noise power information, being prone to the inaccuracies of noise variance estimates.

The most known EVD schemes are the maximum-minimum eigenvalue detection (MMED), the maximum eigenvalue detection (MED), and the ones based on the generalized likelihood ratio test (GLRT) [3]. Among these, the MED needs the noise variance for decision, although it is the asymptotically optimum nonparametric detector under the Neyman-Pearson criterion for the case of a single unknown signal immersed in Gaussian noise with known variance [3].

The realistic situation in which the noise and received signal powers may be different among the SUs' receivers, possibly fluctuation over time and space, can be referred to as the nonuniform-dynamical noise and signal. This situation can be significantly detrimental to the spectrum sensing performance [4], motivating the development of robust detection strategies. In this letter, a simple and powerful test statistic is developed to cope with the nonuniform-dynamical noise and signal scenario.

System Model: The m SUs in cooperation collect mn samples (n samples per SU) of the signal received from s PU transmitters. The mn samples are sent to the FC, where they are arranged as $\mathbf{Y} \in \mathbb{C}^{m \times n} = \mathbf{H}\mathbf{X} + \mathbf{V}$, where $\mathbf{H} \in \mathbb{C}^{m \times s}$ is the channel matrix with elements h_{ij} , $i = 1, 2, \dots, m$, $j = 1, 2, \dots, s$, representing the channel gains between the j -th PU and the i -th SU. These gains are constant during the sensing interval, and independent and identically distributed (i.i.d.) between sensing rounds. In order to model a flat and slow Ricean fading channel, $\mathbf{H} = \mathbf{G}\mathbf{A}$, where $\mathbf{A} \in \mathbb{C}^{m \times s}$ has elements $\alpha_{ij} \sim \mathcal{CN}[\sqrt{K/(2K+2)}, 1 - K/(K+1)]$ that guarantee unitary second moment of the fading magnitude, where K is the Rice factor. The matrix $\mathbf{G} \in \mathbb{R}^{m \times m} = \text{diag}(\sqrt{\mathbf{p}}/p_{\text{avg}})$, where $\mathbf{p} = [p_1, p_2, \dots, p_m]^T$ is the vector with the received signal powers in each SU, and $p_{\text{avg}} = \frac{1}{m} \sum_{i=1}^m p_i$ is the average received signal power over all SUs. Each PU transmits with a constant power given by p_{avg}/s . The matrix $\mathbf{X} \in \mathbb{C}^{s \times n}$ represents the PU signals; its elements can be zero-mean i.i.d. complex Gaussian random variables (the PU signals are white noise) or they are drawn from a zero-mean baseband quaternary phase-shift keying (QPSK) signal with τ samples per symbol ($\tau = 1$ for i.i.d. samples; $\tau = n$ for n.i.i.d. samples having maximum correlation). In the case of uniform noise, $\mathbf{V} \in \mathbb{C}^{m \times n} \sim \mathcal{NC}(0, \sigma^2 \mathbf{I}_m)$, with \mathbf{I}_m being

the identity matrix of order m . In the nonuniform noise situation, the elements on the i -th row of \mathbf{V} have variance σ_i^2 , $i = 1, \dots, m$. If the average noise variance is $\sigma_{\text{avg}}^2 = \frac{1}{m} \sum_{i=1}^m \sigma_i^2$, the received signal-to-noise ratio, in dB, averaged over all SUs, is $\text{SNR} = 10 \log_{10}(p_{\text{avg}}/\sigma_{\text{avg}}^2)$.

Proposed Detector: The Gini coefficient, sometimes referred to as Gini index or Gini ratio, is a statistical dispersion metric widely used in economic and social sciences, for example to measure income, wealth or educational inequalities across populations. In one of its mathematical forms [5, p. 3400], the Gini index G is half of the relative mean absolute difference, which is the ratio between the mean absolute difference and the arithmetic mean: $G = (\sum_{i=1}^N \sum_{j=1}^N |x_i - x_j|) / (2N \sum_{i=1}^N x_i)$. This index was originally thought to operate on the values x_i obtained from a frequency distribution. Moreover, $G \geq 0$.

Here, the Gini index is applied to the received signal sample covariance matrix (SCM) of order m , $\mathbf{R} = \mathbf{Y}\mathbf{Y}^\dagger/n$, where \dagger denotes the Hermitian operation. The discrepancy between the shapes of \mathbf{R} when the PU signals are absent (hypothesis \mathcal{H}_0) and present (hypothesis \mathcal{H}_1) is checked using the inverse of the Gini index adapted to complex entries. Thus, the proposed Gini index detector (GID) test statistic is

$$T_{\text{GID}} = \frac{2(m^2 - m) \sum_{i=1}^m |r_i|}{\sum_{i=1}^m \sum_{j=1}^m |r_i - r_j|}, \quad (1)$$

where r_i is the i -th element of the vector \mathbf{r} formed by stacking all columns of \mathbf{R} . The constant $2(m^2 - m)$ does not influence the decision, but conveniently makes $T_{\text{GID}} = 1$ when $\mathbf{R} = \mathbf{I}_m$, which is the limiting situation under \mathcal{H}_0 . Thus, T_{GID} tends to increase when \mathbf{R} departs from the identity, yielding the decision rule: decide in favor of \mathcal{H}_1 if $T_{\text{GID}} \geq \xi$, where ξ is the decision threshold; decide \mathcal{H}_0 otherwise.

Fig. 1 shows empirical probability density functions (PDFs) of T_{GID} under \mathcal{H}_0 and \mathcal{H}_1 , for two noise levels. Besides demonstrating the capability of T_{GID} for signal detection, Fig. 1 also shows that the GID has the constant false alarm rate (CFAR) property, since the support of the PDF under \mathcal{H}_0 does not change under different noise levels.

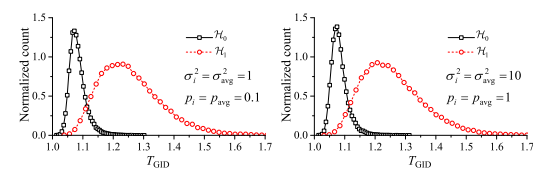


Fig. 1 Empirical PDFs of T_{GID} for $\sigma_i^2 = 1$ (left) and $\sigma_i^2 = 10$ (right), for $m = 5$, $n = 100$, $\text{SNR} = -10$ dB, $K = 10$, and $p_i = p_{\text{avg}}$.

Numerical Results: A typical tool for analyzing the spectrum sensing performance is the receiver operating characteristic (ROC) curve, which trades the probability of detection P_d and the probability of false alarm P_{fa} by varying the decision threshold. These metrics can be concisely assessed by means of the area under the ROC curve (AUC), which is adopted hereafter and computed by Monte Carlo simulations.

The state-of-the-art competing detectors chosen for comparisons are the eigenvalue-based GLRT, the MED, the MMED [3], the arithmetic to geometric mean (AGM) detector [6], the traditional ED, the Hadamard ratio (HR) detector [7], the volume-based detector 1 (VD1) [8], and the Gershgorin radii and centers ratio (GRCR) detector [4]. The first four detectors were not developed to cope with the nonuniform-dynamical noise and signal powers, whereas the last four are robust ones. These competing test statistics are given in Table 1, where $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_m$ are the eigenvalues of \mathbf{R} , $\det(\mathbf{R})$ is the determinant of \mathbf{R} , y_{ij} are the elements of \mathbf{Y} , r_{ij} are the elements of \mathbf{R} , and $\mathbf{E} = \text{diag}(\mathbf{d})$, where $\text{diag}(\mathbf{d})$ is the diagonal matrix whose diagonal is $\mathbf{d} = [d_1, d_2, \dots, d_m]$, with $d_i = \|\mathbf{R}(i, :)\|_2$, where $\|\cdot\|_2$ is the Euclidian norm.

Table 1: Competing test statistics.

$T_{\text{GLRT}} = \sum_{i=1}^m \frac{\lambda_i}{\lambda_i}$	$T_{\text{MED}} = \frac{\lambda_1}{\sigma_{\text{avg}}^2}$
$T_{\text{AGM}} = \frac{\frac{1}{m} \sum_{i=1}^m \lambda_i}{(\prod_{i=1}^m \lambda_i)^{\frac{1}{m}}}$	$T_{\text{HR}} = \frac{\det(\mathbf{R})}{\prod_{i=1}^m r_{ii}}$
$T_{\text{MMED}} = \frac{\lambda_1}{\lambda_m}$	$T_{\text{ED}} = \sum_{i=1}^m \frac{1}{\sigma_i^2} \sum_{j=1}^m y_{ij} ^2$
$T_{\text{GRCR}} = \frac{\sum_{i=1}^m \sum_{j \neq i} r_{ij} }{\sum_{i=1}^m r_{ii}}$	$T_{\text{VD1}} = \log [\det(\mathbf{E}^{-1} \mathbf{R})]$

In the uniform noise and signal powers situation (Unif. for short), $\sigma_i^2 = \sigma_{avg}^2 = 1$, and $p_i = p_{avg}$ according to the SNR. For nonuniform-dynamical noise and signal powers (Nunif. for short), $\sigma_i^2 \sim \mathcal{U}[0.05\sigma_{avg}^2, 1.95\sigma_{avg}^2]$ and $p_i \sim \mathcal{U}[0.05p_{avg}, 1.95p_{avg}]$ in each sensing round. QPSK PU signals with $\tau = 5$ samples per symbol were adopted for all reported results.

Fig. 2 shows the AUC versus the Rice factor K under Unif. (left) and Nunif. (right) situations. Notice that the MED, the GLRL, the AGM and the MMED are not robust, whereas the other ones are. Moreover, it can be observed the superiority of the GID for $K \geq 2$, even beating the ED and the MED (recall that these detectors need the noise variance information). The advantage of the GID becomes even more pronounced when the number of PUs increases, as shown in Fig. 3, where it can be seen that most of the detectors are also penalized at very low K .

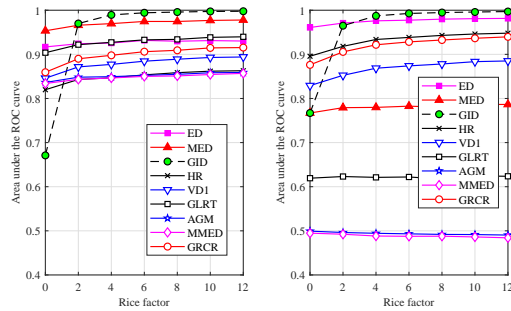


Fig. 2 AUC versus K under Unif. (left) and Nunif. (right) conditions, for $s = 1$ QPSK PU signal, $m = 10$ SUs, $n = 50$ samples, and SNR = -10 dB.

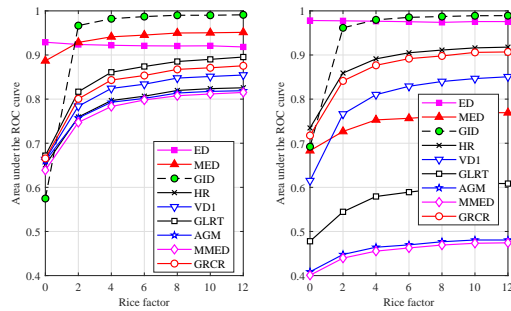


Fig. 3 AUC versus K under Unif. (left) and Nunif. (right) conditions, for $s = 5$ QPSK PU signals, $m = 10$ SUs, $n = 50$ samples, and SNR = -10 dB.

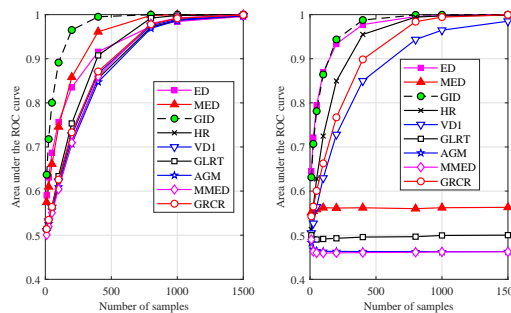


Fig. 4 AUC versus n under Unif. (left) and Nunif. (right) conditions, for $s = 5$ QPSK PU signals, $m = 10$ SUs, $K = 6$, and SNR = -15 dB.

The results in Figs. 4 and 5 are given for $s = 5$ PUs and Rice factor $K = 6$. These figures show the AUC as a function of the number of samples n and the average SNR, respectively. It can be seen that the GID

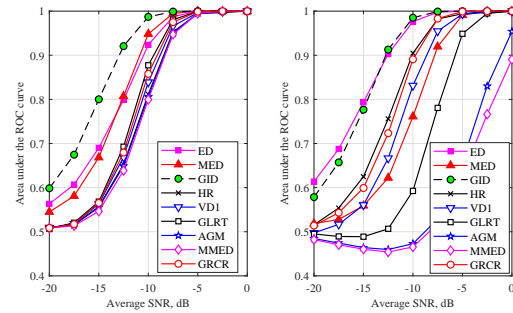


Fig. 5 AUC versus SNR under Unif. (left) and Nunif. (right) conditions, for $s = 5$ QPSK PU signals, $m = 10$ SUs, $n = 50$ samples, and $K = 6$.

unveils comparable or superior performances for any SNR or number of samples. It is slightly outperformed by the ED at SNRs below -15 dB.

Computational Complexity: The GID has roughly the same complexity of the GRCC, which is $\mathcal{O}(nm^2)$ [4], owed mainly by the computation of the SCM. The ED, which is the simplest detector, has complexity $\mathcal{O}(nm)$; the complexities of the GLRT, the MMED, the MED, the AGM, the HR and the VD1 are around $\mathcal{O}(nm^2) + \mathcal{O}(m^3)$ [4]. Thus, along with the GRCC, the GID is the second less complex robust detector available, to the best of the author's knowledge.

Conclusion: This letter proposed the Gini index detector for cooperative spectrum sensing. It was demonstrated that the detector is robust against nonuniform-dynamical noise and signal powers, is better suited to line-of-sight channels having mild dominant signal component (although the need for dominance is relaxed if the number of primary transmitters increases), can outperform most of the state-of-the-art robust detectors under a variety of system parameters, is blind, has the constant false alarm rate property, and is one of the simplest detectors available so far. It is worth mentioning that very similar conclusions were obtained when considering that the primary signals are white noise. The corresponding results were omitted for conciseness.

Acknowledgment: This work was supported by Finep, with Funntel resources, through the Radiocommunications Reference Center (CRR) project of Inatel, under Grant 01.14.0231.00, Brazil.

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