Abstract—Recently, radar frequency bands have been drawing much attention as possible candidates for cognitive radio, since their utilization was revealed to be relatively low. One important functionality of a cognitive radio system is the spectrum sensing, which detects the presence of a primary user (PU) signal. The pulse radar signal, which is the PU signal in radar bands, is characterized by its sparsity in the time domain. Exploiting this sparsity, we present a spectrum sensing technique which combines the intermediate detections from multiple small sliding sensing windows of the received radar signal into a final decision. We also show by computer simulation that it can achieve improvement of the sensing performance in comparison with the conventional single-window based spectrum sensing approach.

Index Terms—Cognitive radio, cooperative spectrum sensing, radar, GRCR.

I. INTRODUCTION

The demand for radio spectrum is explosively growing as a consequence of the emergence of numerous wireless communication systems and services, each one aiming at serving a massive number of users. However, the spectrum scarcity combined with the traditional static spectrum allocation policy places an obstacle to the increasing demand in the near future. In order to address this obstacle, a lot of research efforts have been spent. Among them, the cognitive radio (CR) concept [1] is being considered one of the most promising ones. In short, it is a spectrum sharing technique that allows for a secondary user (SU) to access a frequency band whenever it is not occupied by the primary user (PU), leading to higher spectrum utilization [2], [3].

Recently, radar bands have been attracting much attention as possible candidates for cognitive radio, since they were found to be underutilized and wideband [4]. For instance, the Federal Communications Commission (FCC) has regulated dedicated and nonexclusive band pairs for different types of radar devices. Furthermore, the spectrum utilization [2], [3].

A. Signal Model

Adapting the model of [11] to the radar signal detection problem, consider a cooperative spectrum sensing scheme in which $m$ cognitive SUs collect $mn$ samples (n samples per SU) of the signal received from a radar transmitter during the sensing interval. At the fusion center (FC) of the secondary network, the received signal matrix $Y \in \mathbb{C}^{m \times n}$ is given by

$$Y = hx^T + V,$$  \hspace{1cm} (1)

where the superscript $T$ denotes transposition, and $h = [h_1, h_2, \ldots, h_m]^T$ is the channel vector with $h_i$ representing the complex channel gains between the radar transmitter and the $i$-th SU receiver, for $i = 1, \ldots, m$. These gains are assumed...
to be constant during the sensing interval, and independent and identically distributed (i.i.d.) between consecutive sensing rounds. The channel vector is modeled as \( \mathbf{h} = \mathbf{G} \mathbf{a} \), where \( \mathbf{a} \in \mathbb{C}^{m \times 1} \) is the vector whose elements are zero-mean complex Gaussian random variables having unitary second moment to represent flat and slow Rayleigh fading channels. The matrix \( \mathbf{G} \in \mathbb{R}^{m \times m} \) is a diagonal gain matrix given by \( \mathbf{G} = \text{diag}(\sqrt{p_1/p_{\text{avg}}} \cdot \ldots \cdot \sqrt{p_m/p_{\text{avg}}}) \), with \( p = [p_1, p_2, \ldots, p_m]^T \) being the vector with the received signal power levels in each SU, where \( p_{\text{avg}} = \frac{1}{m} \sum_{i=1}^{m} p_i \) is the average received signal power over all SUs. If the overall channel power gain is unitary (without loss of generality), the radar signal is transmitted with a constant power \( p_{\text{avg}} \). The matrix \( \mathbf{V} \in \mathbb{C}^{m \times n} \) contains independent zero mean complex Gaussian noise samples. To consider the possibility of nonuniform noise variances across the SUs' receivers, the elements in the \( i \)-th row of \( \mathbf{V} \) have variance \( \sigma_i^2 \), \( i = 1, \ldots, m \). Then, denoting the average noise variance as \( \sigma_{\text{avg}}^2 = \frac{1}{m} \sum_{i=1}^{m} \sigma_i^2 \), the received signal-to-noise ratio, in dB, averaged over all SUs, is \( \text{SNR} = 10 \log_{10} \left( \frac{p_{\text{avg}}}{\sigma_{\text{avg}}^2} \right) \).

Finally, the vector \( \mathbf{x} \in \mathbb{R}^{m \times 1} \) in (1) represents the samples of the radar signal. In practice, a pulse radar signal is formed by a series of short time pulses having duration of about 1-5 microseconds [5]. The time between the start of consecutive pulses is typically on the order of 1 millisecond, yielding a very low duty-cycle waveform. From the perspective of the detector, the received signal from a rotating radar is seen as a series of bursts of pulses. The time from the start of one burst to the start of the next one is typically on the order of 1-10 seconds. That said, in base-band representation the radar signal vector model as seen by the SUs’ receivers looks like \( \mathbf{x} = [\ldots, v, v, \ldots, v, 0, 0, \ldots, 0, v, v, \ldots, v, 0, 0, \ldots, 0, \ldots]^T \), where \( v \) is set according to the desired radar signal power \( p_{\text{avg}} \) and duty-cycle \( D \), i.e., \( v = \sqrt{p_{\text{avg}}/D} \). The number of consecutive \( v \)'s (i.e., the pulse duration expressed in terms of a number of samples per pulse) and consecutive \( 0 \)'s (i.e., the pulse spacing, also expressed in samples) are set to yield the desired duty-cycle. The number of pulses within the sensing interval is assumed to be a uniform random variable \( U \sim [1, N] \) to represent the asynchronous operation between the radar bursts and the spectrum sensing interval, where \( N \) is the number of pulses per burst.

### B. GRCR Detector

At the FC, the sample covariance matrix of the received signal is computed as \( \mathbf{R} = \frac{1}{n} \mathbf{Y} \mathbf{Y}^\dagger \), where \( \dagger \) denotes the complex conjugate and transpose. The Gerschgorin radii and centers ratio (GRCR) test statistic defined in [11] is

\[
T_{\text{GRCR}} = \frac{\sum_{j=1}^{m} \sum_{i=1}^{m} |r_{ij}|}{\sum_{i=1}^{m} r_{ii}}, \tag{2}
\]

where \( r_{ij} \) is the element in the \( i \)-th row and \( j \)-th column of \( \mathbf{R} \), for \( i, j = 1, \ldots, m \).

The performance metrics often used to assess the spectrum sensing performance are the probability of detection and the probability of false alarm, respectively defined as \( \Pr\{\text{decision} = \mathcal{H}_1 | \mathcal{H}_1 \} \) and \( \Pr\{\text{decision} = \mathcal{H}_1 | \mathcal{H}_0 \} \), where \( \mathcal{H}_1 \) and \( \mathcal{H}_0 \) are the hypotheses of the presence (i.e. \( \mathbf{Y} = \mathbf{h} \mathbf{x}^\dagger + \mathbf{V} \)) and absence (i.e. \( \mathbf{Y} = \mathbf{V} \)) of the radar signal, respectively, and \( \Pr\{\cdot\} \) is the probability of the underlying event. Notice that the absence of the radar signal might be declared if no radar signal is being transmitted or if the sensing interval falls in between radar signal bursts.

### III. PROPOSED SLIDING GRCR APPROACH

The motivation behind the Sliding GRCR approach goes as follows: for the successful detection of a pulse radar signal, it could be better to adopt a divide-and-conquer strategy in which the whole sensing interval (single sensing round) is split into multiple smaller sensing windows (multiple sensing rounds), since the signal has a small duty-cycle. By doing so, the large pulse energy can be captured in one of the sensing rounds, affected by a noise energy smaller than the one present in the whole sensing interval. However, a simple partition of the sensing interval into adjacent windows may be ineffective, since a pulse might not entirely fall into a given window due to its random position, reducing the detection capability. A more effective approach is to slide a sensing window through the whole sensing interval as an attempt to hit the radar pulse with higher probability. An intermediate decision on the presence or absence of the radar signal is then made for each step of the sliding window. When the window reaches the end of the sensing interval, a logic operation is made among the intermediate decisions to yield the final global decision.

The Sliding GRCR configuration is depicted in Figure 1 which illustrates the operation of the proposed Sliding GRCR spectrum sensing with some hypothetical numerical values. In this figure, a single radar burst with \( N \) pulses (3 in this example) during the sensing interval is shown. The beginning of the first pulse is a random variable, since the radar burst and the sensing interval are not synchronous to each other. Each pulse lasts an interval corresponding to \( W_p \) samples (20 in this example). The sliding window has a size of \( S_w \) samples and moves through the sensing interval (600 samples in this example) in steps of size \( S_s \) samples.

The number of samples collected by each SU, \( n \), the number of radar pulses per burst, \( N \), the radar pulse width, \( W_p \), and the radar signal duty-cycle, \( D \), are related via

\[
n = \frac{W_p N}{D}. \tag{3}
\]

Depending on the sliding window size \( S_w \), the number of steps \( S_n \in \mathbb{N}_+ \), which is the total number of sensing rounds during the whole sensing interval, is given by

\[
S_n = \frac{n - S_w^{\text{avg}}}{S_s} + 1. \tag{4}
\]

In each sensing round, the GRCR test statistic (2) is formed from a sample covariance matrix \( \mathbf{R} \) computed from a received signal matrix \( \mathbf{Y} \) similar to (1), but of order \( m \times S_w \) instead of \( m \times n \). The final global decision is reached via the logic OR operation among the intermediate decisions made in all
rounds, which is equivalent to say that the decision will be made in favor of the presence of the radar signal if \(\max\{T_{\text{GRCR}}(1), T_{\text{GRCR}}(2), \ldots, T_{\text{GRCR}}(S_n)\} > \gamma\), where \(\gamma\) is the decision threshold.

The computational complexity of calculating a sample covariance matrix from a received signal matrix \(Y\) of order \(m \times S_w\) is \(O(S_w m^2)\). Then, the complexity of the Sliding GRCR is \(O(S_n S_w m^2)\) due to the repeated sensing rounds. It can be noticed from (4) that \(S_n S_w \geq n\). Since the complexity of the original GRCR is \(O(n m^2)\) \([11]\), the Sliding GRCR will exhibit a complexity greater than or equal to the original GRCR, with the equality holding when the step size is equal to the window size, i.e., \(S_i = S_w\). Notice that the energy consumption due to the existence of multiple sensing windows is related to the computational burden at the FC. The multiplicity of windows does not affect both the energy spent for sensing at each SU, and the energy due to the transmission over the report channel.

The proposed Sliding GRCR approach has its parameters fully configurable to allow for a trade-off between the spectrum sensing performance and overall sensing speed. For instance, if the window size \(S_w\) fits the pulse width \(W_p\), the maximum pulse energy is captured, improving the sensing performance. Moreover, if the step size is \(S_i = 1\), the chance of hitting a radar pulse is maximized at the expense of a large number of sensing rounds and, thus, a large processing time.

Finally, it is worth emphasizing that the average SNR of the configuration according to the system model described in Section II, i.e., \(\text{SNR} = 10 \log_{10} \left( \frac{\sigma_{\text{avg}}^2}{\sigma_{\text{avg}}^2} \right)\) will be higher than the actual SNR due to the random number of radar pulses seen by the SUs during the sensing interval. It can be shown that the actual SNR can be well approximated by

\[
\text{SNR} \approx 10 \log_{10} \left( \frac{(N+1)\sigma_{\text{avg}}^2}{2N\sigma_{\text{avg}}^2} \right). \tag{5}
\]

The reason why (5) is not exact is that, based on the model depicted in Fig. 1, a radar pulse can be partially included in the sensing interval.

### IV. Numerical Results

A typical tool for analyzing \(P_d\) and \(P_{fa}\) simultaneously is the receiver operating characteristic (ROC) curve, which trades \(P_{fa}\) versus \(P_d\) by varying the decision threshold \(\gamma\). The results shown hereafter are given in terms of ROC curves obtained from 20000 Monte Carlo events in which the received signal matrix \(Y\) is generated under the \(H_1\) and \(H_0\) hypotheses.

We assumed \(m = 5\) SUs and a radar signal with duty-cycle \(D = 5\%\), with a maximum of \(N = 4\) pulses per burst during the sensing interval under the \(H_1\) hypothesis. The random interval shown in Fig. 1 was made uniformly distributed in \([0, n - W_p - 1]\) samples, meaning that at least a single entire pulse is present during the whole sensing interval under \(H_1\).

To consider a scenario of practical significance, nonuniform noise variances and nonuniform received powers were assumed. In each sensing, they were independently drawn from a uniform distribution such as \(\sigma_i^2 \sim \mathcal{U}[0.05\sigma_{\text{avg}}^2, 1.95\sigma_{\text{avg}}^2]\) and \(p_i \sim \mathcal{U}[0.05p_{\text{avg}}, 1.95p_{\text{avg}}]\), respectively.

Figure 2 shows the performance of the Sliding GRCR for a radar pulse width \(W_p = 30\) samples, window size \(S_w = 30\) samples, and window step sizes \(S_i = 1\) (\(S_w = 2371\) sensing rounds) and \(S_i = W_p = 30\) (\(S_i = 80\) sensing rounds), under different SNR values, for \(n = 2400\) samples collected by each SU. The corresponding performances of the original GRCR (\(S_w = n, S_i = 1\)) are also shown. From this figure it can be readily noticed that the Sliding GRCR outperforms the original GRCR for any SNR. Notice from (4) that, with \(S_i = S_w\), then \(S_n S_w = n\), meaning that the complexity of the Sliding GRCR is equal to the complexity of the GRCR. In the case of \(S_i = 1\), the largest performance gain over the original GRCR is attained, at the expense of a higher computational complexity.

In Figure 3, the SNR is kept fixed at \(-15\) dB, and the window size \(S_w\) is varied, again for \(m = 5, n = 2400, W_p = 30, D = 5\%, N = 4, S_i = W_p = 30\). This figure illustrates that the performance improves as the window size decreases from \(S_w = n = 2400\) to \(S_w = W_p = 30\). In this case, the performance is traded against the number of sensing rounds \(S_n\), but the computational complexity remains unchanged.

Figure 4 is similar to Fig. 3, but now the pulse width was reduced to \(W_p = 5\), which in practice is equivalent to a reduction in the received signal sampling rate. The other system parameters are \(m = 5, n = 400, D = 5\%, N = 4\),
and SNR = −10 dB. One can see that the reduction of the window size has brought improvements up to $S_w = 5W_p = 25$. However, the sensing performance for the case of $S_w = W_p = 5$ became even worse than the single-round sensing made by the original GRCR. This behavior is due to the poor sample covariance matrix estimation when $S_w$ is small, because the order of $Y$ is $m \times S_w$.

Even when the sampling rate is small, causing the reduction of the number of samples during the pulse existence, it is still possible to recover the monotonic performance improvement of the Sliding GRCR as the window size approaches the pulse width. Figure 5 exemplifies this situation. The difference from Fig. 4 is that the number of SUs has been increased from $m = 5$ to $m = 30$. Then, if the number of samples obtained from a radar pulse is small, increasing the number of SUs in cooperation may reestablish the desired behavior of the sliding spectrum sensing approach.

V. CONCLUSION

Exploiting the temporal sparsity of a pulse radar signal, we presented a novel spectrum sensing technique that combines intermediate detections from multiple small sliding sensing windows into a final global decision. Simulation unveiled that the sliding window approach can provide significant performance improvement over the conventional single-window spectrum sensing. Moreover, it can be combined with any spectrum sensing scheme, and is flexible by allowing full control over the spectrum sensing performance and computational burden. If the SUs are equipped with multiple antennas, the Sliding GRCR can be performed in each SU for local decision, allowing for the implementation of a decision fusion strategy.

REFERENCES


Fig. 3. Performances of the Sliding GRCR and the original GRCR ($S_w = n$, $S_0 = 1$) for detecting a radar signal under different values of the window size $S_w$, for $m = 5$, $n = 2400$, $W_p = 30$, $D = 5\%$, $N = 4$, and SNR = −15 dB.

Fig. 4. Performances of the Sliding GRCR and the original GRCR ($S_w = n$, $S_0 = 1$) for detecting a radar signal under different values of the window size $S_w$, for $m = 5$, $n = 400$, $W_p = 5$, $D = 5\%$, $N = 4$, and SNR = −10 dB.

Fig. 5. Performances of the Sliding GRCR and the original GRCR ($S_w = n$, $S_0 = 1$) for detecting a radar signal under different values of the window size $S_w$, for $m = 30$, $n = 400$, $W_p = 5$, $D = 5\%$, $N = 4$, and SNR = −17 dB.