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A Survey on Spectrum Sensing Techniques for Cognitive Radio

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Abstract—Spectrum Sensing is an important functionality of Cognitive Radio (CR). Accuracy and speed of estimation are the key indicators to select the appropriate spectrum sensing technique. Conventional spectrum estimation techniques which are based on Short Time Fourier Transform (STFT) suffer from familiar problems such as low frequency resolution, high variance of estimated power spectrum and high side lobes/leakages. Methods such as Multi Taper Spectrum Estimation successfully alleviate these infarctions but exact a high price in terms of complexity. On these accounts, it appears that the filter bank spectrum estimation formulated by F. Boroujeny and wavelet based spectrum estimates are the most promising and pragmatic approaches for CR applications. This article surveys and appraises available literature on various spectrum sensing techniques and discusses spectrum sensing as a key element of CR system design.

I. INTRODUCTION

Cognitive Radio (CR) [1] is an intelligent wireless communication system that is cognizant (hence the name) of its environment, learns from it and adapts its transmission features according to statistical variations in the environment to maximize utilization of premium resources such as spectrum while ensuring good QoS. Two main entities are introduced, namely primary and secondary users. Primary users are the owners of the licensed spectrum while the secondary (unlicensed) users transmit and receive signals over the licensed spectra or portions of it when the primary users are inactive. The secondary users should have the ability to gauge the radio environment and intelligently exploit the unused licensed spectrum and relinquish it when primary users are active.

Key to the successful operation of CR systems is to gauge the wireless environments over wide frequency bands and identify spectrum holes and occupied bands. The challenge is in the identification and detection of primary user signals amidst harsh and noisy environments. In this context, speed and accuracy of measurement are the main metrics to determine the suitable spectrum sensing technique for CR. Speed and accuracy are important to answer the questions of which band is active. Due to uncertainty principle, it is not possible to have the best frequency and time resolution at the same time. Complexity of the mechanism is another issue. Since CRs are envisioned to operate on wireless nodes with small size and power, the spectrum sensing implementation should be kept as simple as possible.

In this paper, spectrum sensing as a crucial aspect in CR system is discussed. Various physical layer techniques available in the literature are catalogued and evaluated. The organization of the article is as follows. In Section II, a review of traditional spectrum sensing technique is provided. This includes periodogram estimate and its variants as an example for non parametric estimation. Section III describes spectrum estimation technique proposed for CR. The techniques are based on estimation through detection of pilots, features or energy. The highlight of this section is the introduction of the Filter Bank Spectrum Estimation proposed by F. Boroujeny [2, 3]. Section IV explores the possibility of applying wavelet theory for spectrum sensing. The article rounds up with a summary in Section V.

II. GENERAL REVIEW ON SPECTRUM ESTIMATION

In general, spectrum estimation can be categorized into direct and indirect methods. In classical direct method (usually recognized as frequency domain approach), the power spectrum is estimated directly from signal being estimated. On the other hand, in indirect method, also known as time domain approach, the autocorrelation function of the signal being estimated is calculated. From this autocorrelation value, the power spectrum density can be found by applying the Discrete Fourier Transform.

Another way to categorize spectrum estimation method is by classifying it into parametric or non-parametric methods. Parametric method is basically model based approach [4]. In this method, a signal is modeled by Auto Regressive (AR), Moving Average (MA) or Auto Regressive Moving Average (ARMA) process. Once the signal is modeled, all parameters of the underlying model can be estimated from the observed signal. Estimator based on parametric method generally has capability to distinguish higher degree of detail. The disadvantage of parametric method is that if the signal is not sufficiently and accurately described by the model, the result is less meaningful. Non Parametric methods, on the other hand, do not have any assumption about the shape of the power spectrum and try to find acceptable estimate of the power spectrum without prior knowledge about the underlying stochastic approach.

The most commonly known spectrum sensing technique is periodogram which is classified as a non
parametric estimate [4]. The procedure starts by calculating the Discrete Fourier Transform (DFT) of the random signal being estimated, followed by taking the square of it and then dividing the result with the number of samples N. The main issue in periodogram is the use of rectangular windowing of waveform to obtain finite length samples. This windowing introduces a discontinuity (illustrated in Figure 1) between the original signal and the aliased version produced by a DFT transformation. In the frequency domain, the rectangular window has resulted in a Dirichlet Kernel described by the width of the main lobe and the size of side lobe [4], [5]. The width of the main lobe is related to the frequency resolution of the power spectra, and the size of side lobe is related to the ratio between maximum and minimum spectral power that is distinguishable by the estimator. The rectangular window compromises the frequency resolution, producing leakage and a biased estimate.

Various optimizations can be brought about to improve the periodogram. For example, the power variance in the periodogram estimate can be reduced by averaging (Bartlett Method). The samples are divided into several segments and the periodograms of each segment is averaged [6]. The important thing is to identify a trade off between number of samples per segments and number of segments. In theory, the number of segments should be maximized in order to minimize the variance of estimated power. However, this also means lowering the number of samples in each sub-sequence resulting in larger bias and smaller frequency resolution.

\[
\begin{align*}
\text{STFT} \{x(t)\} &= X(\tau, f) = \int_{-\infty}^{\infty} [x(t)w(t-\tau)] e^{-j2\pi ft} dt \\
&= X(\tau, f) e^{-j2\pi ft} \int_{-\infty}^{\infty} \pi^\tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \ta
coherent detection are the susceptibility of the detection threshold to noise, in-band interference and fading [8].

\[
\begin{align*}
D \text{ qk} & \quad \text{for} \quad f(n) = f_i
\end{align*}
\]

\[
\text{is given as:} \quad \kappa_k(f_i) = D \text{ qk}^H x(n)
\]

\[
\begin{align*}
\text{Based on (3), the MTSE is formulated as:} \\
S_{MTSE}(f_i) = \frac{1}{K} \sum_{k=0}^{K-1} |\kappa_k(f_i)|^2
\end{align*}
\]

Indeed if there is only a vector \(q_0\) containing 1’s as its elements, (4) becomes periodogram with rectangular window. If we manipulate the elements in \(q_k\), (4) becomes the windowed periodogram with window type determined by \(q_k\). Hence, (4) can generally be interpreted as average of several periodograms with different windows. The averaging process in (4) is conducted on the data set in its entirety and in this sense MTSE is different from the Welch approach [7] where the data samples are segmented and averaged. Equation (3) can be regarded as Fourier Transform of convolution between the received samples \(x(n)\) and a filter having \(q_k\) as its impulse response. Therefore, \(\kappa_k(f_i)\) can be viewed as the output of \(k^{th}\) bandpass filter of a group of bandpass filter banks with different filter response.

\[
\begin{align*}
\text{C. Cyclostationary Feature Detection} \\
\text{This method takes advantage of the cyclostationarity of the modulated signal [8]. Generally, the transmitted data is taken to be a stationary random process. However, when it is modulated with sinusoid carriers, cyclic prefixes (as in OFDM) and code or hopping sequences (as in CDMA), a cyclostationarity is induced i.e. the mean, autocorrelation and statistics show periodic behavior. This feature is exploited in a detector (depicted in figure 4) that measures a signal property called Spectral Correlation Function (SCF). When parameter \(\alpha\) in figure 4 (called cycle frequency) is 0, the SCF yields the PSD.}
\end{align*}
\]

\[
\begin{align*}
\text{D. Multi Taper Spectrum Estimation} \\
The periodogram estimate may be viewed as the output of several filters banks with each point in the power spectrum estimate corresponding to a filter’s output. The filter bank is constructed by modulating a single prototype filter. The Multi Taper Spectrum Estimator (MTSE), proposed by Thomson [10], works somewhat similar but it uses multiple orthogonal prototype filters to improve the variance of estimated power and reduce the leakage. The process is initiated by collecting the last \(M\) received samples in a vector \(x(n) = [x(n) x(n-1) … x(n-M+1)]^T\) and representing it as a set of orthogonal slepian base vectors [2], [10]:
\end{align*}
\]

\[
\begin{align*}
x(n) = \sum_{i=0}^{K-1} \kappa_i(f_i) D q_k
\end{align*}
\]

In (2), \(\kappa_i(f_i)\) is the expansion coefficients, \(K\) is the number of frequency points \(f_i\) that have to be estimated, \(q_k\) is the set of orthogonal slepian basis vectors and \(D\) is a diagonal matrix with the diagonal elements of 1, \(e^{j2\pi f_i}\), \(…., e^{j2\pi (M-1)}f_i\). \(\kappa_i(f_i)\) is given as:

\[
\begin{align*}
\kappa_i(f_i) = (D q_k)^H x(n)
\end{align*}
\]

\[
\begin{align*}
\text{Indeed if there is only a vector } q_0 \text{ containing 1’s as its elements, (4) becomes periodogram with rectangular window. If we manipulate the elements in } q_k, \text{ (4) becomes the windowed periodogram with window type determined by } q_k. \text{ Hence, (4) can generally be interpreted as average of several periodograms with different windows. The averaging process in (4) is conducted on the data set in its entirety and in this sense MTSE is different from the Welch approach [7] where the data samples are segmented and averaged. Equation (3) can be regarded as Fourier Transform of convolution between the received samples } x(n) \text{ and a filter having } q_k \text{ as its impulse response. Therefore, } \kappa_k(f_i) \text{ can be viewed as the output of } k^{th} \text{ bandpass filter of a group of bandpass filter banks with different filter response.}
\end{align*}
\]

\[
\begin{align*}
\text{Since every point } f_i \text{ in the power spectrum estimate is related to outputs of a group of bandpass filters in the same band, the filter’s pass band } \Delta f \text{ gives its frequency resolution. Hence, (4) is the estimate of the power over the frequency band } (f_i-\Delta f/2, f_i+\Delta f/2). \text{ For a given resolution } \Delta f, \text{ the prototype low pass filters should have pass band between } (-\Delta f/2, +\Delta f/2) \text{ and minimum energy at stop band to minimize leakage. The variance of the estimate is reduced by taking advantage of the presence of multiple prototype (prolate) filters having impulse responses derived from the vectors } q_k. \text{ For given frequency band, the output of each band pass filter corresponding to different prototype filter is collected and averaged. The output of each band pass filter should be independent from each other to effectively reduce the variance of estimated power. This is achieved from the orthogonality of the Slepian sequences. Minimax theorem is used to derive the Slepian sequences [2]. Firstly, the autocorrelation matrix } R \text{ of the observation vector } x(n) \text{ is computed. The set of eigenvalues } \lambda_0 > \lambda_1 > ... > \lambda_{M-1} \text{ of correlation matrix } R \text{ and the corresponding eigenvectors } q_0, q_1, ..., q_{M-1}, \text{ are obtained through the following optimizations [2]}:
\end{align*}
\]

\[
\begin{align*}
\lambda_{\max} = \lambda_0 = \max \left\{ \left[ q_0^T x(n) \right]^2 \right\} \\
\lambda_k = \max \left\{ \left[ q_i^T x(n) \right]^2 \right\} \text{ for } i = 1, 2, 3, ..., M-1
\end{align*}
\]

\[
\begin{align*}
\lambda_{\max} = \max \left\{ \left[ q_0^T x(n) \right]^2 \right\} \text{ for } 0 \leq k < i \text{ in (5) and (6) is the Euclidean norm of vector } q_k. \text{ The last step is basically to choose the } K \text{ eigenvectors out of } M \text{ eigenvectors of the correlation matrix } R. \text{ These } K \text{ eigenvectors corresponds to the largest } K \text{ eigenvalues.}
\end{align*}
\]

\[
\begin{align*}
\text{While } K \text{ prototype filters having minimum energy in stop band are expected, not all of the prototype filters fulfill the expectation. The filter having } q_k \text{ as its impulse response tends to have minimum energy in stop band [2],}
\end{align*}
\]
However, the filter having \( q_1 \) as its impulse response does not have a stop band attenuation as good as that of \( q_0 \). The reason for this can be explained as follows. A filter with impulse response \( q_0 \) has the best stop band attenuation since it is chosen to maximize the corresponding eigenvalue (as expressed in equation (5)) without any constraint. On the other hand, a filter with impulse response \( q_1 \) is chosen to maximize the corresponding eigenvalue in (5) but with additional constraint \( q_1^T q_0 = 0 \) mentioned in (6). The performance of the next derived filter has more deterioration. With regard to the need for having minimum leakage, a careful treatment is needed when the outputs of each filter corresponding to different prototype filters are averaged. Obviously, they should not have the same weight. The output of filter having better stop band attenuation should be given more weight. Thomson offers an iterative algorithm to compute the estimate of power spectrum [10].

### E. Filter Bank Spectrum Sensing

Farhang-Boroujeny in [2] proposed filter bank spectrum estimation (FBSE) for CR by using a pair of matched root Nyquist-filter. The proposal is based on the assumption that multicarrier modulation is used as the underlying communication technique [2] [3]. In [2], every point in power or frequency spectrum is considered as the output of single filter (or multiple filters operating at the same band). Hence, the frequency spectrum is considered as the output of multiple filters (operating at different bands). In MTSE, every point in the spectrum is obtained by averaging the output of different prototype filters at the same frequency band. In the FBSE method, MTSE is simplified by introducing only one prototype filter for each band. The idea is to assume that the filters at the receiver and transmitter side are a pair of matched root-Nyquist filters \( H(z) \) in Fig. 5 [11]. The received signal is demodulated by \( i^\text{th} \) subcarrier and then forwarded to the root-Nyquist filter as illustrated in Fig. 5.

![Figure 5. The demodulation of received signal with respect to \( i^\text{th} \) subcarrier before it is processed through root-Nyquist filter [2].](image)

Considering Figure 5, it can be shown that:

\[
S_{y, y} (f) = S_x (f + f_i) |H(e^{j2\pi f_i})|^2
\]

(7)

where \( S_{y, y} (f) \) is power spectrum density of \( y(n) \). Assuming that \( H(z) \) is narrowband, \( S_x (f + f_i) \) can be approximated as \( S_x (f_i) \). Based on this approximation, we can try to write (7) in \( z \) domain as:

\[
\Phi_{y, y} (z) = S_x (f_i) H(z) H(z^{-1})
\]

(8)

\( S_x (f_i) \) in (8) is constant. It can be noted that the correlation coefficients of \( y(n), \phi_{y, y} (k) \), can be obtained from the inverse \( Z \)-transform \( \Phi_{y, y} (z) \). According to [2], \( H(z) \) is designed as root-Nyquist \((N)\) filter \((N \text{ gives number of zero-crossings of the filter in an of interval of } N \text{ samples})\). Consequently, \( G(z) = H(z)H(z^{-1}) \) is Nyquist \((N)\) filter. It is required that \( G(z) \) satisfies [2]:

\[
g(n) = \begin{cases} 
1, & \text{if } n = 0 \\
0, & \text{if } n = mN 
\end{cases}
\]

(9)

As a result, the correlation \( \phi_{y, y} (k) \) bears a resemblance to Nyquist \((N)\) sequence \( g(n) \), where the subscript \( N \) indicates the number of zero-crossing of the autocorrelation function. For an observation vector \( y(n) = [y(n), y(n-L), \ldots, y(n-(K-1)L)] \) of size \( L \) the correlation matrix is given as [2]:

\[
R_{y, y} = S_x (f_i) A
\]

(10)

with \( A \) given as:

\[
A = \begin{bmatrix} 
g_s(0) & g_s(L) & \ldots & g_s(K-1)L 
g_s(-L) & g_s(0) & \ldots & g_s(K-2)L 
\vdots & \vdots & \ddots & \vdots 
g_s(-(K-1)L) & g_s(-(K-2)L) & \ldots & g_s(0) 
\end{bmatrix}
\]

(11)

An eigenvalue decomposition is then performed on matrix \( A \). These resultant eigenvalues are used to measure the degree of freedom which can later be used to adjust the variance of the estimates.

A comparative analysis of MTSE and FBSE can be summarized as follows. Firstly, the magnitude response of the root-Nyquist filter proposed by [2] is comparable to the magnitude response of the best prototype filters used in MTSE. Recall that MTSE uses multiple prototype filters for a given frequency band and only the one derived with respect to largest eigenvalue of the autocorrelation matrix of the observation vectors gives the best response. As a result, FBSE outperforms MTSE (in terms of variance) over frequency band with low power spectrum density. Due to the better magnitude response of its prototype filter, FBSE introduces lower leakages compared to MTSE. Secondly, FBSE is usually better when the number of samples is large while MTSE is superior for estimations with a small sample space. With respect to speed issue, MTSE is faster due to smaller window size for a given certain sampling rate. However, the use of iterative process in MTSE increases the complexity making it non-viable for CR.

### IV. WAVELET BASED EDGE DETECTION

Wavelets and wavelet transform are latest additions to the rich arsenal of communication system tool box [12], [13]. They possess excellent time-frequency localization properties that can serve as powerful mathematical tools to analyze local spectral structure to identify singularities and edges.

In [14] Z. Tian and G.B. Giannakis propose a wavelet based wideband spectrum sensing approach for dynamic
spectrum management. In their approach, the signal spectrum over a wide frequency band is decomposed into elementary building blocks of non-overlapping sub-bands that are well characterized by local irregularities in frequency. Then the entire wideband is modeled as a sequence of consecutive frequency sub-bands, where the power spectral characteristic is smooth within each sub-band but exhibit a discontinuous change between adjacent sub-bands. Information on the locations and intensities of spectrum holes and occupied bands is derived by considering the irregularities in PSD. The main attraction for wavelets in this application is their ability to analyze singularities and irregular structures which can be used to characterize the local regularity and edges of signals. Hence, the method is also called Edge detection.

Assuming a wide band of interest in the frequency range \([f_0, f_s]\) with bandwidth \(B = f_s - f_0\) with \(N\) spectrum bands. The task is to find the occupied and unoccupied bands as well as the frequency boundaries of each band. Hence, we must find \(f_0 < f_1 < f_2 < \ldots < f_{N-1} < f_N\) in which the \(n^{th}\) band is defined as \(B_n = [f \in B_n : f_{n-1} \leq f < f_n], \ n = 1, 2, \ldots, N\) [14]. The last task would be the spectral density estimation for each band. Tian & Giannakis [14] assume that the power spectrum density (PSD) within each band is smooth and flat but exhibit discontinuities and irregularities with the adjacent bands. From the received signals in the wide band of interest \([f_0, f_s]\), the wavelet is used to locate the boundaries of each band indicated by the discontinuities. A dilated (by a factor \(s\)) version of the wavelet function \(\phi(f)\) can be given as [14]:

\[
\phi_s(f) = \frac{1}{s} \phi\left(\frac{f}{s}\right)
\]  

(12)

Then the Continuous Wavelet Transform (CWT) of the PSD can be defined as [12]:

\[
\text{CWT} \{S_r(f)\} = S_r * \phi(f)
\]  

(13)

Since \(S_r(f)\) is the Fourier Transform of the autocorrelation function \(R_r(\tau)\) and by defining \(\Phi_r(\tau) = \text{IFT}\{\phi(f)\} = \Phi(\tau), \ \text{where IFT}\{\}\) denotes Inverse Fourier Transform, (13) is represented as [14]:

\[
\text{CWT} \{S_r(f)\} = \text{FT}\{R_r(\tau)\Phi(\tau)\}
\]  

(14)

FT\{\} in (9) denotes Fourier Transform. The CWT of PSD in (14) is used to locate the irregularities and discontinuities in the wide band of interest especially by investigating the shape of the first and second derivative of (14), described, respectively, as follows [14]:

\[
\text{CWT} \{S_r(f)\} = -s^{\text{FT}}\{R_r(\tau)\Phi(\tau)\}
\]  

(15)

\[
\text{CWT}'' \{S_r(f)\} = s^2 \text{FT}\{R_r(\tau)\Phi(\tau)\}
\]  

(16)

In (15) and (16), \(\text{CWT} \{S_r(f)\}\) and \(\text{CWT}'' \{S_r(f)\}\) describe the first and second order derivatives of \(S_r(f)\) smoothed by the wavelet \(\phi(f)\).

The local maxima of first derivative \(\text{CWT} \{S_r(f)\}\) can be used to indicate the irregularities of the PSD [14]. Therefore, with regard to the assumption that the PSD is smooth within each band, the boundaries of each band are located by the location of the local maxima of the first derivative \(\text{CWT} \{S_r(f)\}\). The same goal can also be achieved by tracking the location of zero crossing of the second derivative \(\text{CWT}'' \{S_r(f)\}\). Both procedures give location of \(f_0, f_1, \ldots, f_{N-1}, f_N\).

The problem with this approach is the possibility of noise induced local maxima in the shape of first order derivative \(\text{CWT} \{S_r(f)\}\). This can be solved by varying the scale variable \(s\). The actual boundaries of each band are described by the local maxima that always presents in \(\text{CWT} \{S_r(f)\}\) for any scales. Tian & Giannakis research has shown that the wavelet approach has successfully identified the number of bands that is occupied within wide band of interest [14]. This method also offers a good dynamic spectrum range. However, issues of the speed of the method remain unexplored.

Another wavelet approach for spectrum sensing is offered by Hur, et al in [15], which basically combines coarse and fine sensing resulting in Multi Resolution Spectrum Sensing. The received signal is correlated with the modulated wavelet to obtain the spectral contents of the input signal at the band around the carrier frequency modulated by the wavelet. The resolution is adjusted by either using wavelet with large or small resolution bandwidth. The coarse sensing is used to examine a wideband spectrum in fast manner and to produce information about candidate unoccupied spectrum segments. Fine sensing is used to further investigate the candidate spectrum segments [15]. In addition to the wavelet based techniques introduced above, there is a great scope for utilizing wavelet theory for spectrum sensing. Wavelet has the requisite properties to dynamically tune the time and frequency resolution by playing with dilated versions of the wavelet and scaling functions. Best time resolution is preferred to locate discontinuities in time-domain signal. On the other hand, time resolution can be compromised and traded-off with high frequency resolution for segments of signal that tend to remain stationary for long time durations (slow varying) [13].

V. CONCLUSIONS

In this paper, spectrum sensing as a crucial aspect in CR system was discussed. Various physical layer techniques available in the literature were catalogued and evaluated. With regard to selecting the right spectrum sensing technique for CR, speed and accuracy of estimation are the two main metrics. Due to the uncertainty principle, it is not possible to simultaneously have the best frequency and time resolutions. This also means that there is a trade-off between the speed (time resolution) and accuracy (frequency resolution) of estimation achievable. A good time resolution is necessary to locate discontinuities in time-domain signal. However, the time resolution can be compromised for segments of the signal which are stationary for long periods of time or slow varying and traded off to get a high frequency resolution. Traditional techniques such as periodogram or STFT based estimators cannot be tuned to vary the time-frequency resolutions according to the demands of the radio environment. Furthermore, these
estimators suffer from drawbacks such as leakage and large variance in power spectrum estimates. Other techniques such as MTSE and FBSE successfully overcome these infarctions but they too lack the keys to adjust and optimally tailor the time-frequency resolution window.

It is here that the theory of wavelets stands out for spectrum estimation applications. Wavelet has properties required to dynamically tune the time and frequency resolution by playing with dilated versions of the wavelet and scaling functions. This facility of the wavelets has been demonstrated by Tian and Giannakis in [14] by convolving the wavelet with the power spectrum density of the received signal (or smoothing the power spectrum density with the wavelet). By taking advantage of the first and second order derivative of this convolution, the location of the frequency boundaries of each band within the wide band of interest is found.

To conclude, the best spectrum sensing approach for Cognitive Radio would be the ones which offer a trade off between time-frequency resolutions with minimum complexity. Existing state-of-art technologies unfortunately do not offer this possibility. Future research should focus on finding means to identify the optimal spectrum sensing technique with flexible tuning capability in terms of time and frequency resolution. On these accounts, it appears that the wavelet based spectrum estimation may hold the answers.

REFERENCES
